



Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA



Symmetry-preserving discretization of Navier-Stokes on unstructured grids: collocated vs staggered

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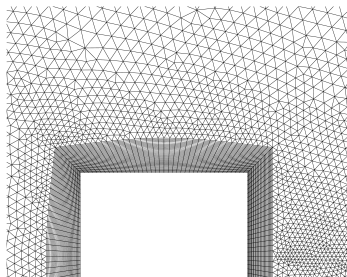
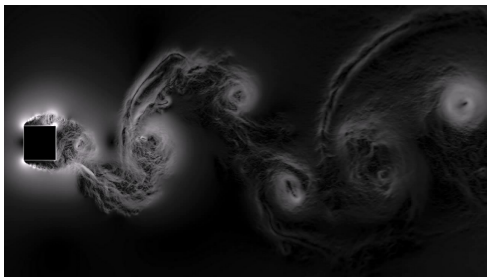
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- 2 Preserving symmetries: collocated vs staggered
- 3 Building a staggered formulation
- 4 Conclusions

Motivation

Research question:

- Can we construct numerical discretizations of the Navier-Stokes equations suitable for **complex geometries**, such that the **symmetry properties** are exactly preserved?



DNS¹ of the turbulent flow around a square cylinder at $Re = 22000$

¹F.X.Trias, A.Gorobets, A.Oliva. *Turbulent flow around a square cylinder at Reynolds number 22000: a DNS study*, **Computers&Fluids**, 123:87-98, 2015.

Motivation

Frequently used general purpose CFD codes:

- STAR-CCM+



SIEMENS

- ANSYS-FLUENT



- Code-Saturne



- OpenFOAM

Open  FOAM®



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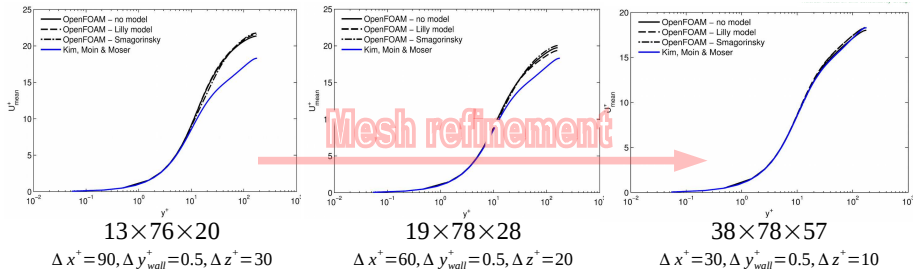


Main common characteristics of LES in such codes:

- **Unstructured finite volume** method, **collocated** grid
- Second-order spatial and temporal discretisation
- Eddy-viscosity type LES models

Motivation

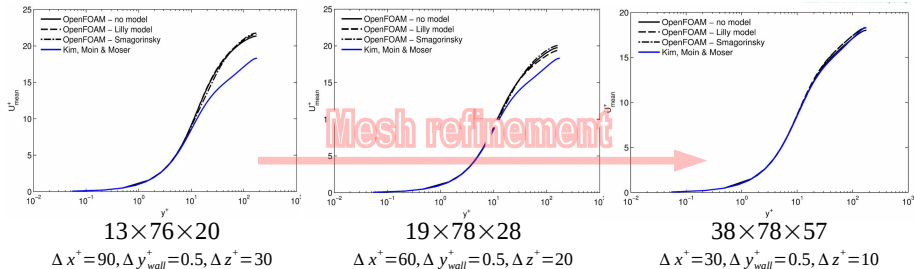
OpenFOAM® LES² results of a turbulent channel for at $Re_\tau = 180$



²E.M.J.Komen, L.H.Camilo, A.Shams, B.J.Geurts, B.Koren. *A quantification method for numerical dissipation in quasi-DNS and under-resolved DNS, and effects of numerical dissipation in quasi-DNS and under-resolved DNS of turbulent channel flows*, **Journal of Computational Physics**, 345, 565-595, 2017.

