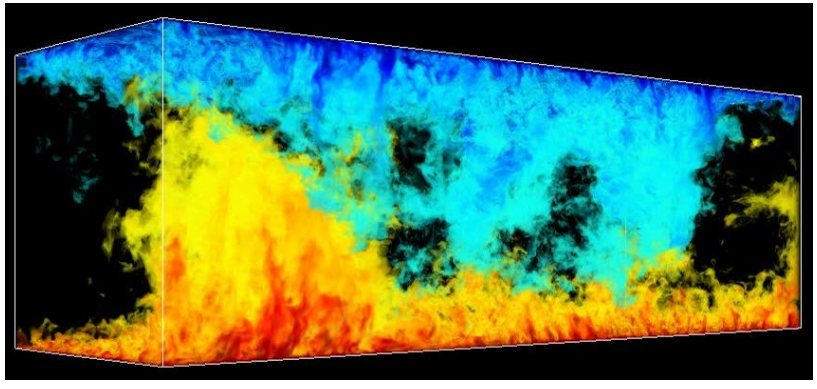
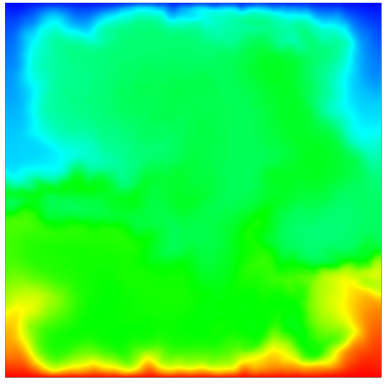
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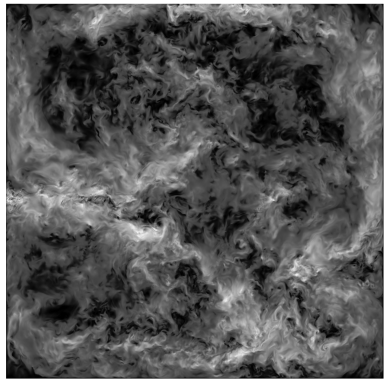
On the effect of Prandtl number to SGS heat flux models

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Andrey Gorobets[2], Assensi Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia
[2]Who cares?







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On the effect of Prandtl number to SGS heat flux models

Is it possible to hit the ultimate regime of turbulence with LES?

F.Xavier Trias[1], Daniel Santos[1], Jannes Hopman[1]
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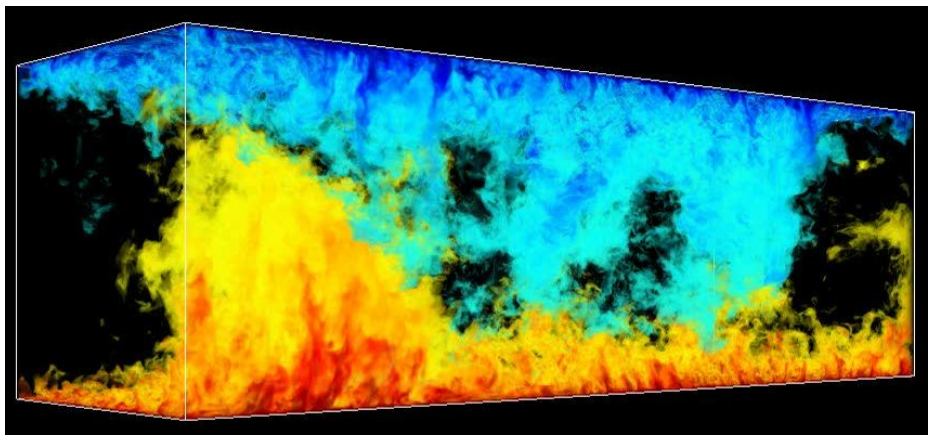
Contents

1. Motivation
2. Modeling the subgrid heat flux
3. Can we reach the ultimate regime?
4. LES results at low-Pr
5. Conclusions

Motivation

Research question #1:

- Can we find a nonlinear SGS heat flux model with **good physical** and **numerical properties**, such that we can obtain satisfactory predictions for a turbulent Rayleigh-Bénard convection?



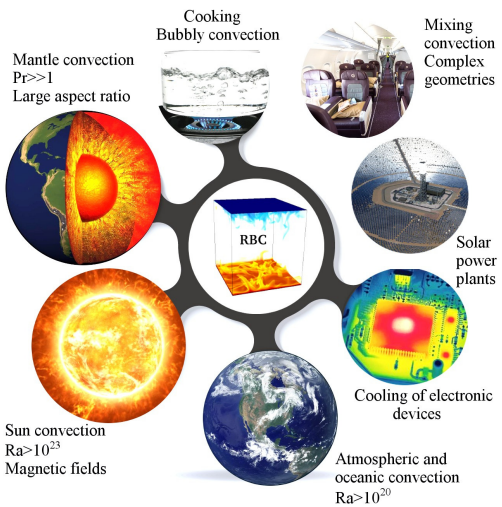
DNS of an air-filled Rayleigh-Bénard convection at $Ra = 10^{10}$

[1] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *On the evolution of flow topology in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 28:115105, 2016.

Motivation

Research question #2:

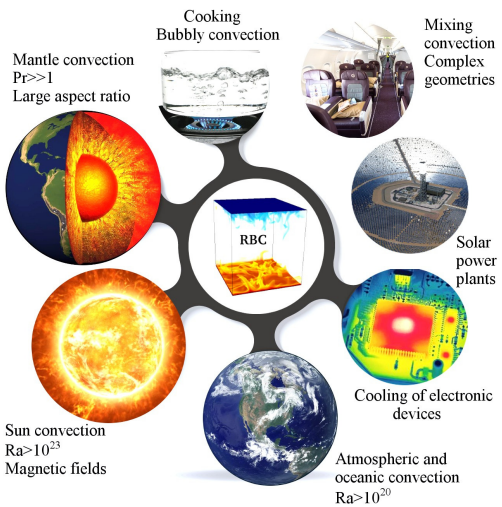
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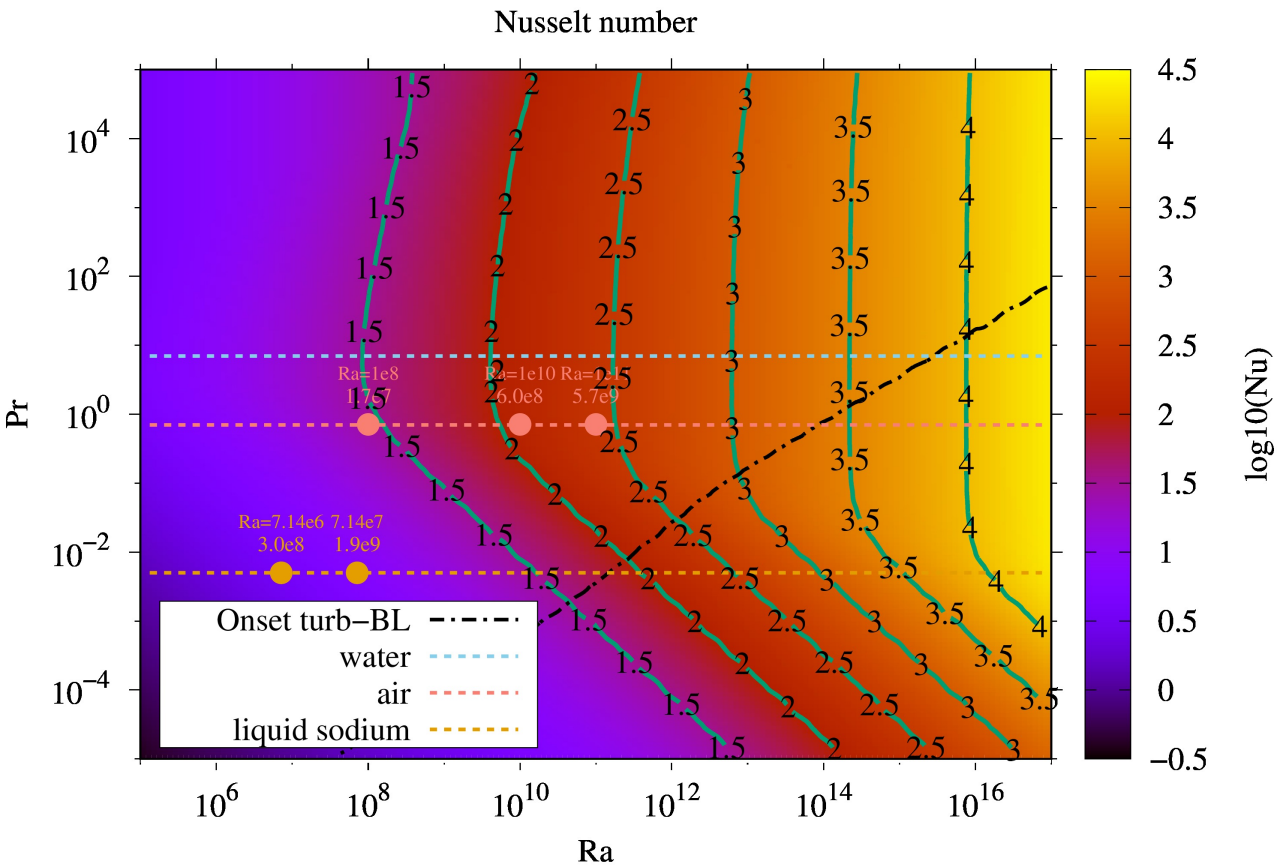


Motivation

Research question #2:

- Is it possible to hit the ultimate regime of turbulence with LES?

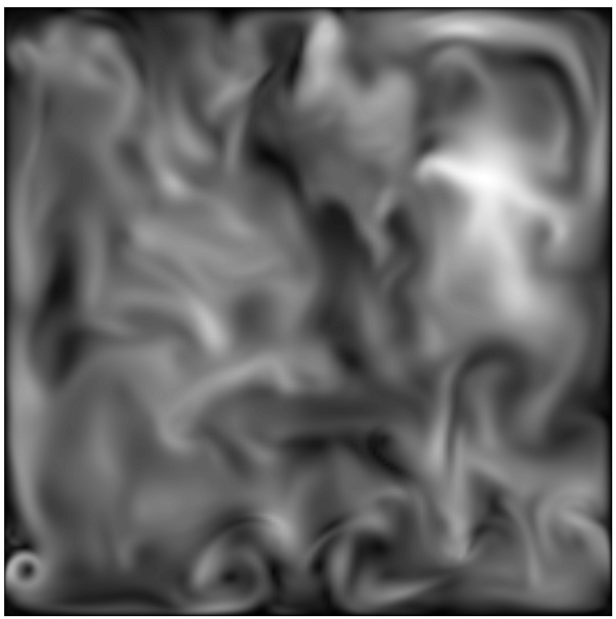




Motivation

Air-filled RB: $Pr = 0.7$

$Ra = 10^8$

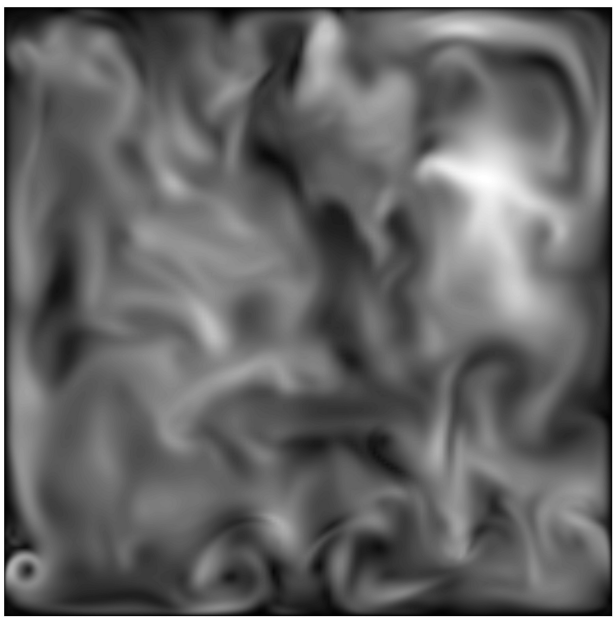


[2]F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **Phys.Rev.Fluids**, 5:024603, 2020.

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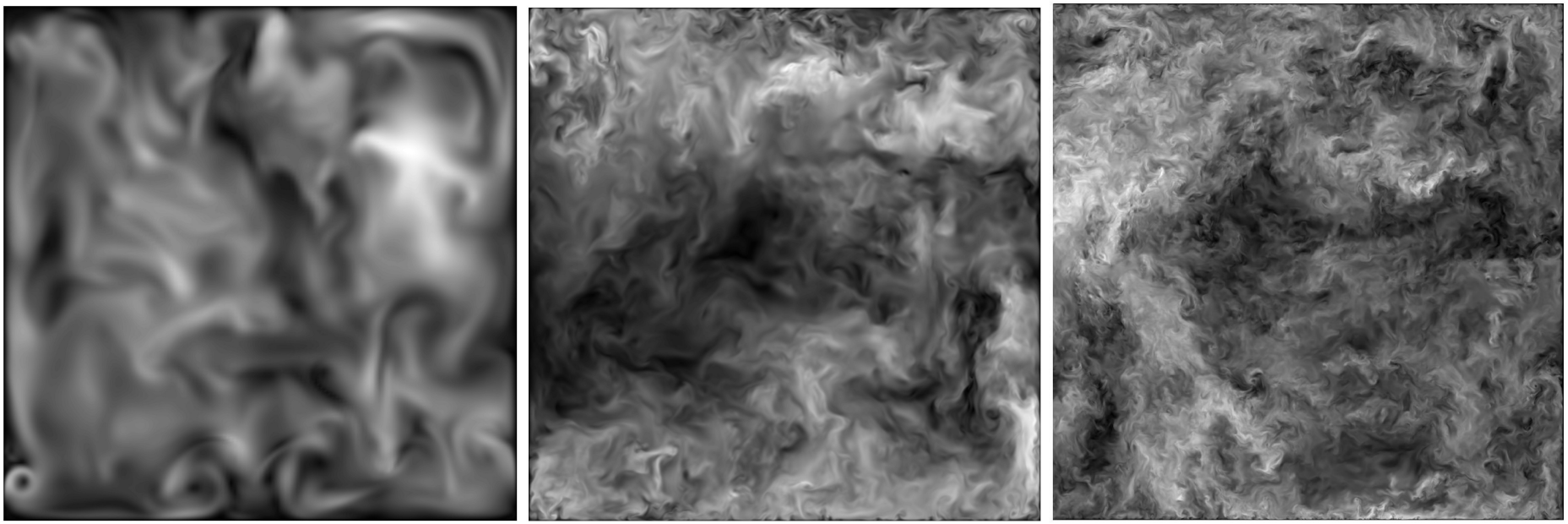



[2]F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **Phys.Rev.Fluids**, 5:024603, 2020.

Motivation

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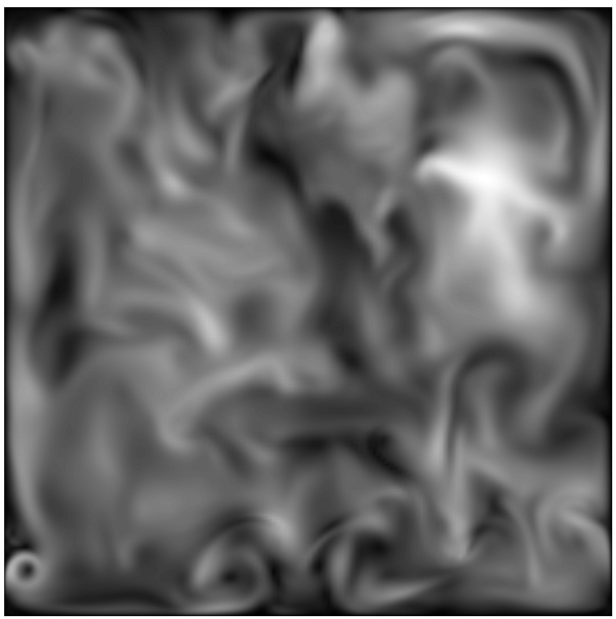
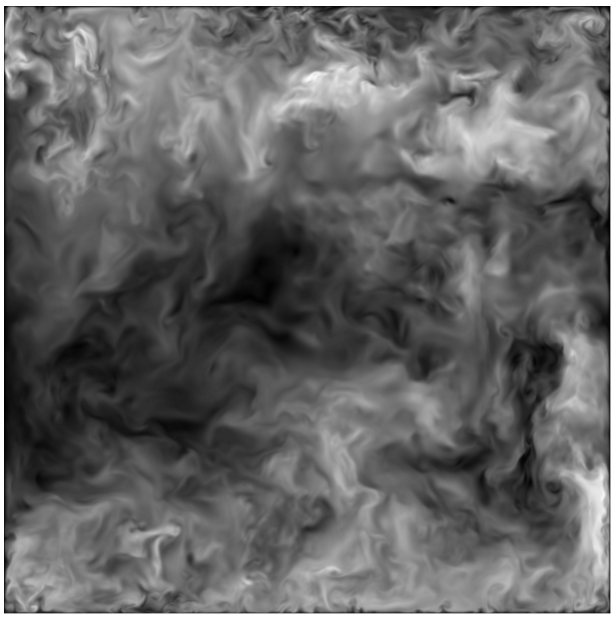
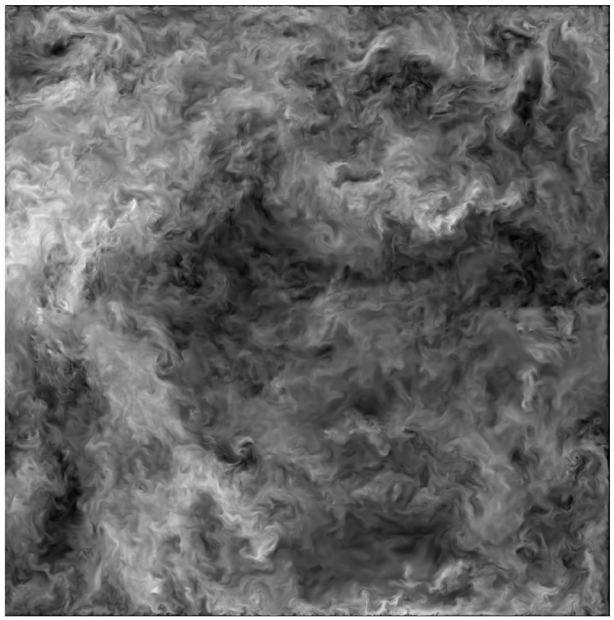


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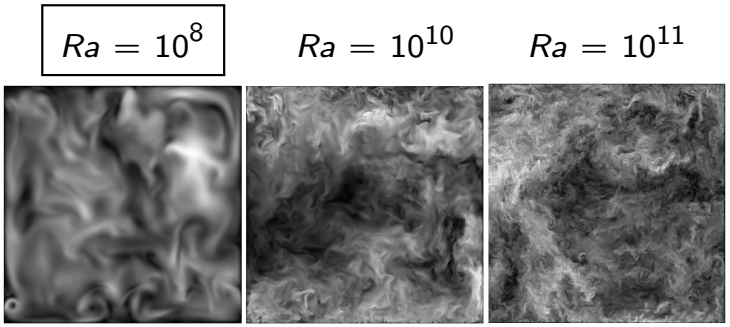


$Ra = 10^8$	$Ra = 10^{10}$	$Ra = 10^{11}$
		
$208 \times 208 \times 400$	$768 \times 768 \times 1024$	$1662 \times 1662 \times 2048$
17.5M	**607M**	**5600M**

[2] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *Flow topology dynamics in a 3D phase space for turbulent Rayleigh-Bénard convection*, **Phys.Rev.Fluids**, 5:024603, 2020.

