Symmetry-preserving discretizations in (unstructured) staggered meshes

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Overview

1. Motivation
2. Laplacian
3. Convection
4. Discussion
5. Conclusions
Context: DNS of Turbulence

- Expensive
- Reliable
- Effective
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Context: The physics

The incompressible Navier-Stokes.

\[ \nabla \cdot \vec{u} = 0 \]

\[ \frac{\partial \vec{u}}{\partial t} + C(\vec{u}, \vec{u}) = -\nabla p + \nu \nabla^2 \vec{u} \]

\[ \Omega \frac{d\vec{u}_f}{dt} + C(\vec{u}_f)\vec{u}_f = -\Omega G\rho_c + L\vec{u}_f \]

Mathematical → physical properties

\[ \int_\Omega abd\Omega = (a, b) \]

\[ (C(\vec{u}, \phi), \rho) = - (\phi, C(\vec{u}, \rho)) \]

\[ (\nabla \cdot \vec{u}, \rho) = (\vec{u}, \nabla \rho) \]

\[ (\nu \nabla^2 \vec{u}, \vec{u}) \leq 0 \]

\[ (\nabla \cdot \vec{u}, p) \leq 0 \]

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\[ D = -\Omega G^T \]

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**Context: existing codes**

Collocated arrangement is preferred by most popular codes.

Collocated

- simple
- \( D \neq -\Omega G^T \)

Staggered

- \( D = -\Omega G^T \)
- complex

**Idea**

Can we reuse collocated codes to construct staggered formulations?

\[
Du_f = 0_c \\
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Operators

Collocated operators defined over and arbitrary unstructured mesh

- Dual gradient $\tilde{\nabla}$
- Curl $\tilde{\mathbf{R}}$
- Divergence $\tilde{\mathbf{D}}$
- Laplacian $\nabla^2$
- Convection $\mathbf{V}_c$

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Collocated operators defined over and arbitrary unstructured mesh
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Rotational formulation: \( \nabla^2 \vec{u} = \nabla \times \nabla \times \vec{u} - \nabla \nabla \cdot \vec{u} \)

\[ L = R\tilde{R} - \tilde{G}D \]
Motivation  Laplacian  Convection  Discussion  Conclusions

Laplacian L

Rotational formulation: \( \nabla^2 \vec{u} = \nabla \times \nabla \times \vec{u} - \nabla \nabla \cdot \vec{u} \)

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Laplacian L

Application to Cartesian grids.

Recovers Harlow and Welch ✓
Laplacian L

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Convection $C(u_f)$

Baseline: Harlow and Welch $\nabla \cdot (\vec{u} \otimes \vec{u})$

Research question: How to define in non-Cartesian meshes?
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**Research question:** How to define in non-Cartesian meshes?
Previous attempts

Rotational formulation \( \nabla \cdot (\ddot{\vec{u}} \otimes \ddot{\vec{u}}) = \ddot{\vec{u}} \times \nabla \times \ddot{\vec{u}} + \frac{1}{2} \nabla (\ddot{\vec{u}} \cdot \ddot{\vec{u}}) \)

Recovers Harlow and Welch \( \times \)

Chain rule does not hold at the discrete level.
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Interpolated formulation $C_s^0(u) = \Gamma_{c\rightarrow s} C_c(u) \Gamma_{s\rightarrow c}$

Recovers Harlow and Welch ×
Larger stencil.
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CF180 - Interpolated

Channel flow at \( Re_\tau = 180 \). Cartesian mesh.
Our attempt

Recovers Harlow and Welch ✓
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Our attempt

Construct the explicit staggered control volume.

\[
\begin{align*}
\vec{F}_c^i &= \bar{u} \, SP_{f \to c}^i \, u_f \\
(\Delta x)^{-1} \hat{n}_f \cdot \sum_{c \in f} \pm \vec{F}_c^i \\
\vec{F}_e^i &= \bar{u}_e \, SP_{f \to e}^i \, u_f \\
(\Delta x S_f)^{-1} \sum_{e \in f} \pm \vec{F}_{ei} \cdot \left( \hat{n}_f \times \vec{f}_e \right) \, L_e \Delta x
\end{align*}
\]

Project over the face normal
Our attempt

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Project over the face normal
Channel flow at $Re_\tau = 180$. Cartesian mesh.
Dispersion relation

\[ C_s = \Gamma_c \rightarrow s \quad C_c \rightarrow c \]

\[ \text{Modified wavenumber} \]

\[ \text{Wavenumber} \]

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Conclusions

- Construction of $L$ in unstructured meshes.
- Construction of $C(u_f)$ is not trivial, but possible.
- Interpolation schemes may not preserve spectral properties.

Future work

- Implement $L$ and $C(u_f)$ in unstructured meshes.
- Assess its performance in canonical flow configurations.
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Closure

Conclusions

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Thank you for your attention.