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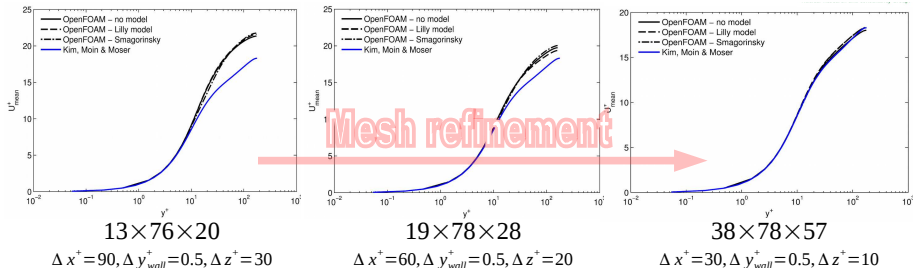


- Are LES results are merit of the SGS model?

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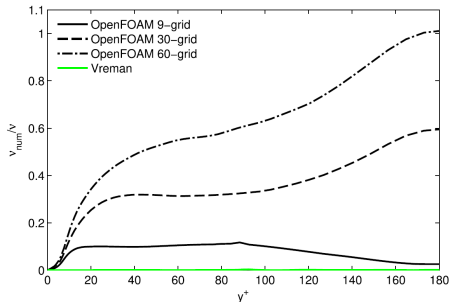
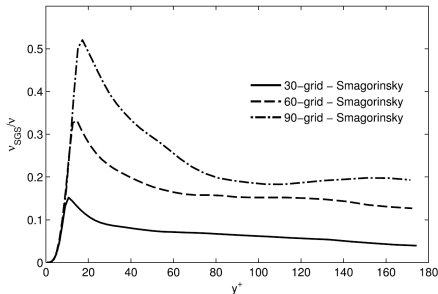


- Are LES results are merit of the SGS model? Apparently **NOT!!!** ✗

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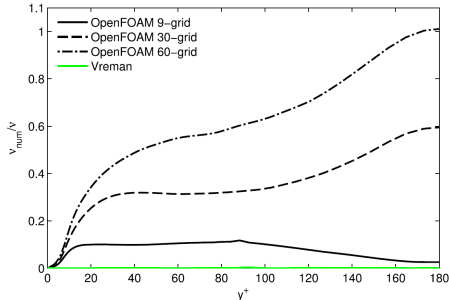
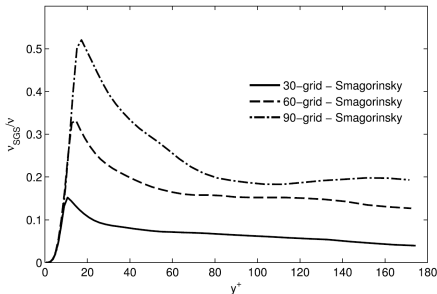


$$\nu_{num} \neq 0$$

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$$\nu_{SGS} < \nu_{num} \neq 0$$

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Symmetry-preserving discretization

Continuous

$$\frac{\partial \mathbf{u}}{\partial t} + \mathcal{C}(\mathbf{u}, \mathbf{u}) = \nu \nabla^2 \mathbf{u} - \nabla p$$

$$\nabla \cdot \mathbf{u} = 0$$

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Discrete

$$\Omega \frac{d\mathbf{u}_h}{dt} + \mathbf{C}(\mathbf{u}_h) \mathbf{u}_h = \mathbf{D} \mathbf{u}_h - \mathbf{G} \mathbf{p}_h$$