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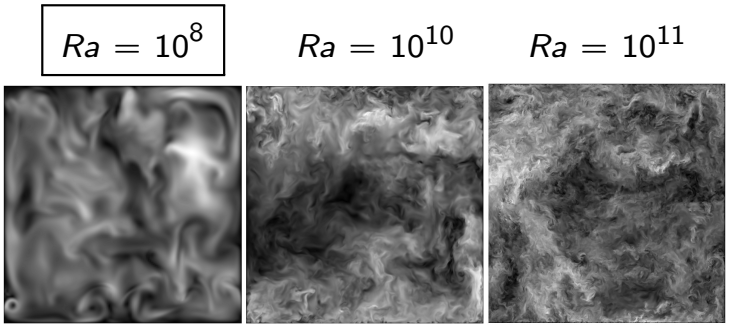
DNS: $208 \times 208 \times 400$



Motivation

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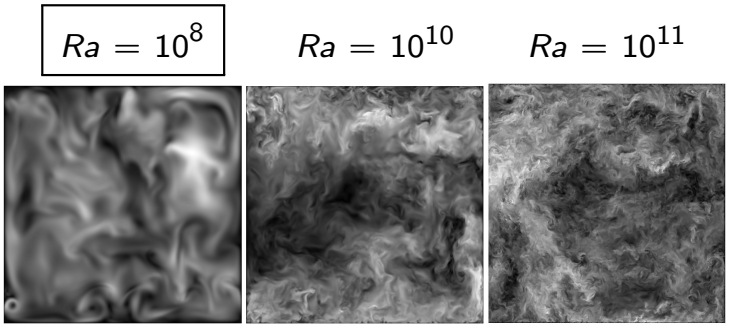
LES: $80 \times 80 \times 120$

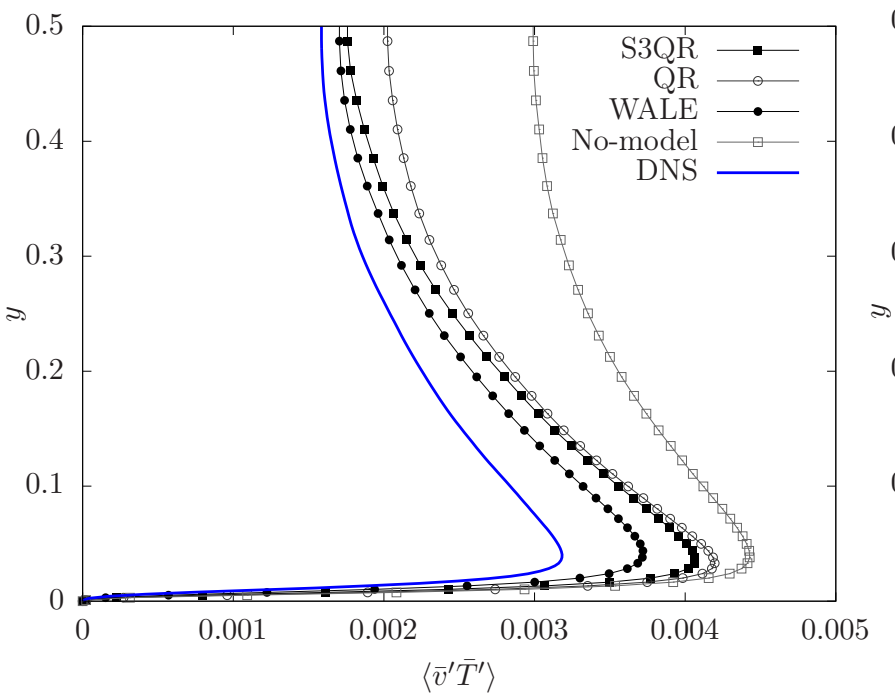


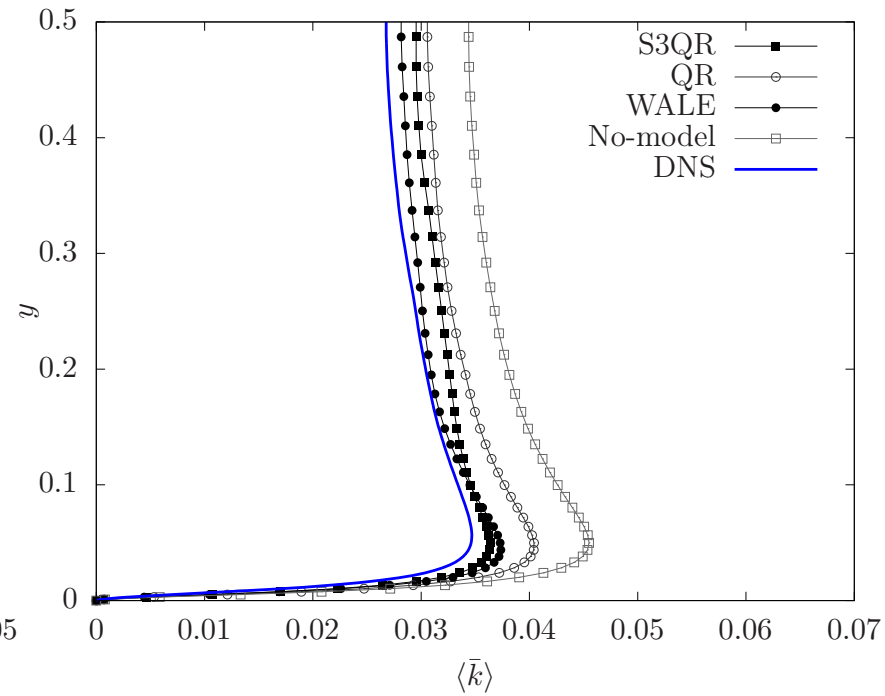
Motivation

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LES: $80 \times 80 \times 120$



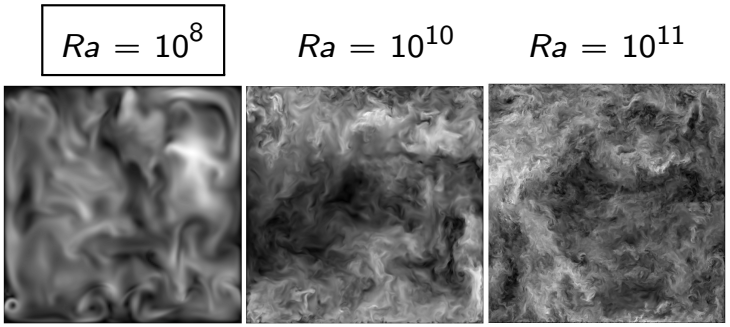


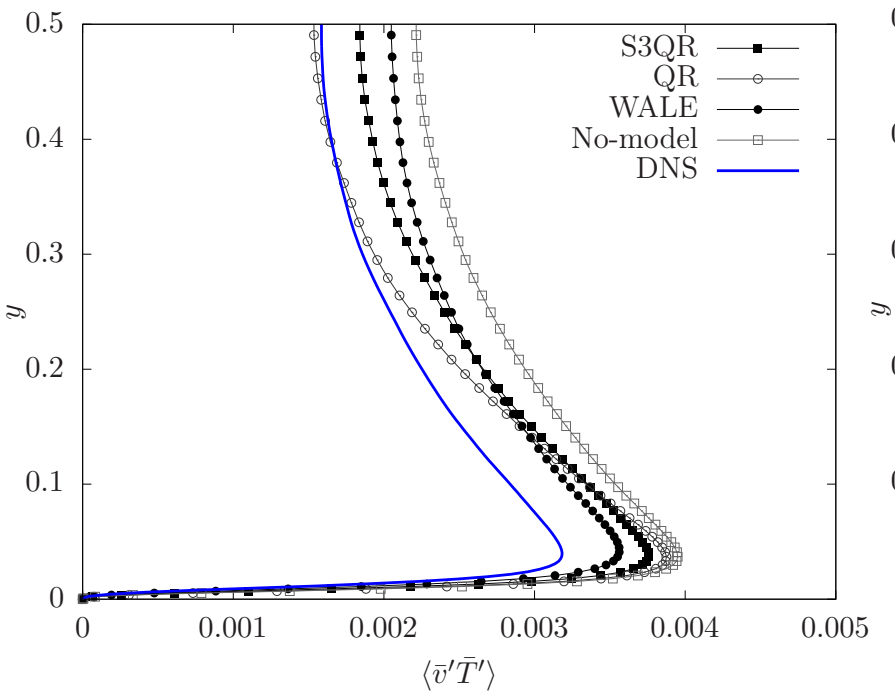


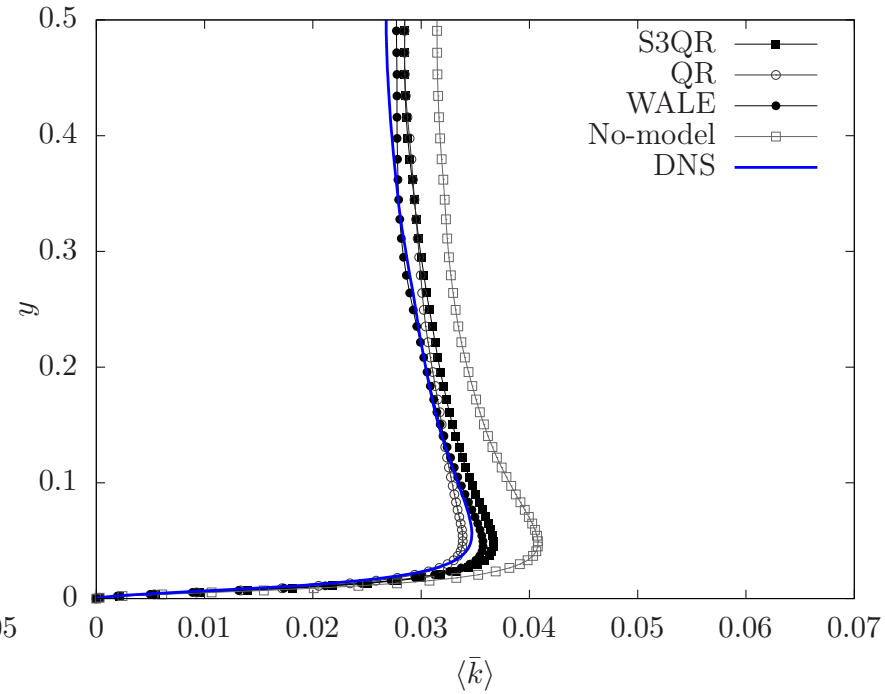
Motivation

DNS: $208 \times 208 \times 400$

LES: $110 \times 110 \times 168$







How to model the subgrid heat flux in LES?

$$\partial_t \overline{u} + (\overline{u} \cdot \nabla)\overline{u} = \nu \nabla^2 \overline{u} - \nabla \overline{p} \qquad - \nabla \cdot \tau(\overline{u}) \;\; ; \qquad \nabla \cdot \overline{u} = 0$$

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$G \equiv \nabla\overline{u}$ $\qquad$ $q = -\dfrac{\delta^2}{12} G\nabla\overline{T} + \mathcal{O}(\delta^4)$

A priori alignment trends[3]

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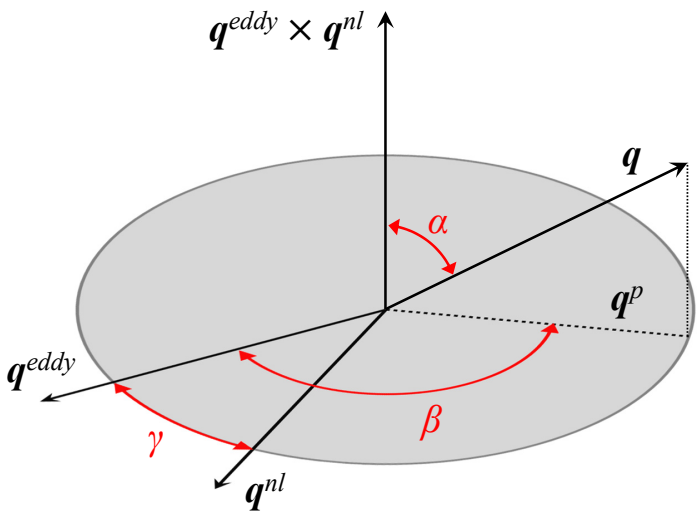
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[3] F.Dabbagh, F.X.Trias, A.Gorobets, A.Oliva. *A priori study of subgrid-scale features in turbulent Rayleigh-Bénard convection*, **Physics of Fluids**, 29:105103, 2017.

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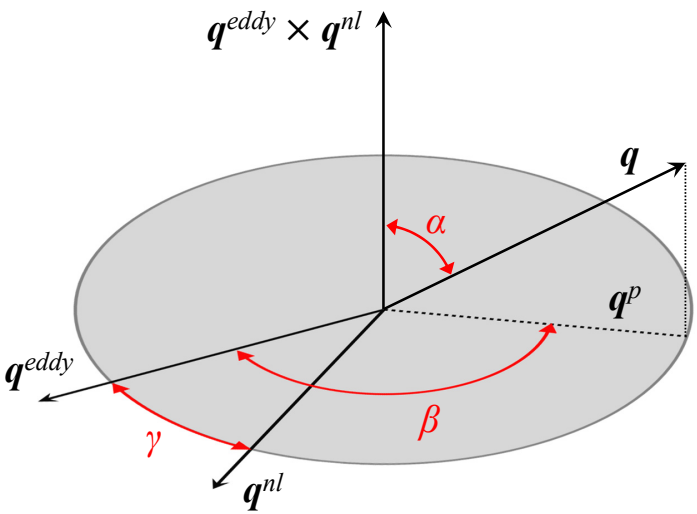


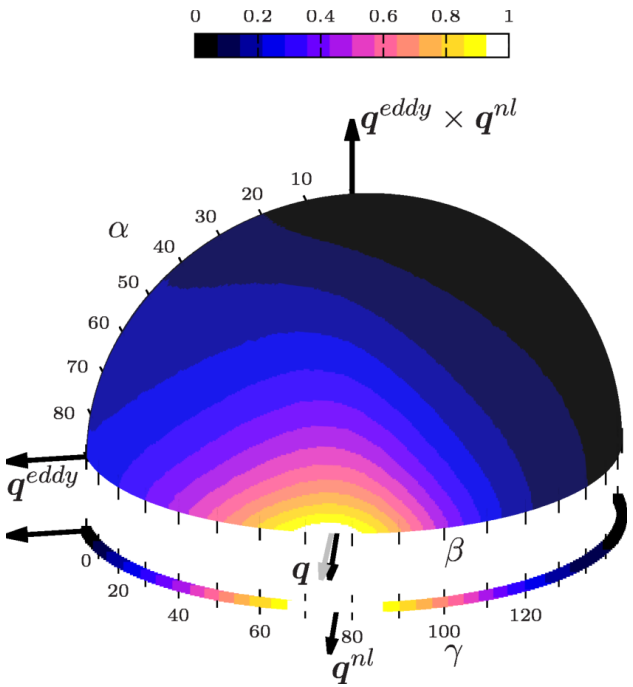
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[4] S.Peng and L.Davidson. **Int.J.Heat Mass Transfer**, 45:1393-1405, 2002.

