# ON A PROPER TENSOR-DIFFUSIVITY MODEL FOR LARGE-EDDY SIMULATIONS OF RAYLEIGH-BÉNARD CONVECTION

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In this work, we plan to shed light on the following research question: *can we find a nonlinear tensorial subgrid-scale (SGS) heat flux model with good physical and numerical properties, such that we can obtain satisfactory predictions for buoyancy driven turbulent flows?* This is motivated by our findings showing that the classical (linear) eddy-diffusivity assumption, $\boldsymbol{q}^{eddy} \propto \nabla\overline{T}$, fails to provide a reasonable approximation for the SGS heat flux, $\boldsymbol{q} = \overline{\boldsymbol{u}T} - \overline{\boldsymbol{u}}\overline{T}$ (see Figure 1, left). This was shown in our work [1] where SGS features were studied *a priori* for a Rayleigh-Bénard convection (RBC) (see Figure 1, middle). We also concluded that nonlinear (or tensorial) models can give good approximations of the actual SGS heat flux. The nonlinear Leonard model, $\boldsymbol{q}^{nl} \propto \nabla\overline{\boldsymbol{u}}\nabla\overline{T}$, is an example thereof. However, this model is unstable and therefore it cannot be used as standalone SGS heat flux model. Apart from being numerically stable we also want to have the proper cubic near-wall behavior. In this regard, the model proposed by Daly and Harlow [2]

$$\boldsymbol{q} \approx -\mathcal{T}_{SGS}\tau\nabla T = -\frac{1}{|\mathsf{S}|}\frac{\delta^2}{12}\mathsf{G}\mathsf{G}^T\nabla T \qquad (\equiv \boldsymbol{q}^{DH}), \tag{1}$$

is stable since it relies on the positive semi-definite tensor $\mathsf{G}\mathsf{G}^T$, where $\mathsf{G}$ is the gradient of the resolved velocity, $\mathsf{G} \equiv \nabla\overline{\boldsymbol{u}}$. However, the DH model does not have the proper near-wall behaviour, *i.e.* $\boldsymbol{q} \propto \langle v'T' \rangle = \mathcal{O}(y^3)$ where $y$ is the distance to the wall. An analysis of the DH model leads to $\mathsf{G}\mathsf{G}^T\nabla\overline{T} \propto \mathcal{O}(y^1)$. Therefore, the near-wall cubic behaviour is recovered if $\mathcal{T}_{SGS} \propto \mathcal{O}(y^2)$. This is not the case of the timescale used in the Daly and Harlow [2] model, *i.e.* $\mathcal{T}_{SGS} = 1/|\mathsf{S}| = \mathcal{O}(y^0)$. At this point it is interesting to observe that new timescales can be derived by imposing restrictions on the differential operators they are based on. For instance, let us consider models that are based on the invariants of the tensor $\mathsf{G}\mathsf{G}^T$

$$\boldsymbol{q} \approx -C_M \left(P^p_{\mathsf{G}\mathsf{G}^T} Q^q_{\mathsf{G}\mathsf{G}^T} R^r_{\mathsf{G}\mathsf{G}^T}\right)\frac{\delta^2}{12}\mathsf{G}\mathsf{G}^T\nabla T \qquad (\equiv \boldsymbol{q}^{S2}), \tag{2}$$

where $P_{\mathsf{G}\mathsf{G}^T}$, $Q_{\mathsf{G}\mathsf{G}^T}$ and $R_{\mathsf{G}\mathsf{G}^T}$ are the first, second and third invariant of the $\mathsf{G}\mathsf{G}^T$ tensor. Then, the exponents $p$, $q$ and $r$ in Eq.(2), must satisfy $-6r-4q-2p = 1$ (units of time) and $6r+2q = s$ ($s$ is the asymptotic near-wall behavior, *i.e.* $\mathcal{O}(y^s)$). It we restrict ourselves to solutions with the proper near-wall scaling, *i.e.* $s = 2$ (blue lines in Figure 1, right), a family of $p$-dependent models follows. Corrections in this regard will be presented together with *a priori/posteriori* studies of nonlinear SGS heat flux models for RBC. Results from LES simulations will be compared with the DNS results at $Ra = 10^{11}$ recently obtained in the PRACE project *"Exploring new frontiers in Rayleigh-Bénard convection"*.

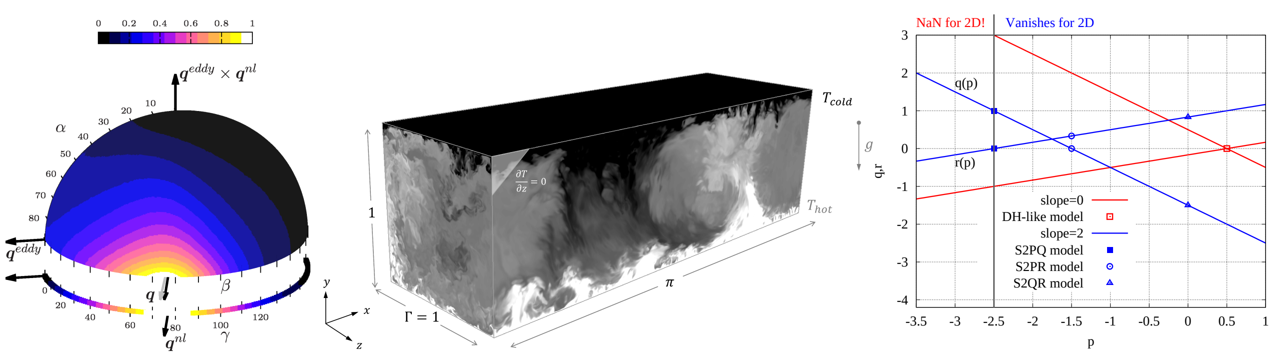


**Figure 1.** Left: alignment trends of the actual SGS heat flux, $\boldsymbol{q}$ (see Ref. [1] for details). Middle: schema of the air-filled RBC studied in Ref. [1] together with an instantaneous temperature field of the DNS at $Ra = 10^{10}$. Right: space of tensor-diffusivity models with the form of Eq.(2). Solutions for slope $s = 0$ (likewise the DH model [2] in Eq. 1) and $s = 2$ (correct slope) are shown.

## References

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[2] B. J. Daly and F. H. Harlow. Transport equations in turbulence. *Physics of Fluids*, **13**:2634, 1970.