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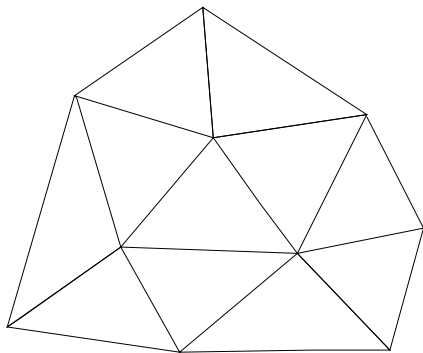
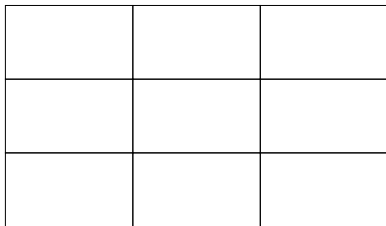
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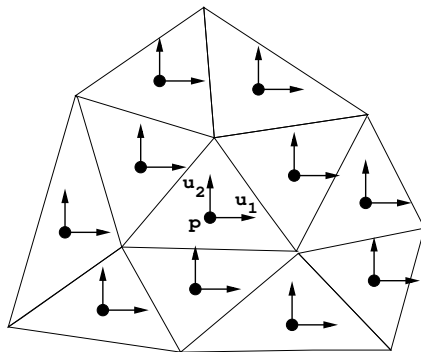
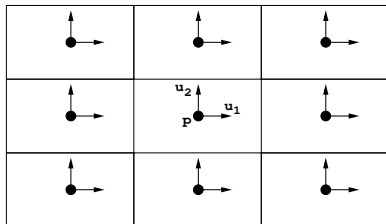
$$\mathbf{D} = \mathbf{D}^T \quad \text{def -}$$

Collocated vs staggered



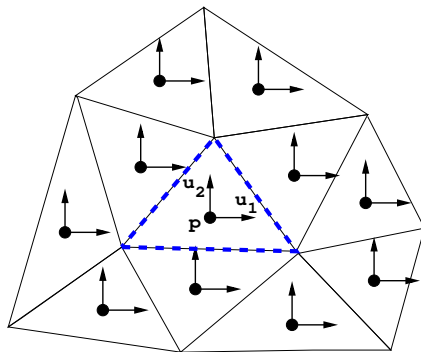
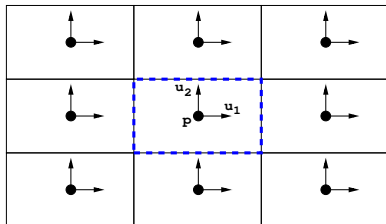
Collocated vs staggered

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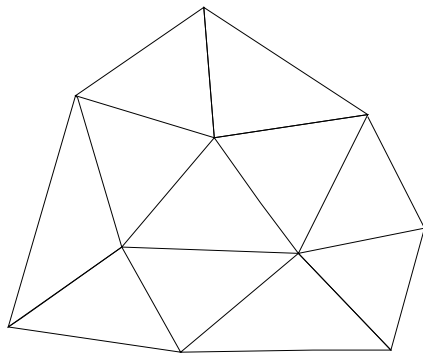
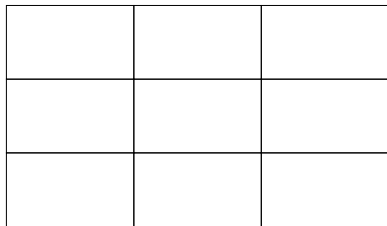


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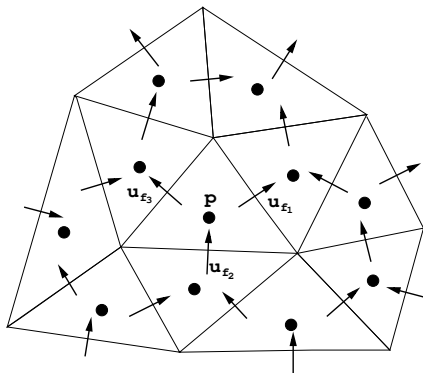
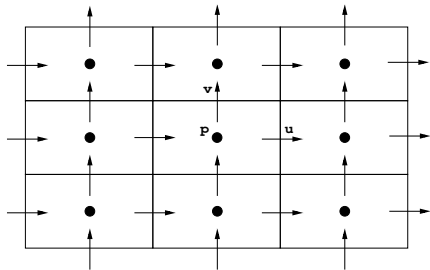