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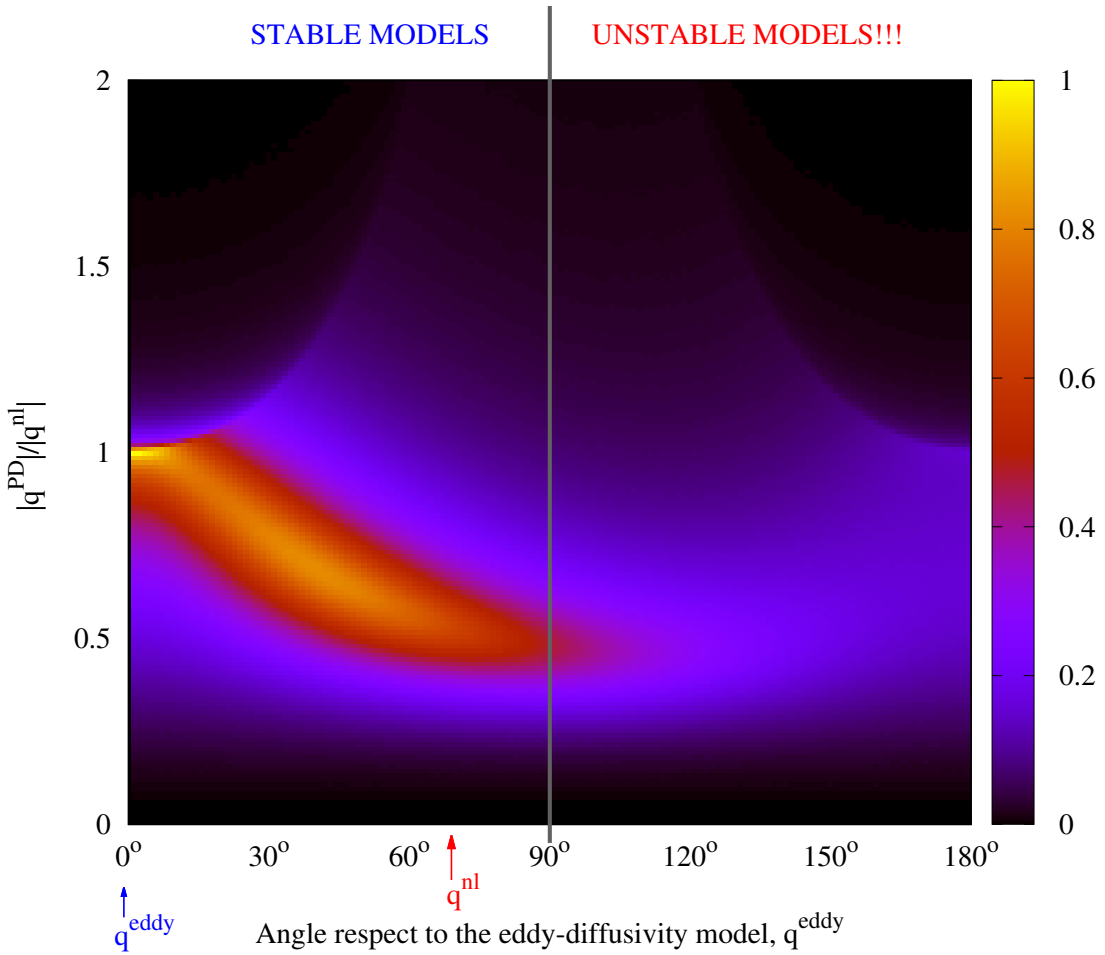
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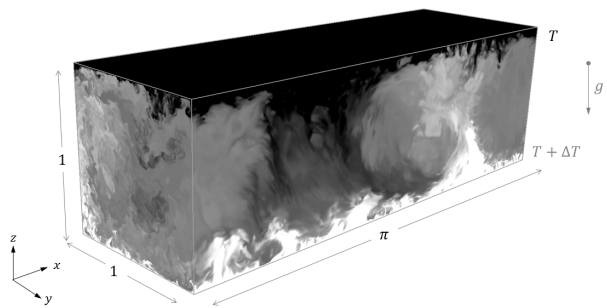
Peng&Davidson[4] $\longrightarrow$ $q \approx -\frac{\delta^2}{12} S\nabla\overline{T}$ $(\equiv q^{PD})$

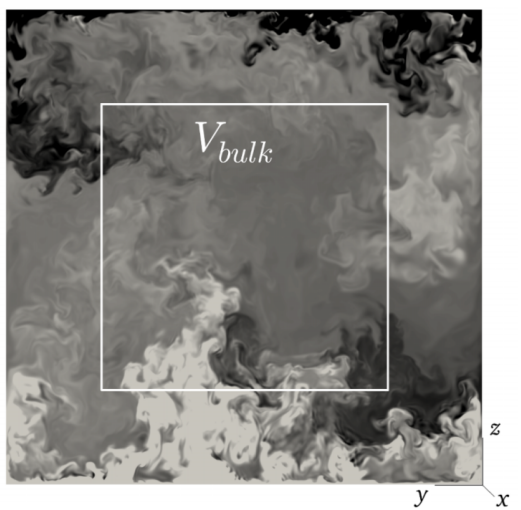
[4] S.Peng and L.Davidson. **Int.J.Heat Mass Transfer**, 45:1393-1405, 2002.

A priori alignment trends

$$q^{nl} \equiv -\frac{\delta^2}{12} G \nabla \overline{T} \qquad \boxed{q^{PD} \equiv -\frac{\delta^2}{12} S \nabla \overline{T}}$$







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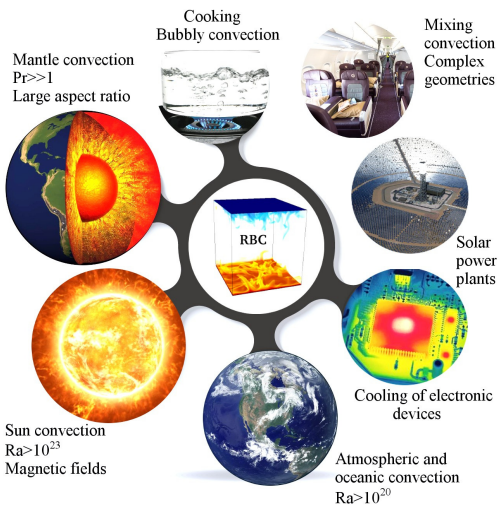
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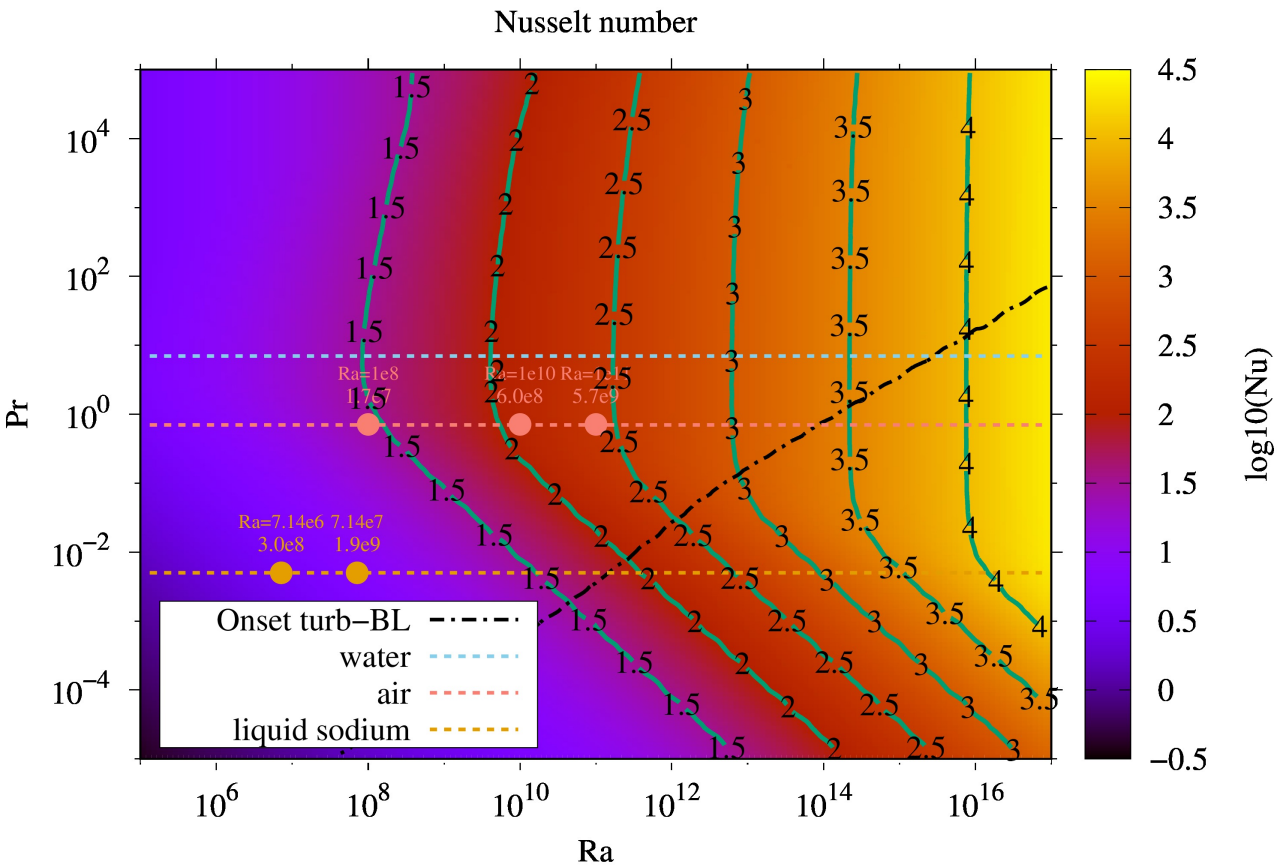
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Motivation

Research question #2:

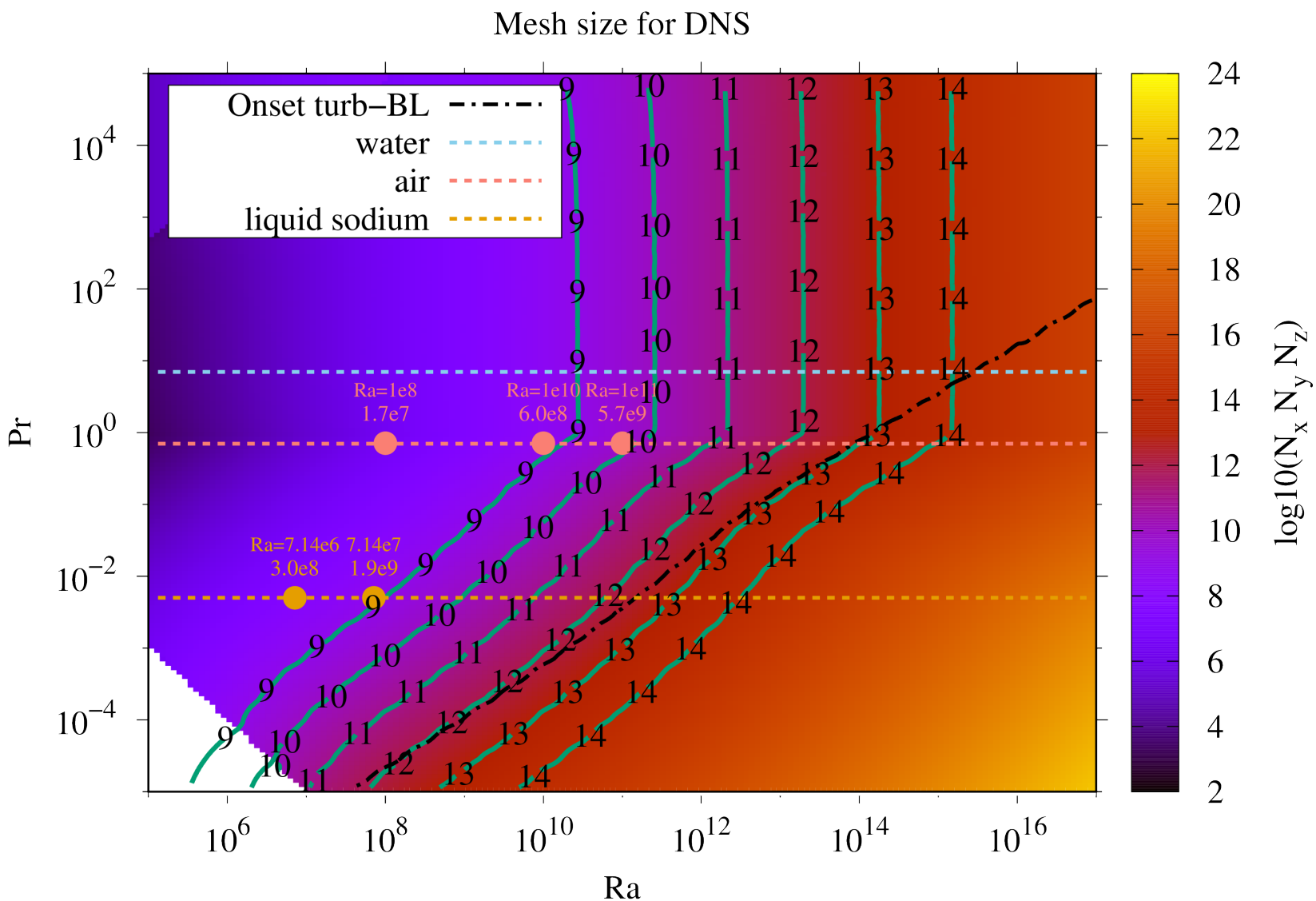
- Is it possible to hit the ultimate regime of turbulence with LES?





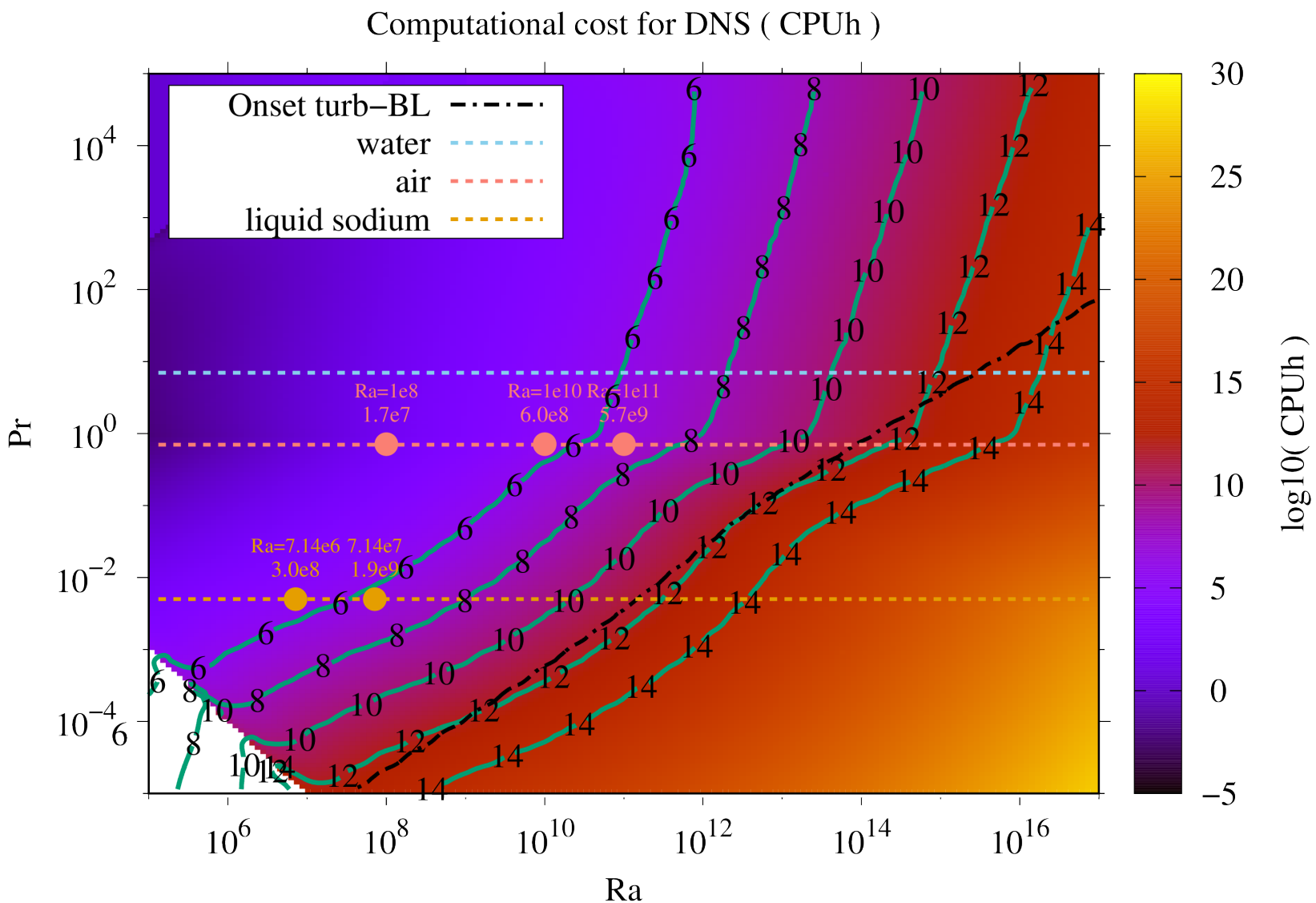
Can we reach the ultimate regime of turbulence?