Collocated vs staggered



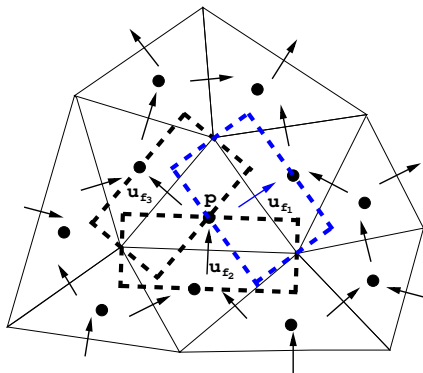
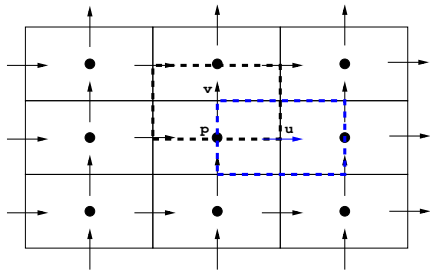
Collocated vs staggered

Staggered



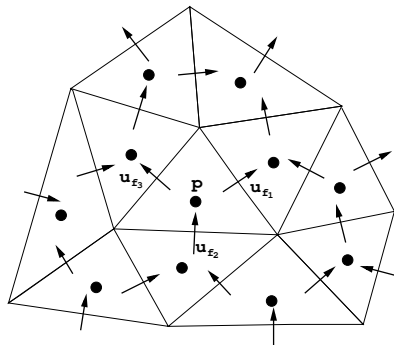
Collocated vs staggered

Staggered



Why staggered?

$$\Omega_s \frac{d\mathbf{u}_s}{dt} + \mathbf{C}(\mathbf{u}_s) \mathbf{u}_s = \mathbf{D} \mathbf{u}_s - \mathbf{G} \mathbf{p}_c; \quad \mathbf{M} \mathbf{u}_s = \mathbf{0}_c$$

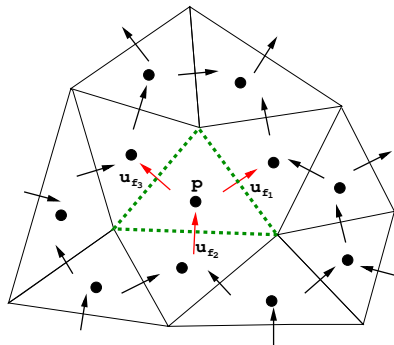


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Let's consider we have $\mathbf{u}_s$ such as

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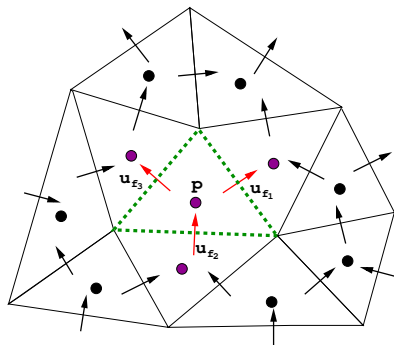
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then, we can easily project $\mathbf{u}_s$

$$\mathbf{u}_s = \mathbf{u}_s - \mathbf{G}\mathbf{p}_c$$



Why staggered?

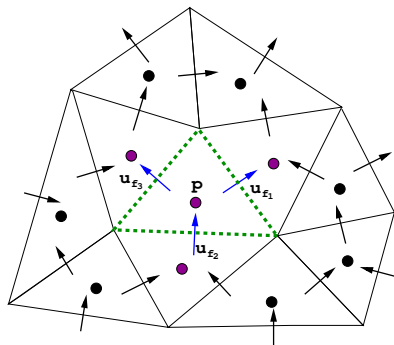
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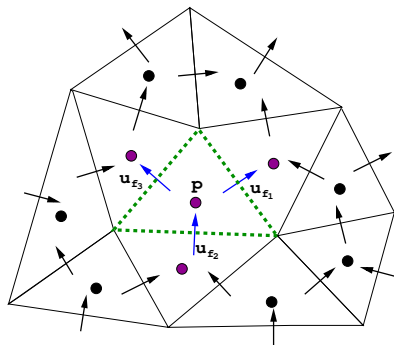
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Finally, this leads to a Poisson eq.

$$\mathbf{M}\mathbf{G}\mathbf{p}_c = \mathbf{M}\mathbf{u}_s$$



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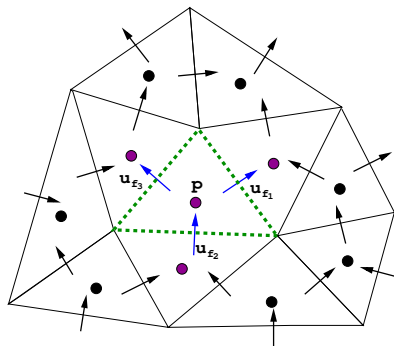
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Finally, this leads to a Poisson eq.

$$\mathbf{M}\mathbf{G}\mathbf{p}_c = \mathbf{M}\mathbf{u}_s$$

$$\text{If } \Omega_s \mathbf{G} = -\mathbf{M}^T$$

$$\langle \nabla \cdot \mathbf{a}, \varphi \rangle = -\langle \mathbf{a}, \nabla \varphi \rangle$$



Why staggered? Everything seems to be in the right place!

$$\Omega_s \frac{d\mathbf{u}_s}{dt} + \mathbf{C}(\mathbf{u}_s) \mathbf{u}_s = \mathbf{D}\mathbf{u}_s - \mathbf{G}\mathbf{p}_c; \quad \mathbf{M}\mathbf{u}_s = \mathbf{0}_c$$

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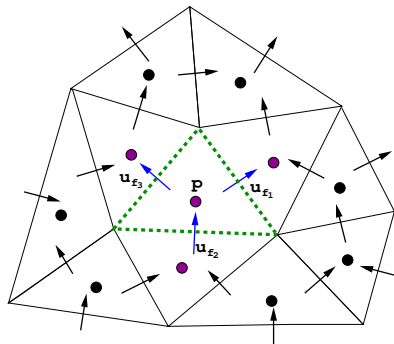
$$\mathbf{M}\mathbf{u}_s \neq \mathbf{0}_c$$

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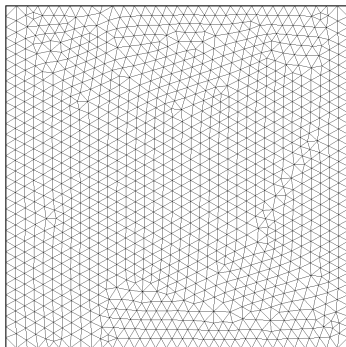


$$\text{If } \Omega_s \mathbf{G} = -\mathbf{M}^T \implies \langle \mathbf{u}_s, \mathbf{G}\mathbf{p}_c \rangle_h = \mathbf{u}_s^T \Omega_s \mathbf{G}\mathbf{p}_c = -(\mathbf{M}\mathbf{u}_s)^T \mathbf{p}_c = 0$$

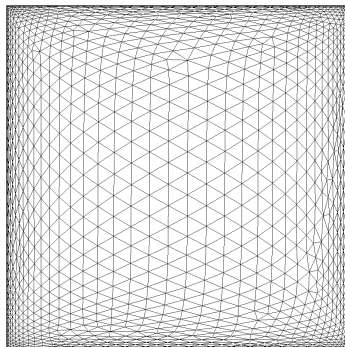
$$\langle \nabla \cdot \mathbf{a}, \varphi \rangle = -\langle \mathbf{a}, \nabla \varphi \rangle \implies \langle \mathbf{u}, \nabla p \rangle = -\langle \nabla \cdot \mathbf{u}, p \rangle = 0$$

But is this discrete Laplacian accurate?