DNS is far too expensive



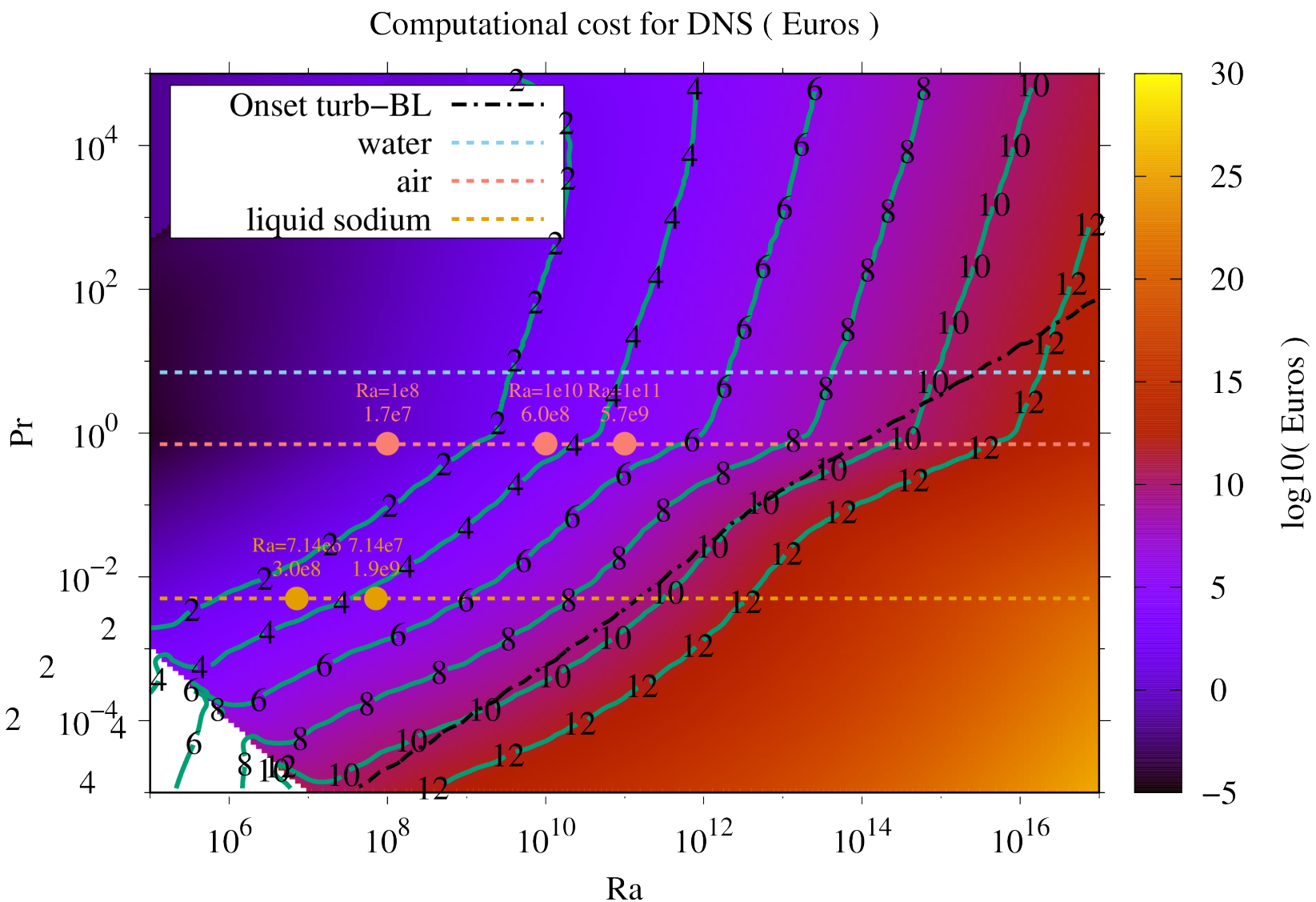
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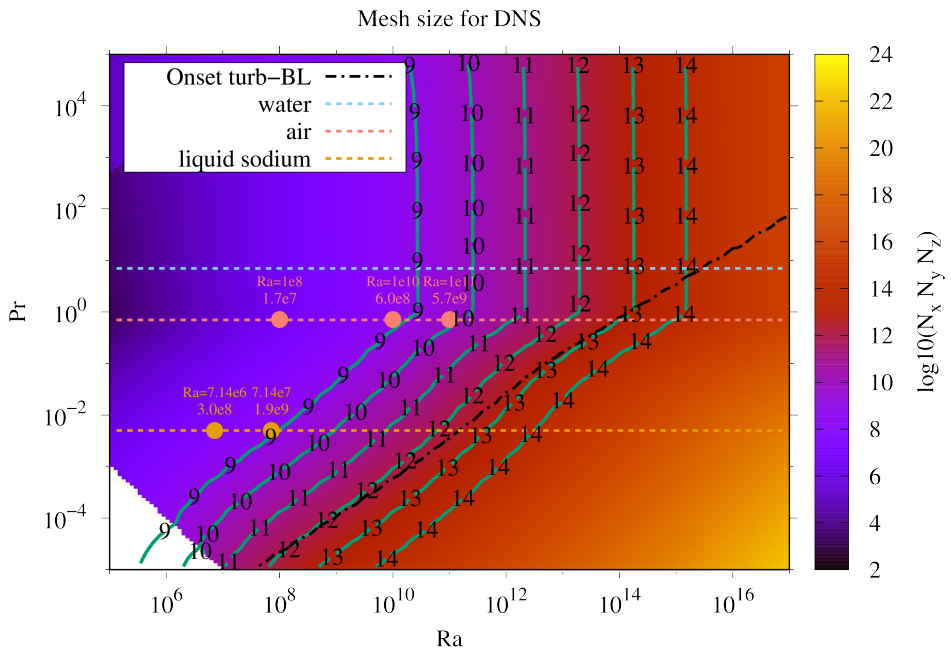
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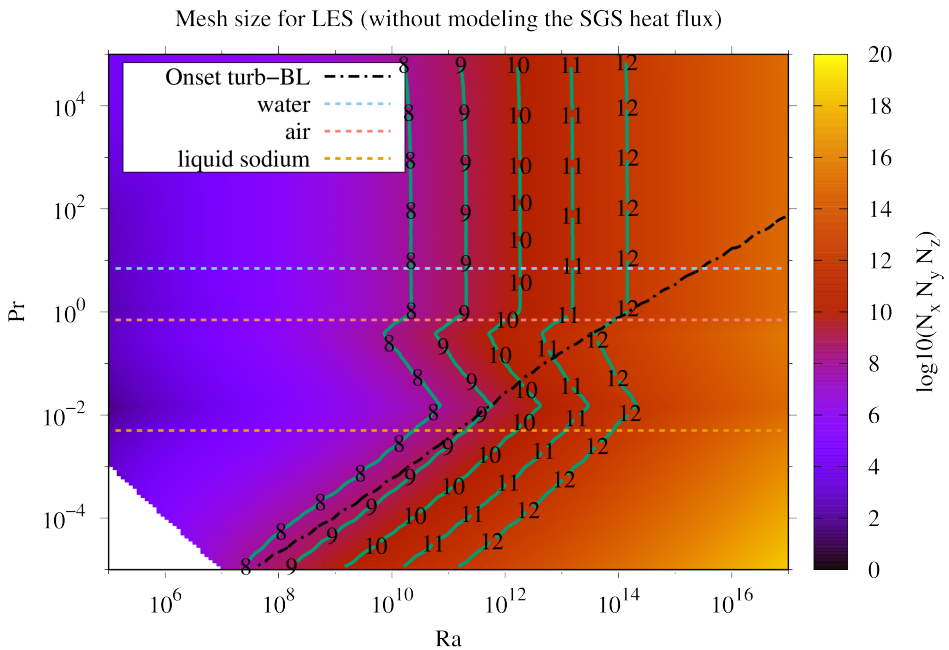
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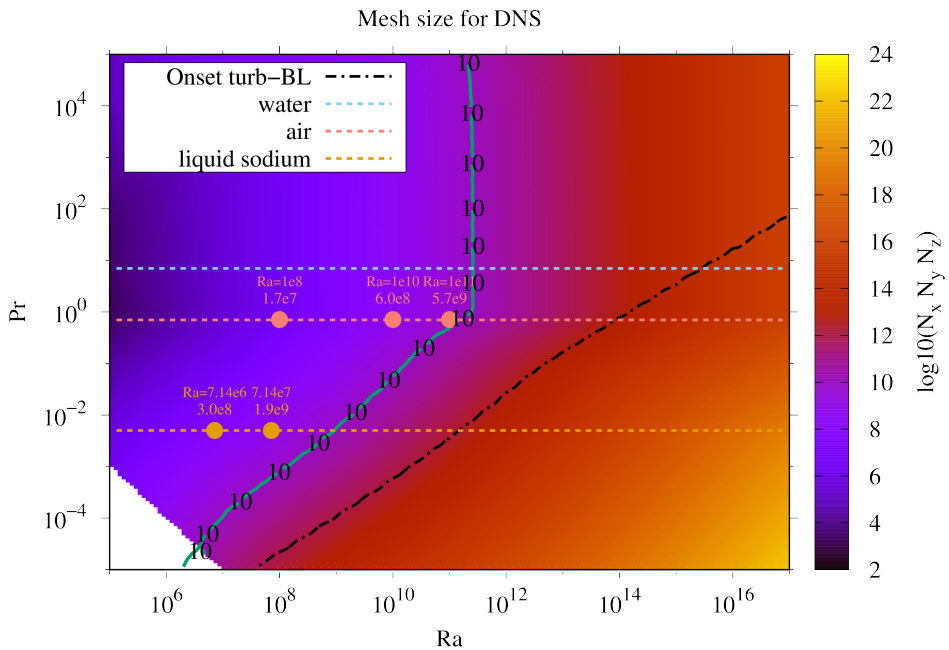
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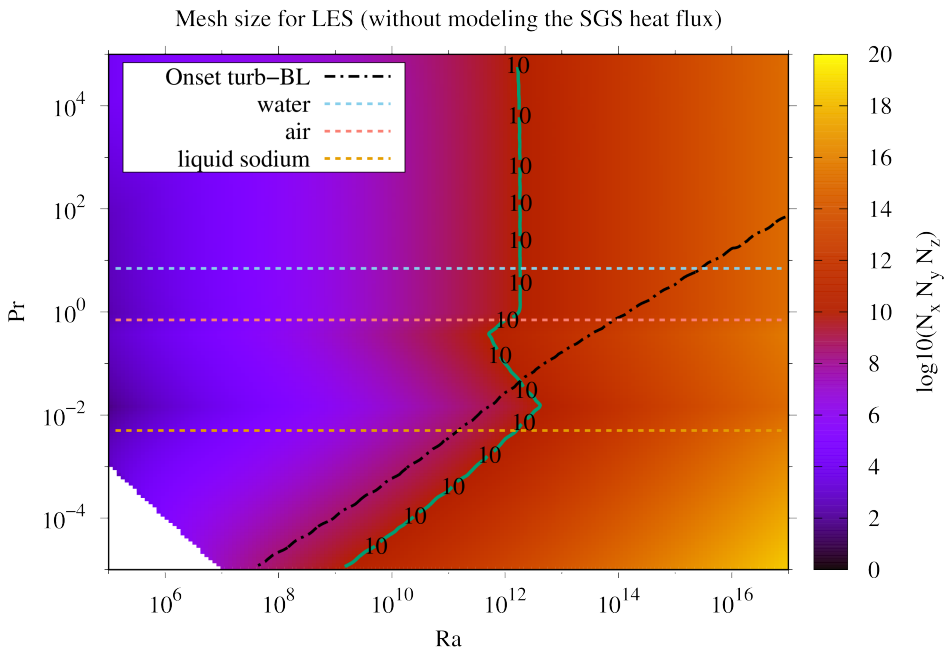




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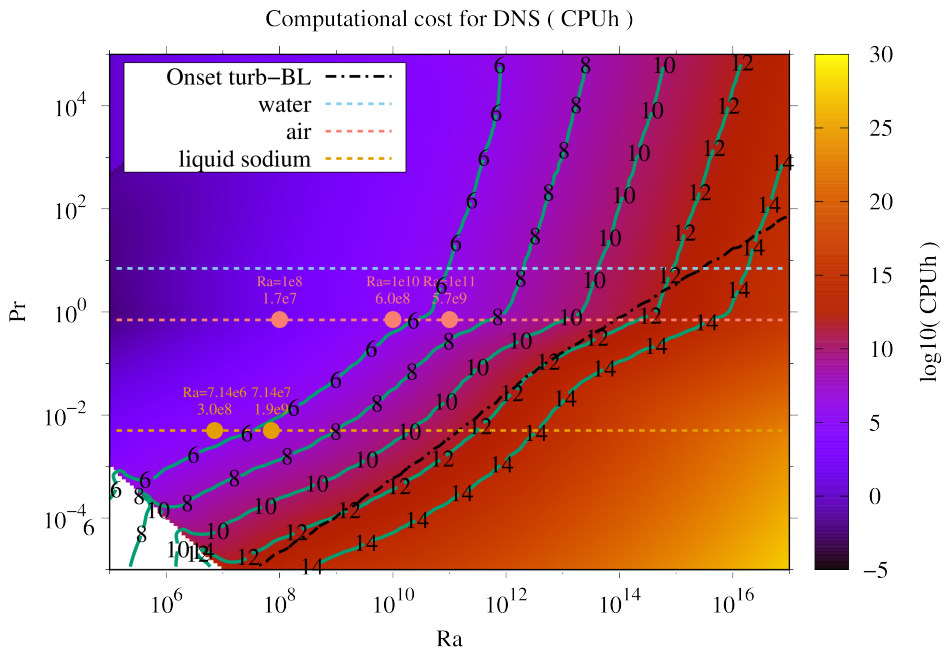
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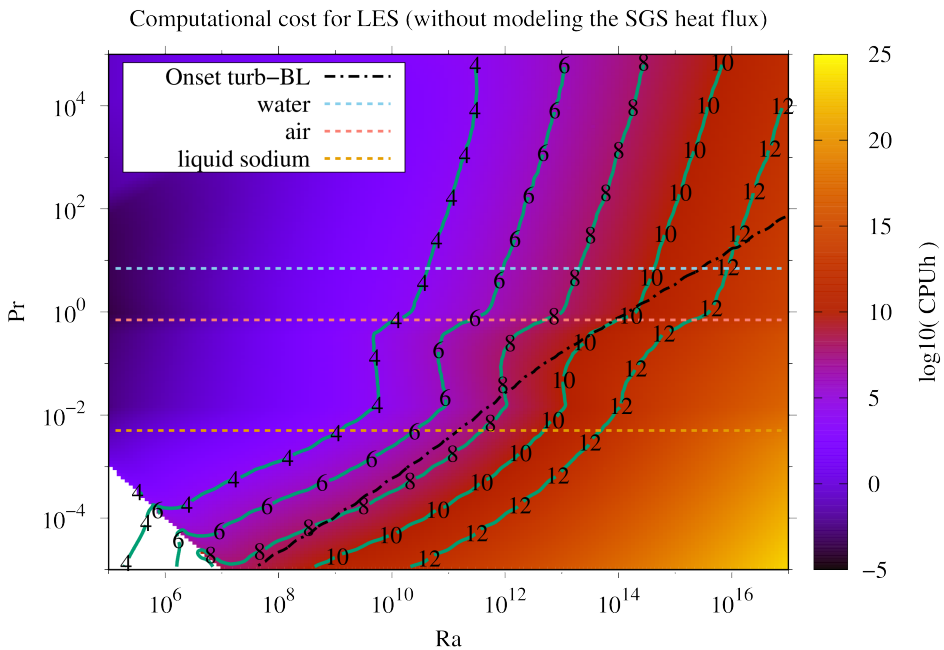




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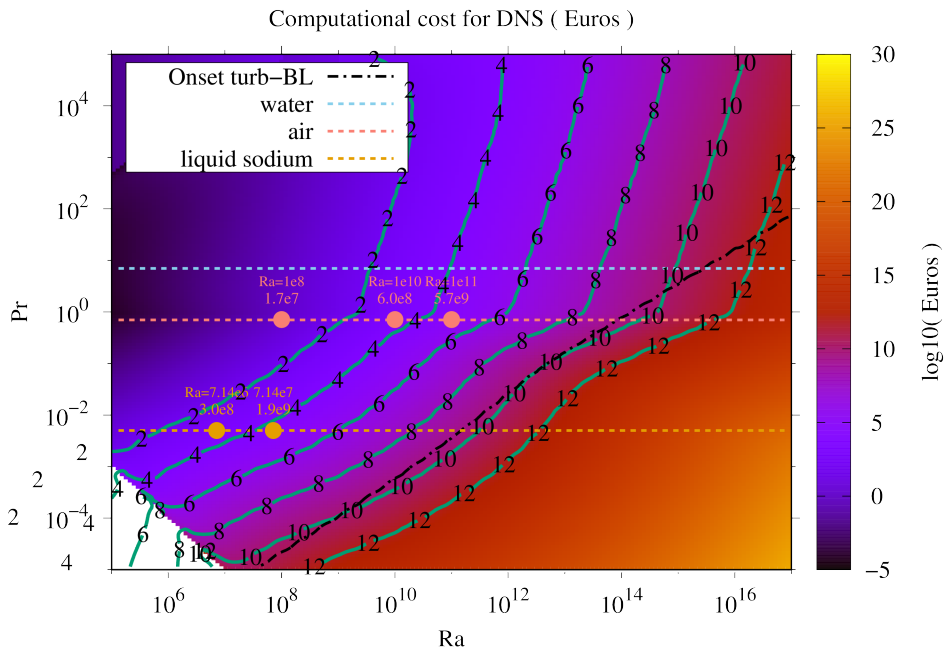
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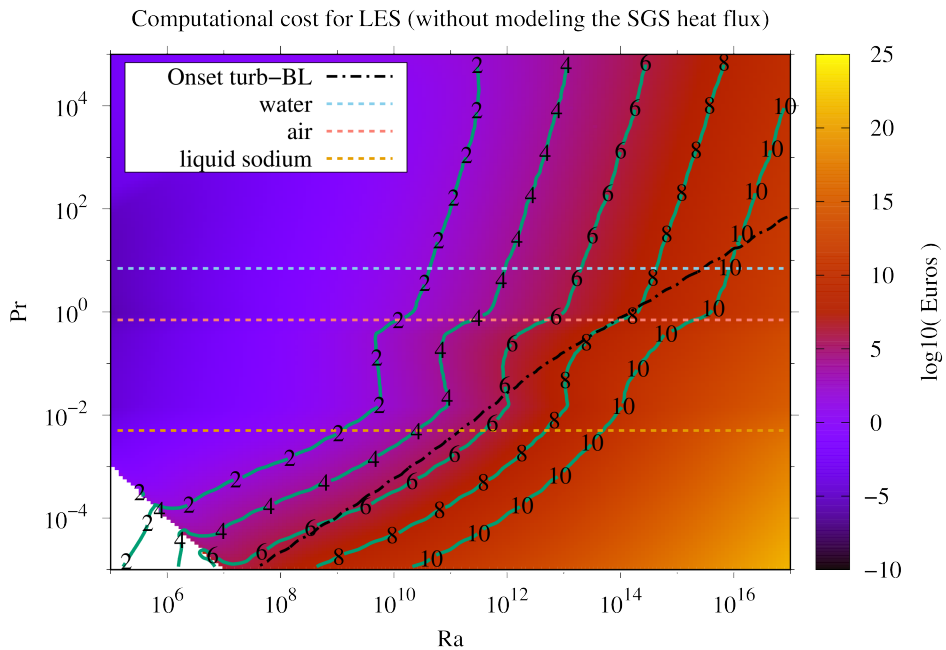




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Can we obtain good a *a posteriori* results?

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⚠ But first we need to answer the following **research question**:

- Are **eddy-viscosity models** for momentum able to provide satisfactory results for turbulent Rayleigh-Bénard convection?

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Idea: let's do an LES for momentum and a DNS for temperature!

DNS at very low Pr number

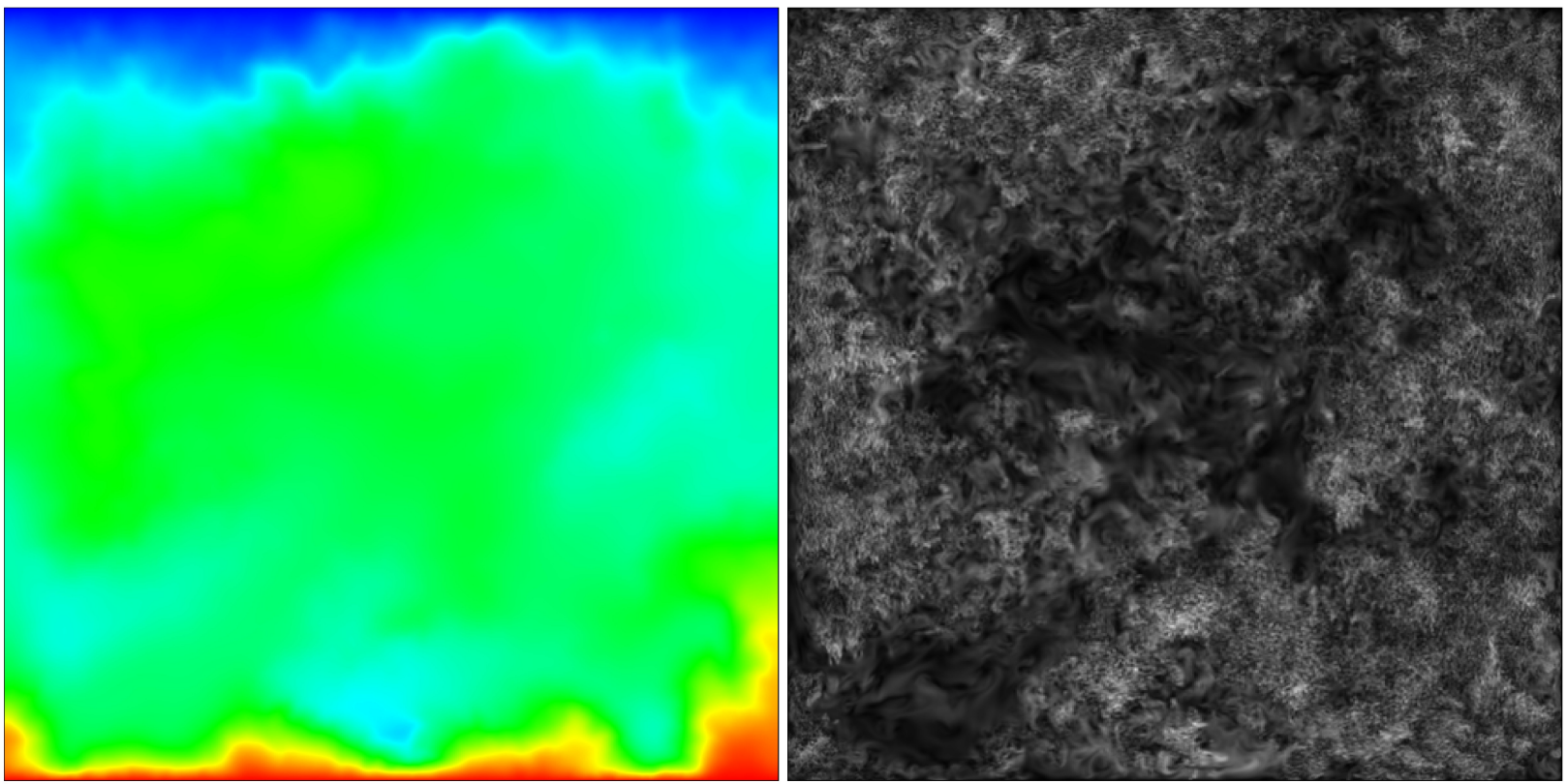
Why? scale separation grows as $\eta_K/\eta_T = Pr^{3/4}$.

η_T: Obukhov-Corrsin scale; η_K: Kolmogorov scale

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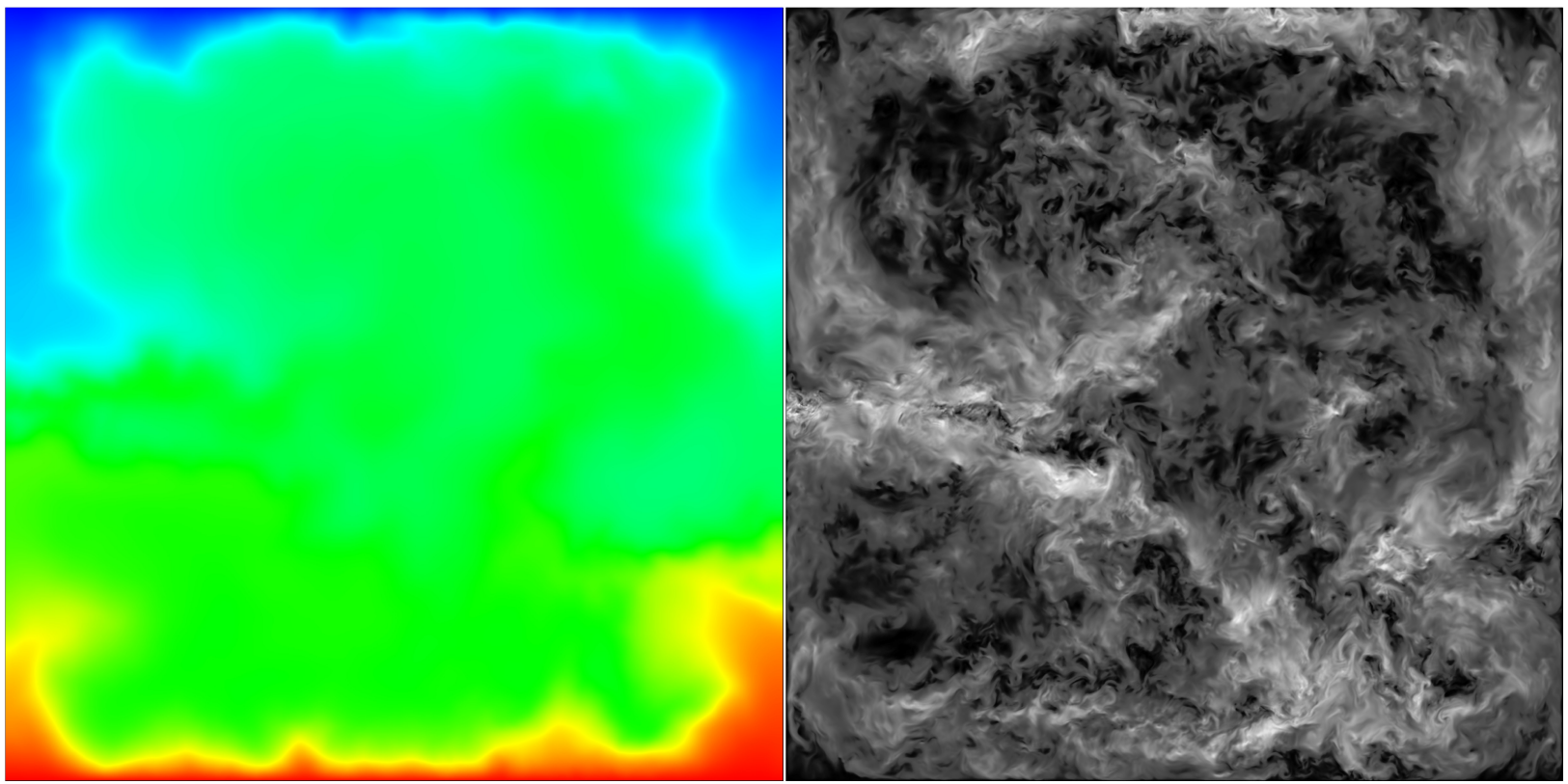
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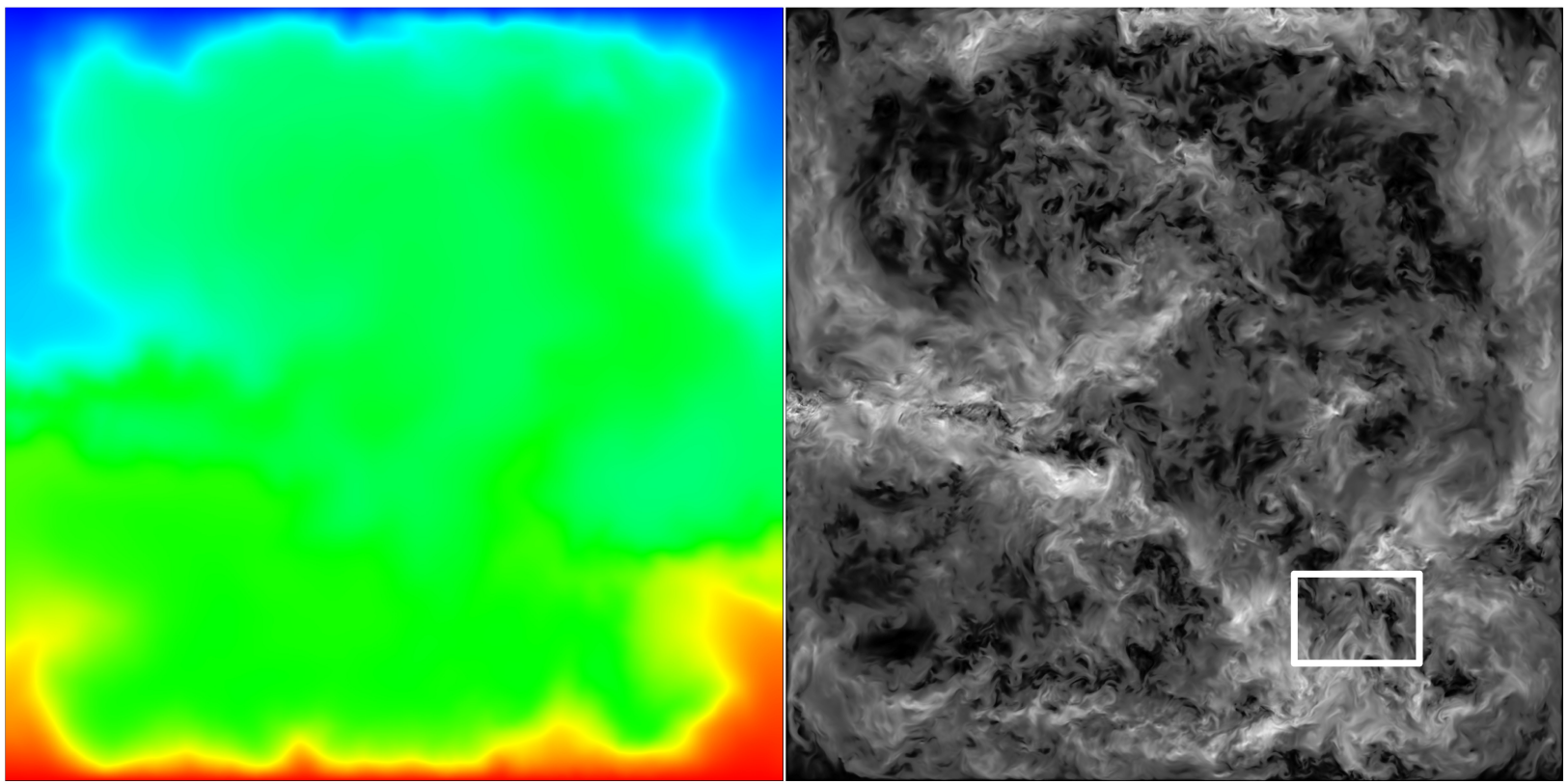
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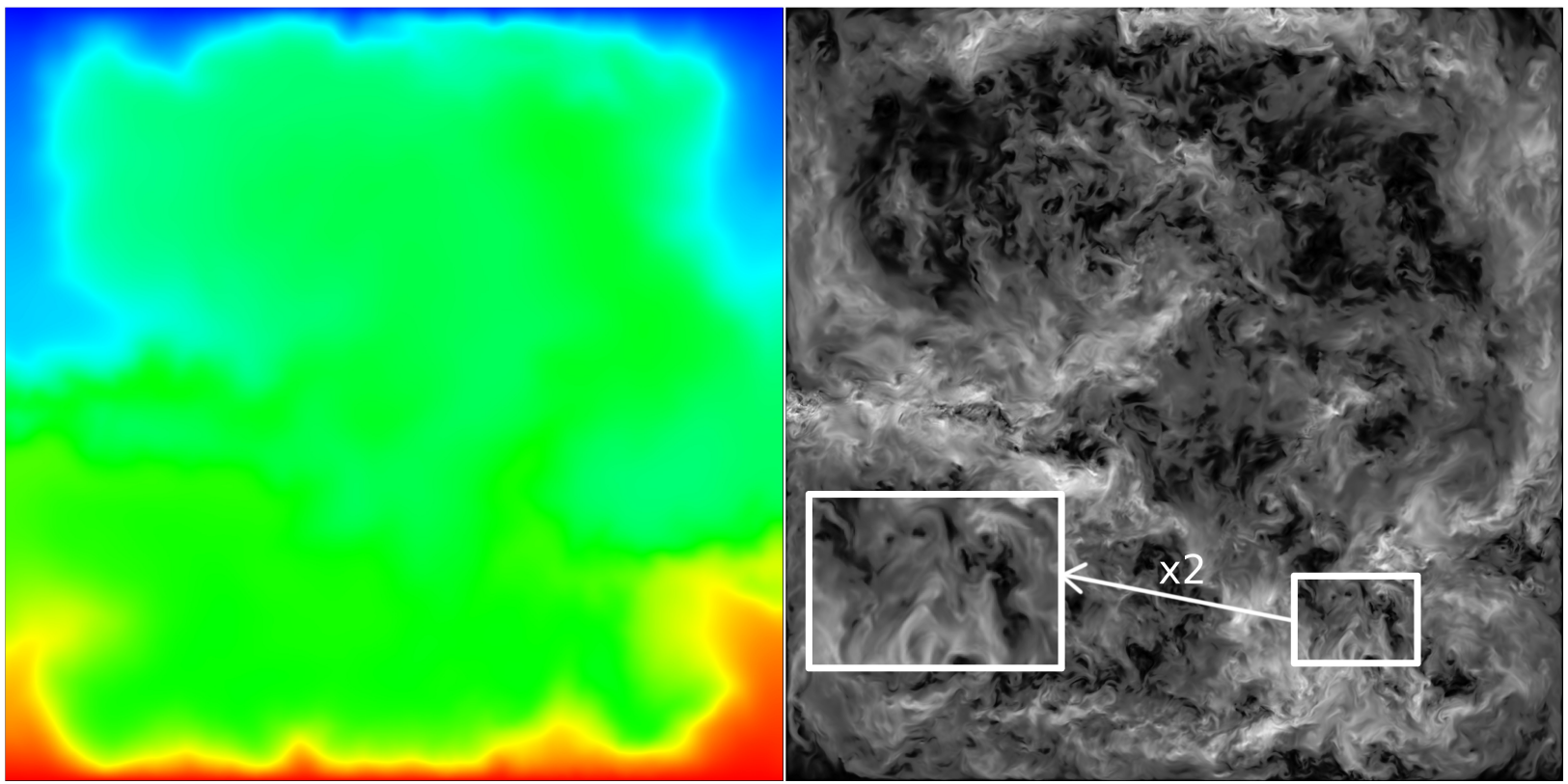
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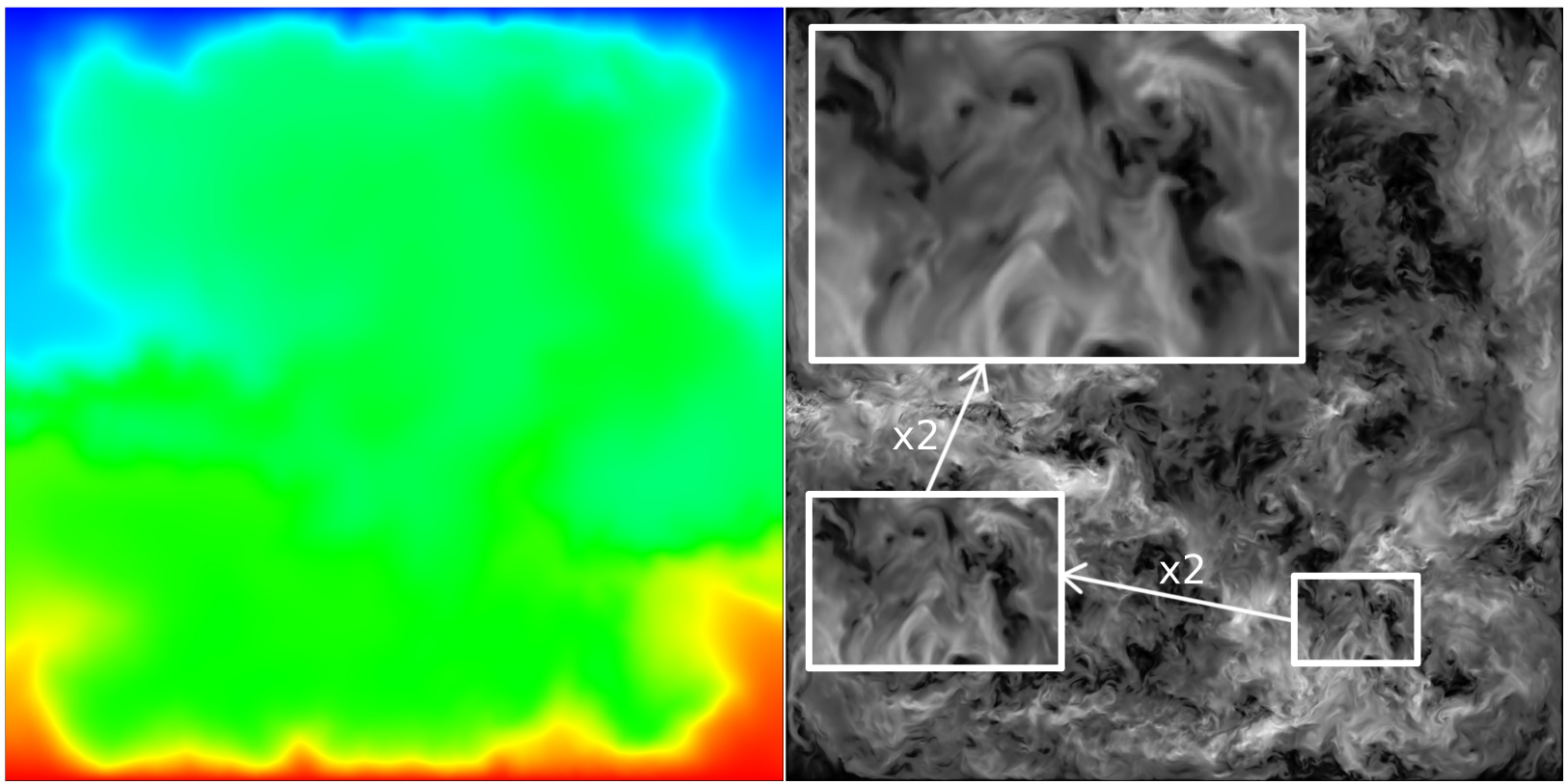
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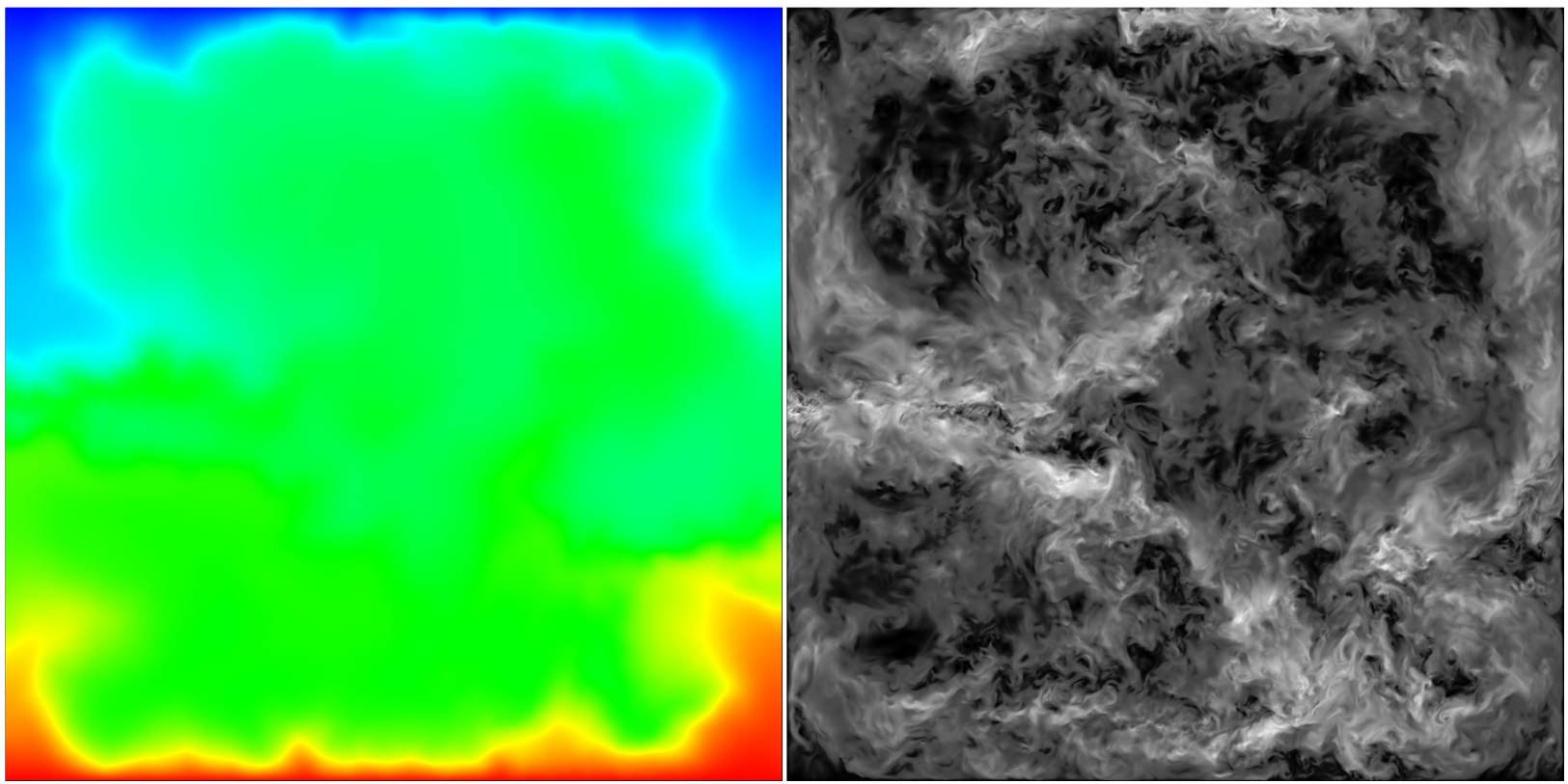
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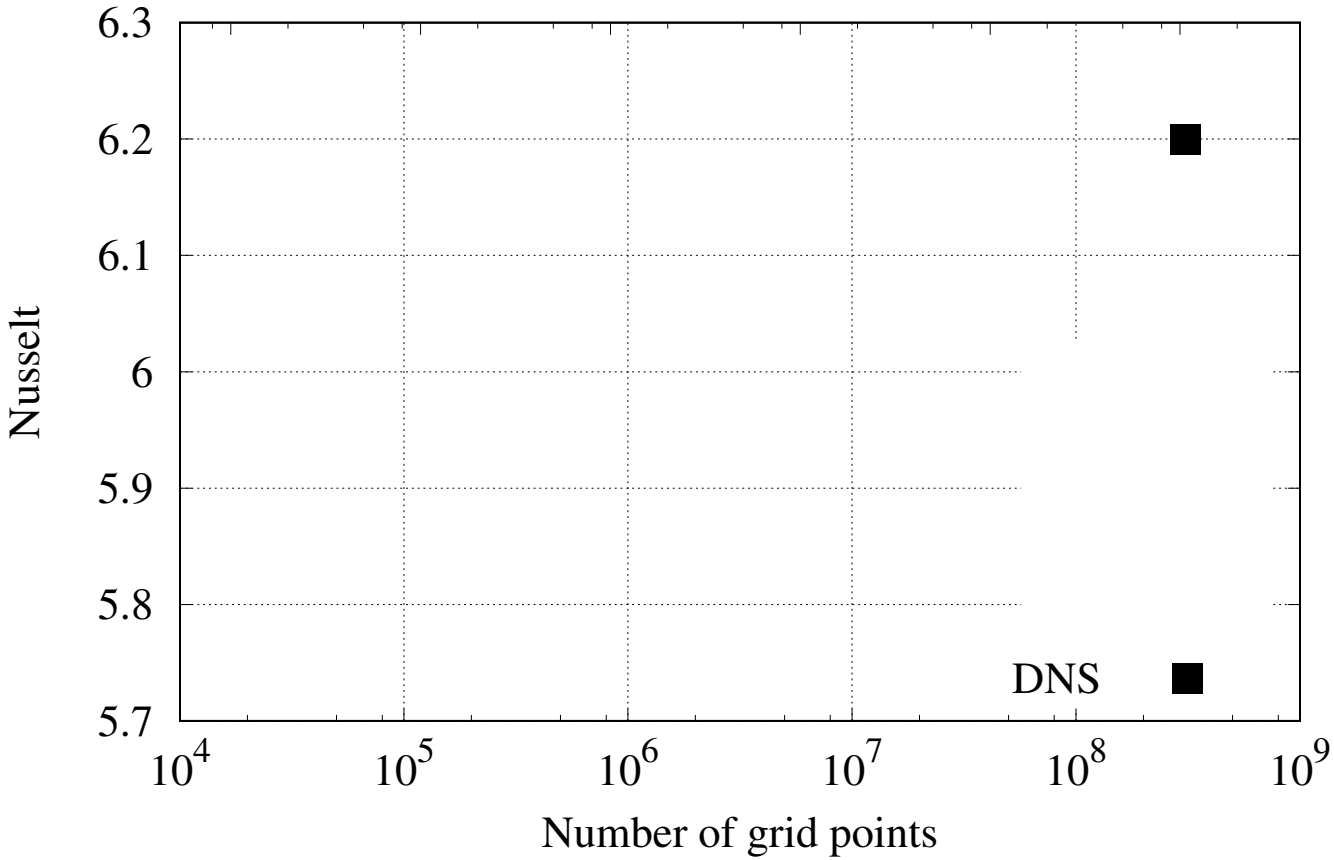


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LES[6] results at very low Pr number

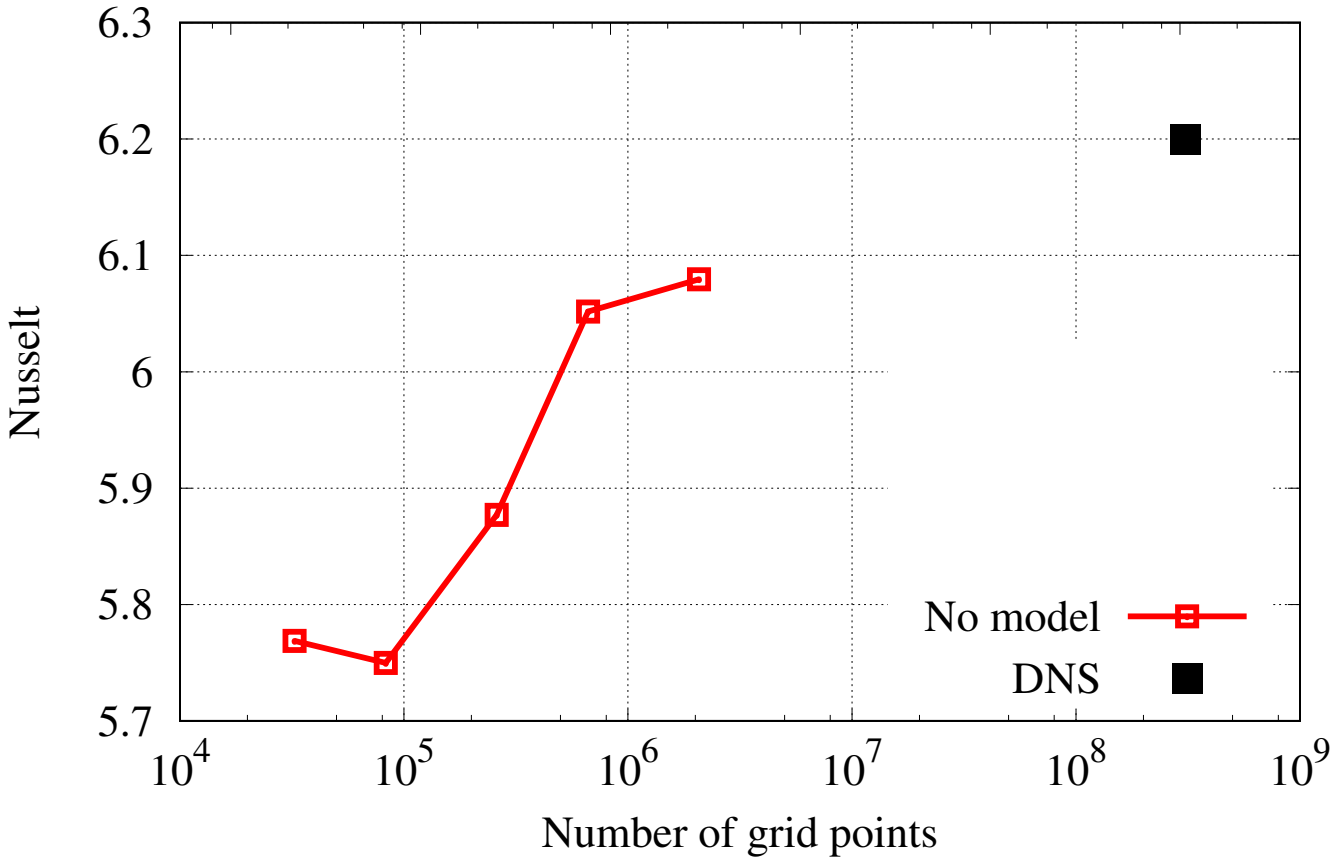
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[6] F.X.Trias, D.Folch, A.Gorobets, A.Oliva. *Building proper invariants for eddy-viscosity subgrid-scale models*, **Physics of Fluids**, 27: 065103, 2015.

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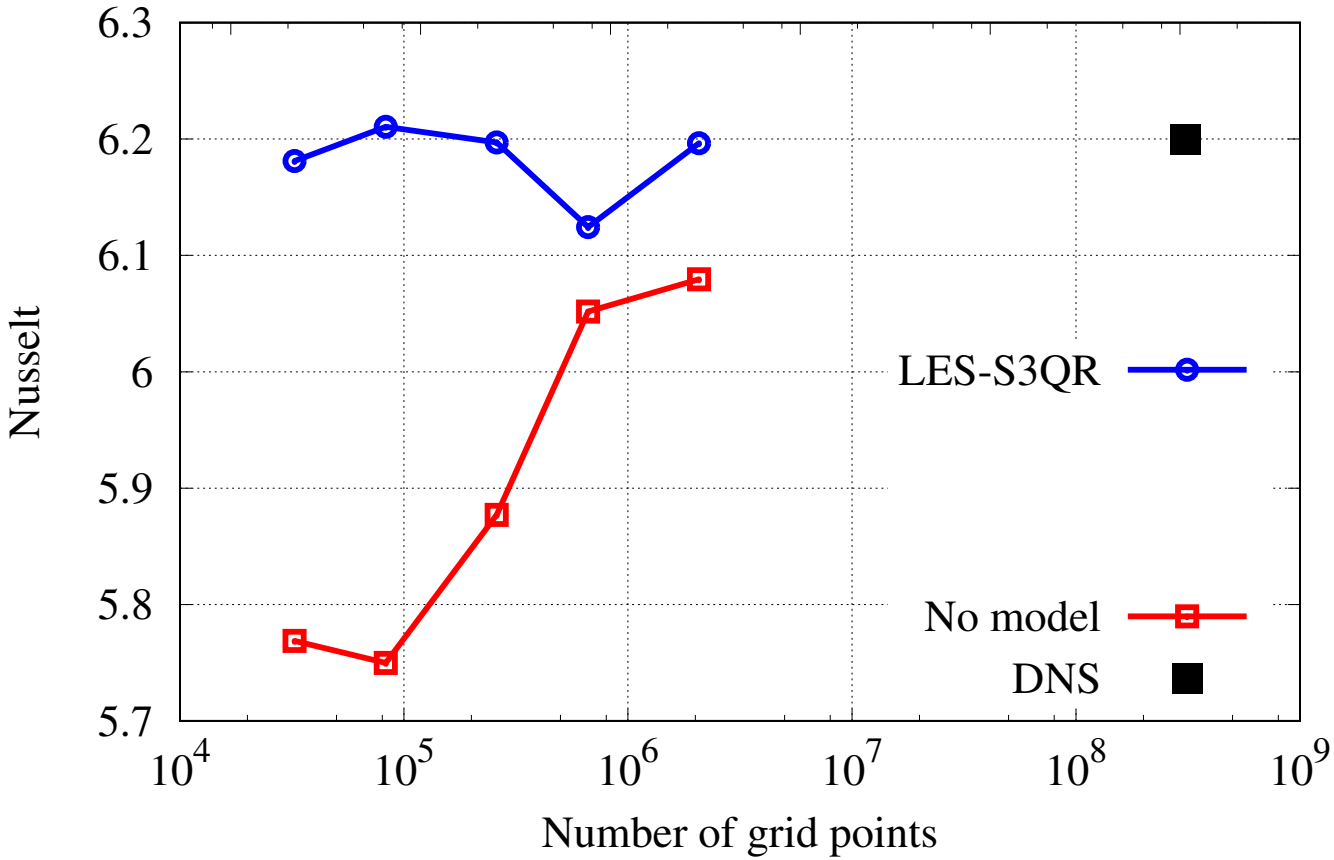
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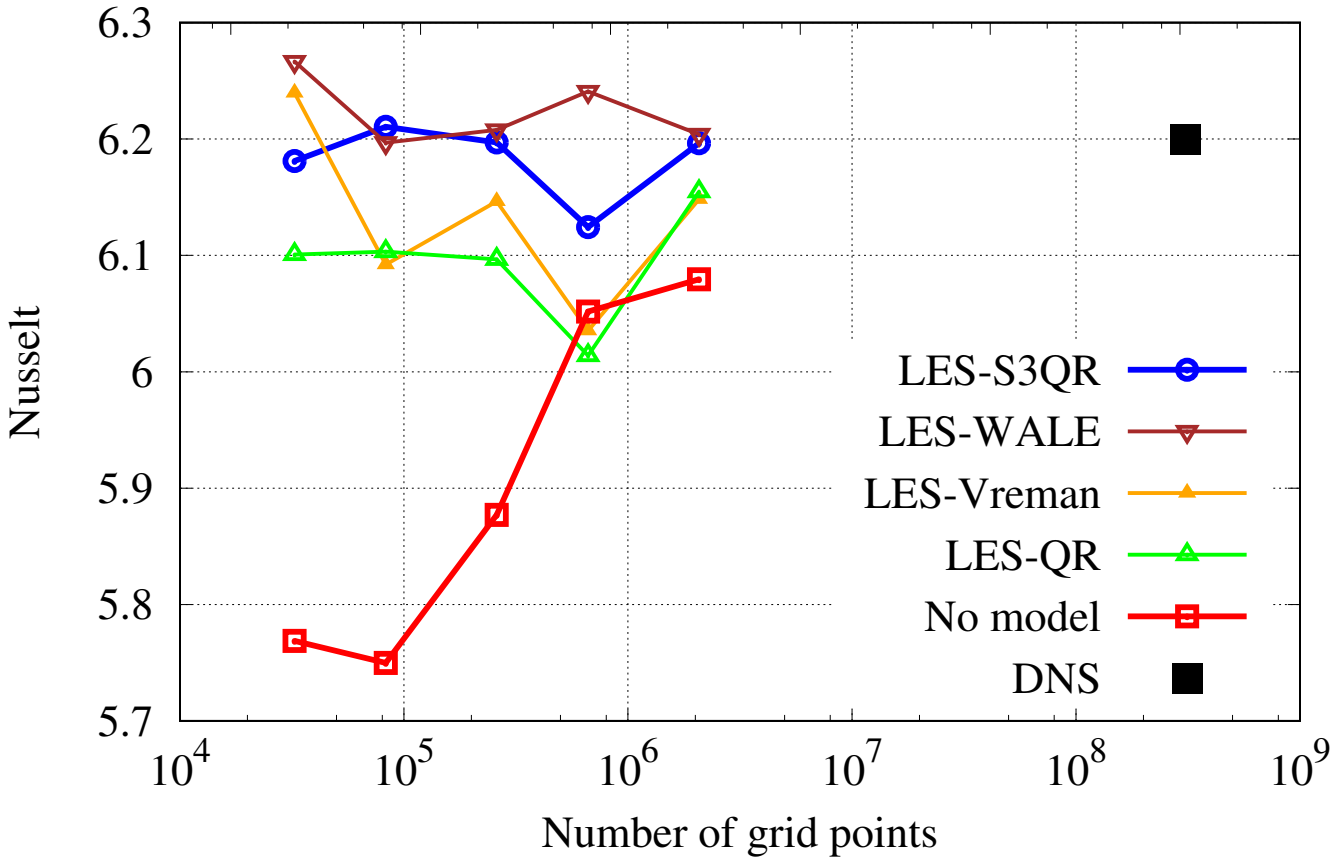
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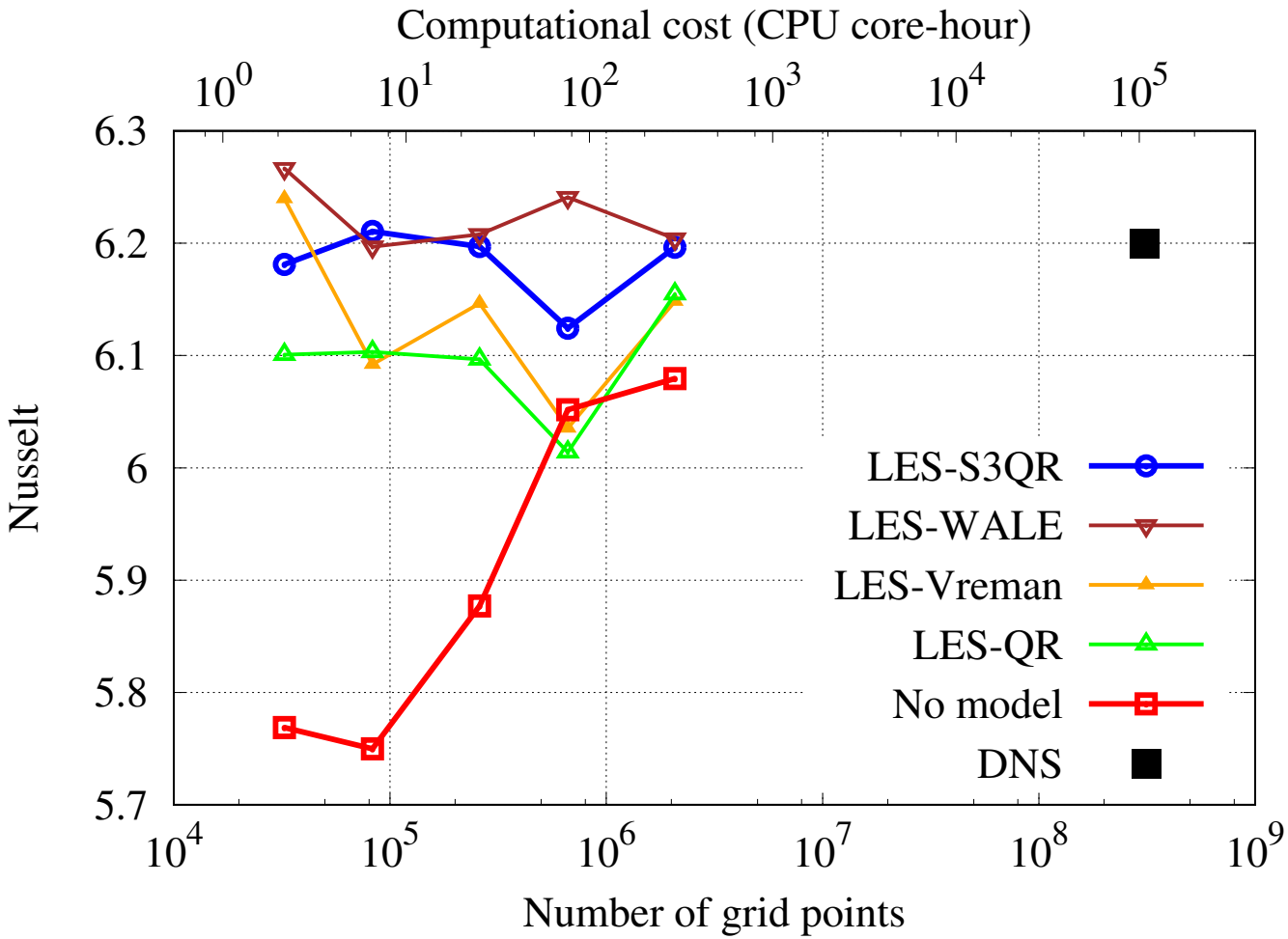
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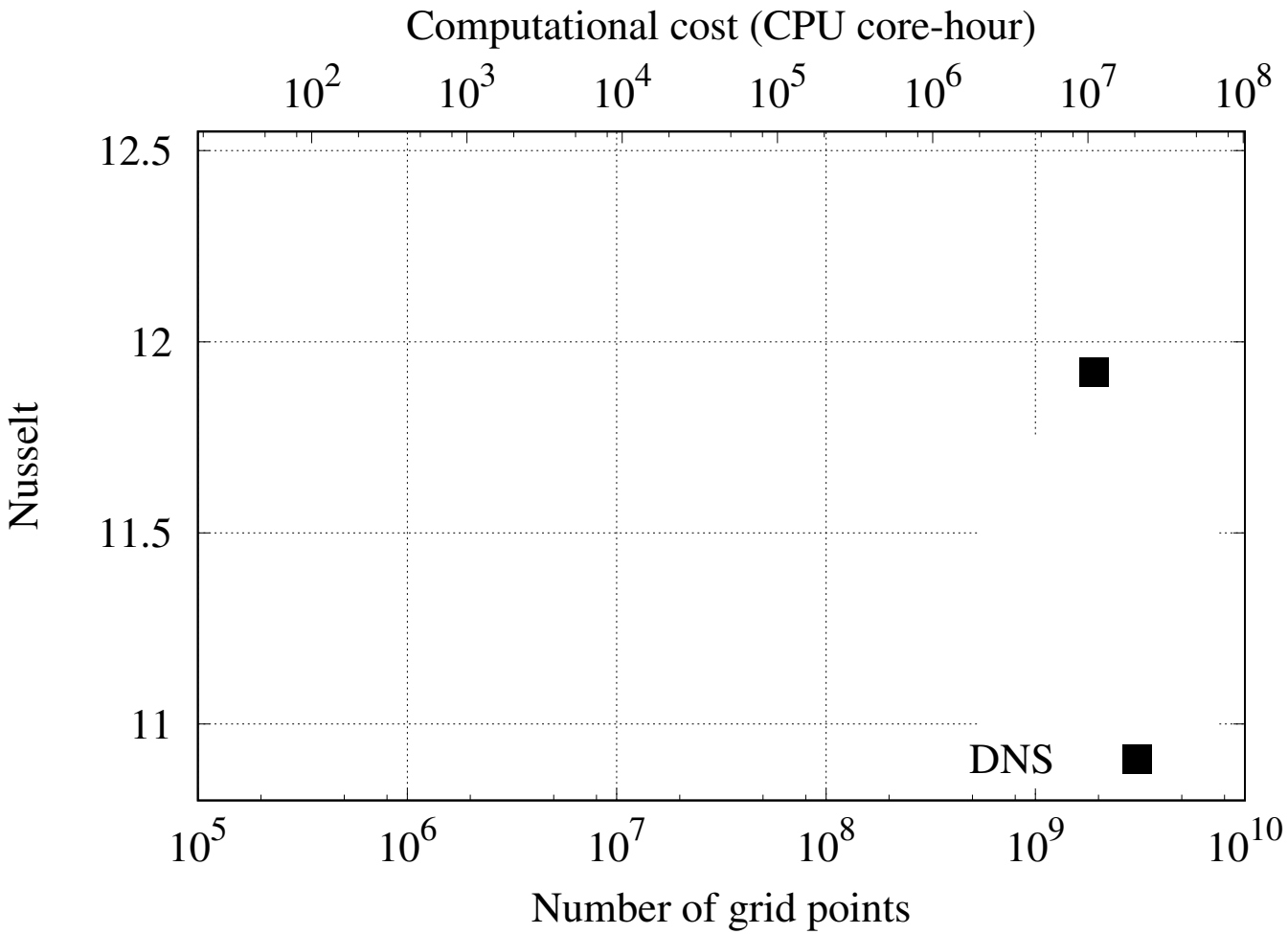
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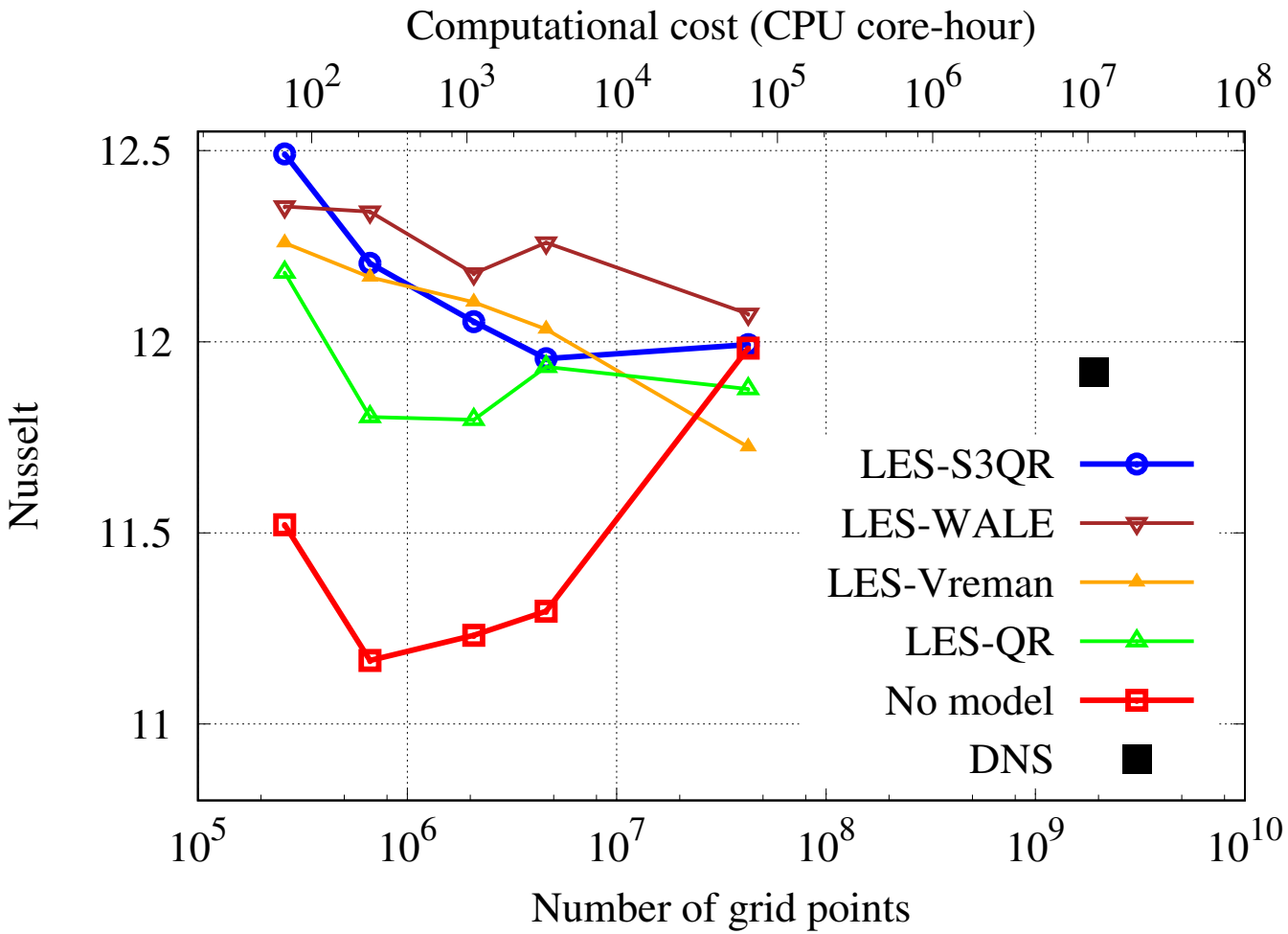
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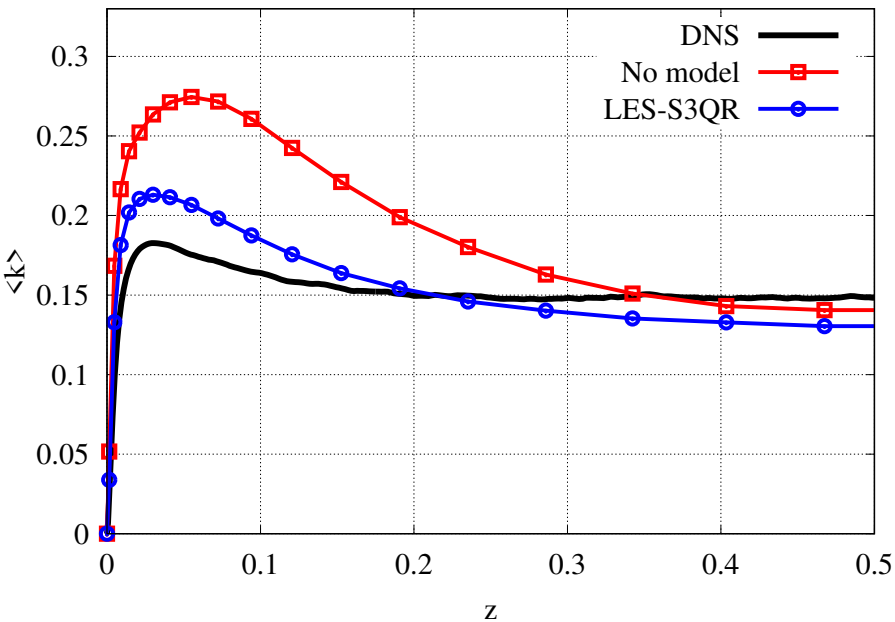
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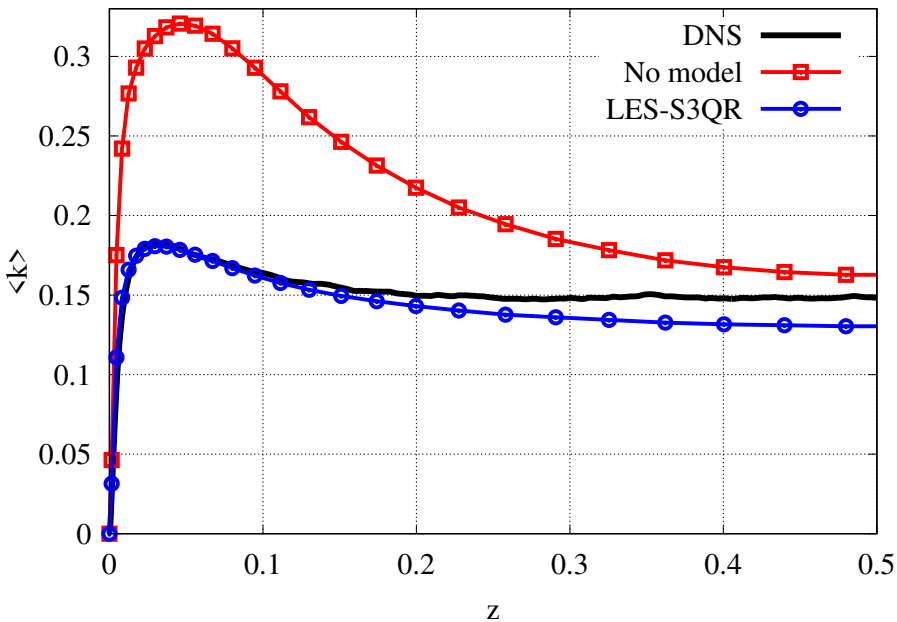


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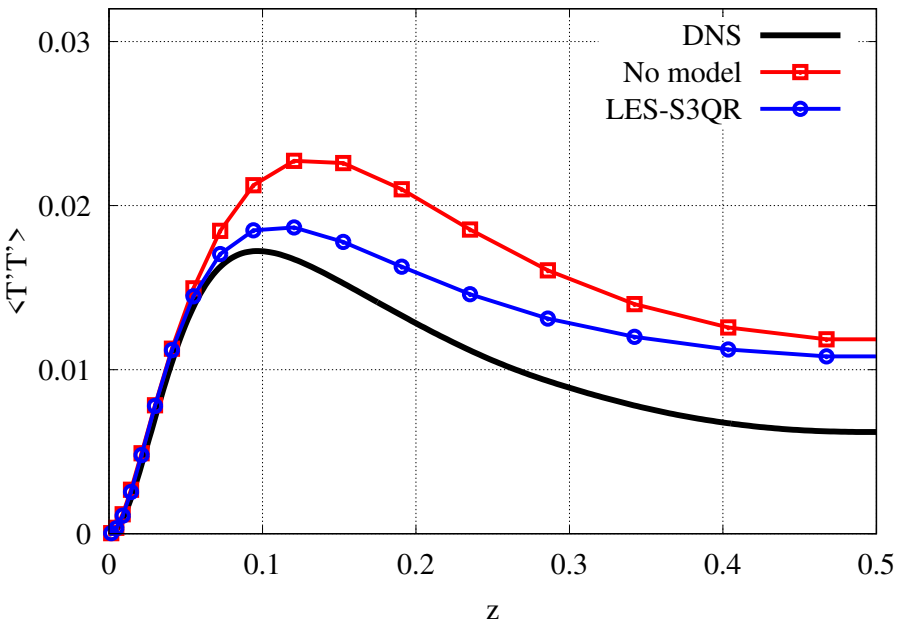
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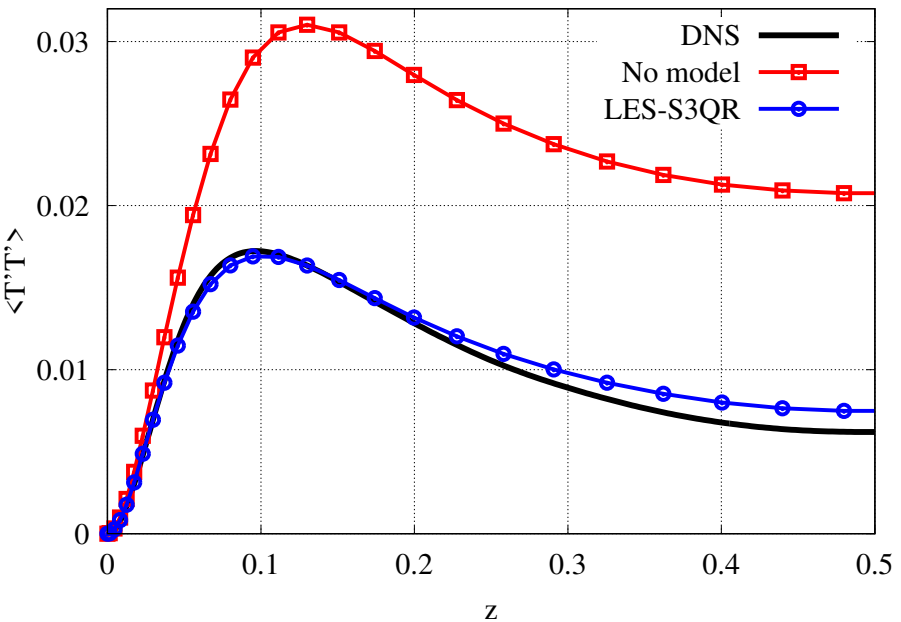
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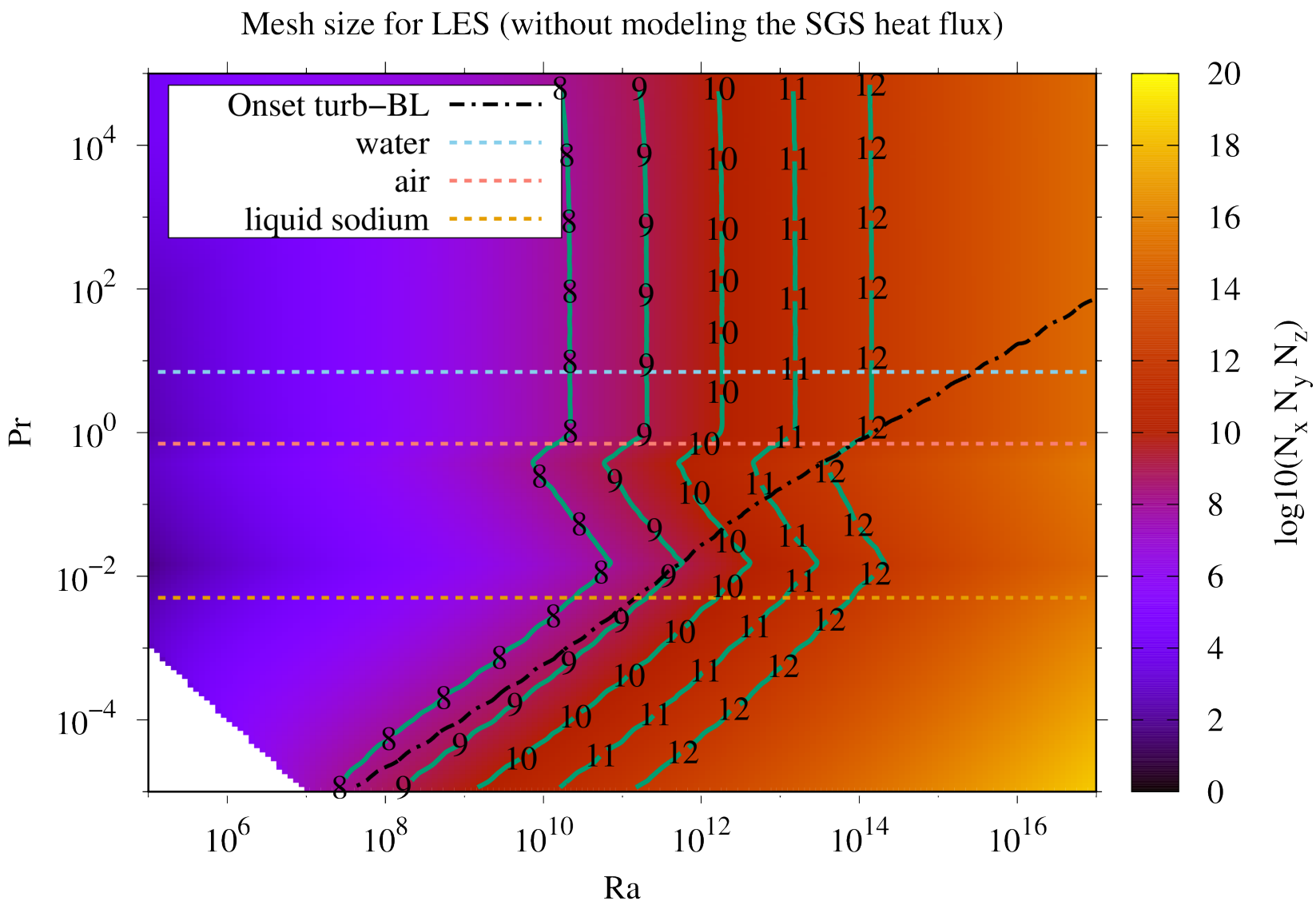


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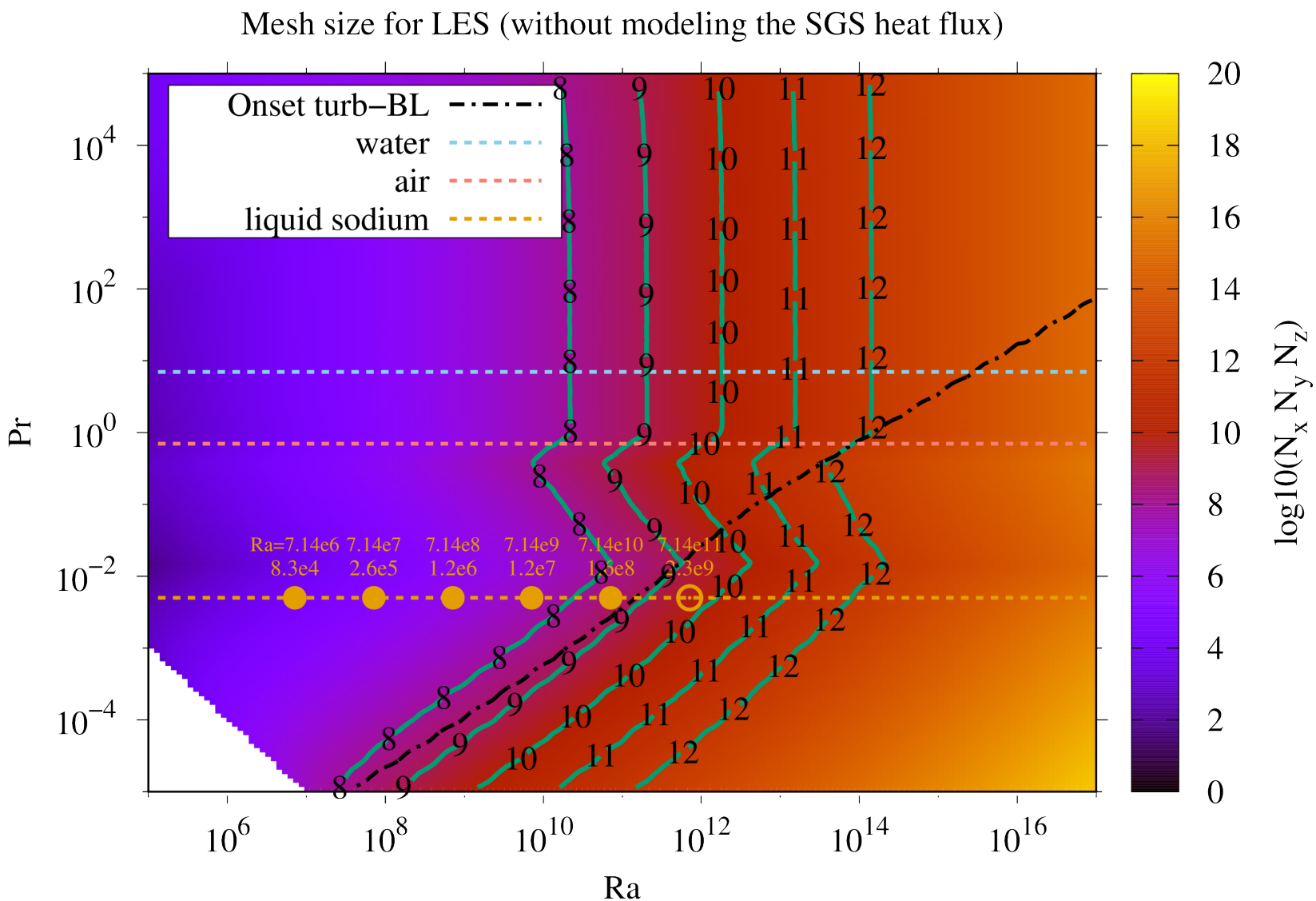


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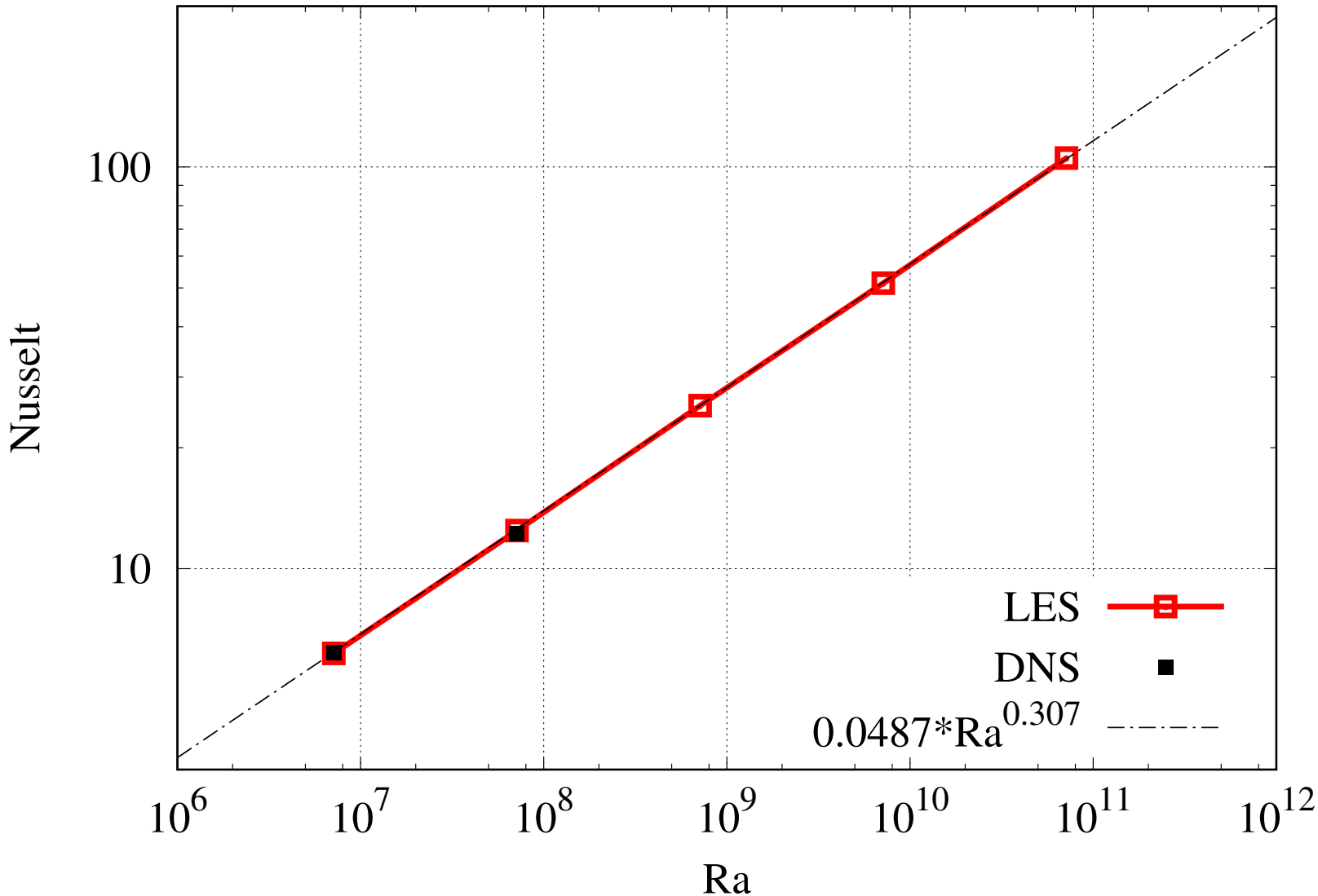
Fresh LES results (still running)



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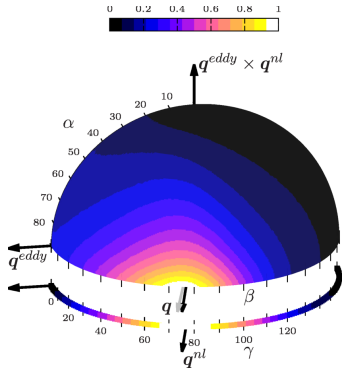


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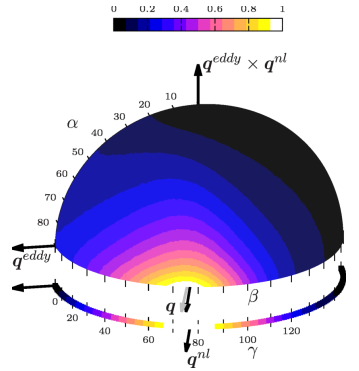
Concluding remarks

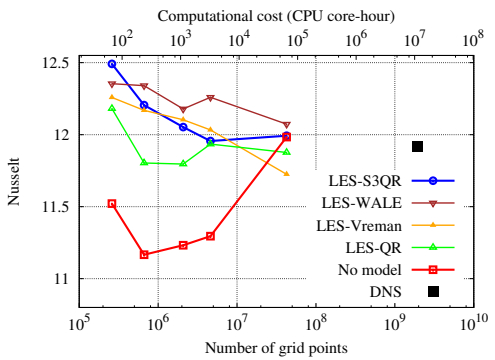
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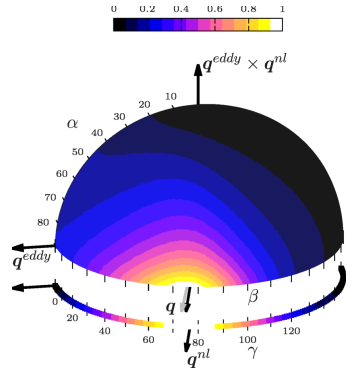
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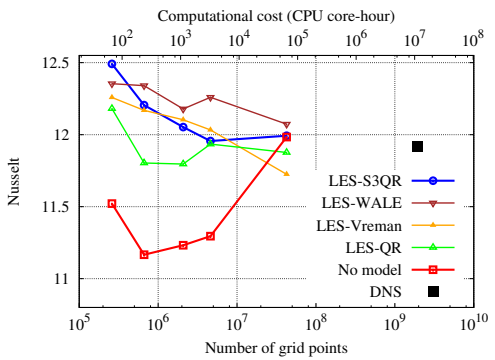


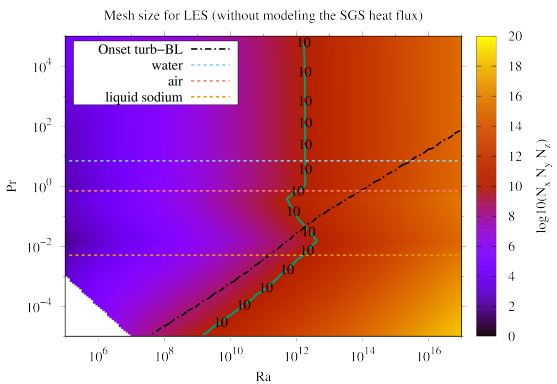


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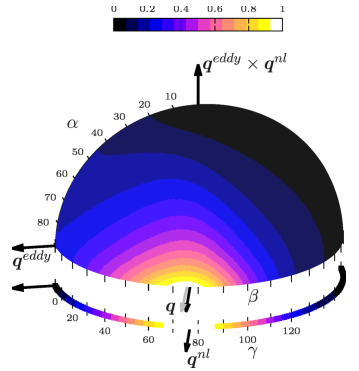


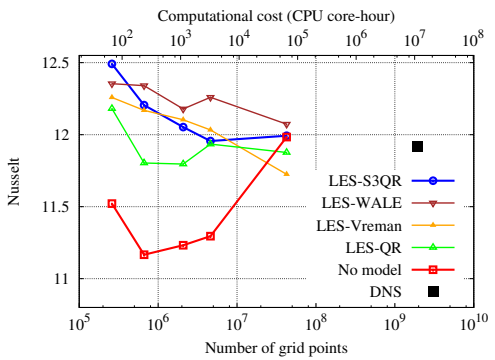
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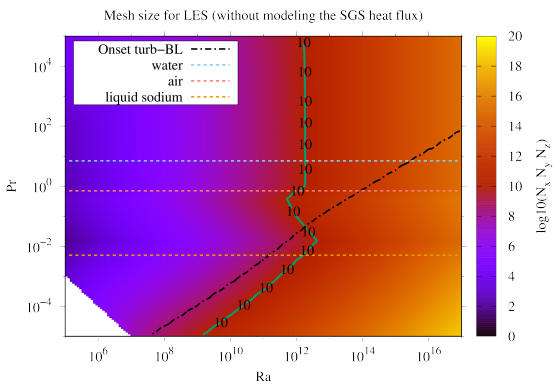
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Future research:

- LES simulations at low-Pr and very large Ra
- *A posteriori* tests at $Pr = 0.7$ for the new non-linear SGS heat flux models proposed[a]







[a] F.X.Trias, F.Dabbagh, A.Gorobets, C.Oliet. *On a proper tensor-diffusivity model for LES of buoyancy-driven turbulence*, **Flow Turbul Combust**, 105:393-414, 2020.

Thank you for your ~~virtual~~ attendance