Without stretching



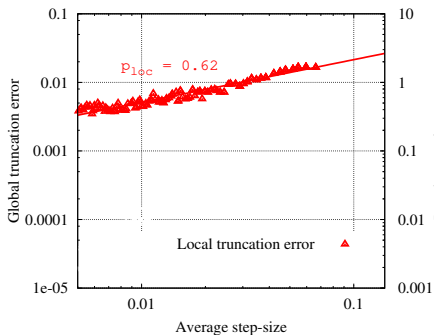
With stretching



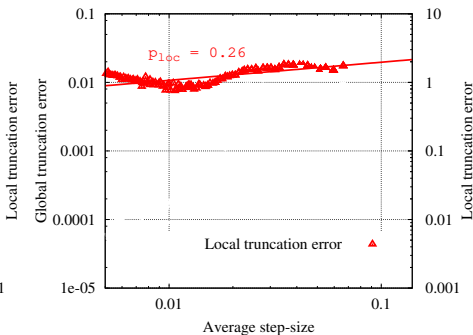
$$\nabla^2 \varphi = f(x, y) \quad \text{with } f(x, y) = \nabla^2(k^{-2} \sin(kx) \sin(ky)) \text{ and } k = 25\pi$$

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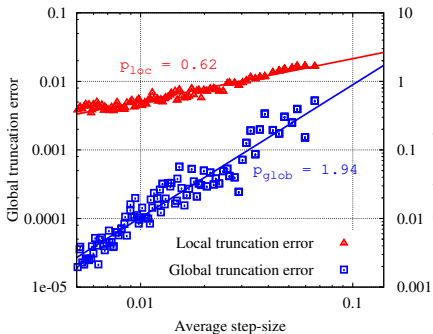


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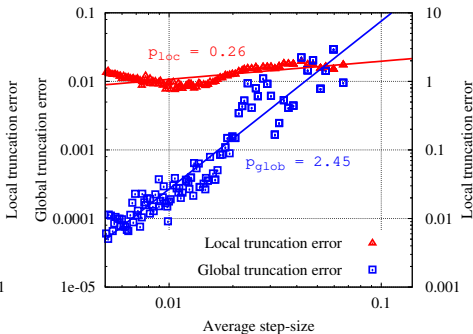
But is this discrete Laplacian accurate?

Yes, even for distorted unstructured meshes! And symmetries are preserved!

Without stretching



With stretching



$$\nabla^2 \varphi = f(x, y) \quad \text{with } f(x, y) = \nabla^2 (k^{-2} \sin(kx) \sin(ky)) \text{ and } k = 25\pi$$

Then, why collocated arrangements are so popular?

- STAR-CCM+



SIEMENS

- ANSYS-FLUENT



- Code-Saturne



- OpenFOAM

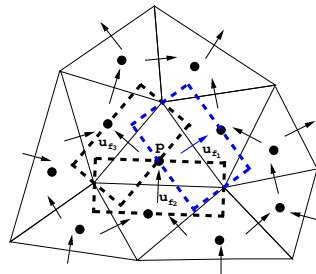
OpenFOAM®



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In staggered meshes

- $p - \mathbf{u}_s$ coupling is naturally solved ✓
- $\mathbf{C}(\mathbf{u}_s)$ and $\mathbf{D}$ difficult to discretize ✗



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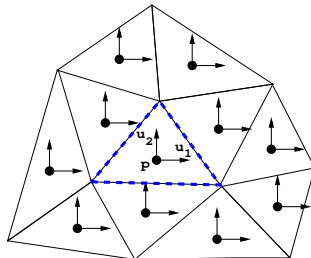
OpenFOAM®



$$\Omega_c \frac{d\mathbf{u}_c}{dt} + \mathbf{C}(\mathbf{u}_s) \mathbf{u}_c = \mathbf{D} \mathbf{u}_c - \mathbf{G}_c \mathbf{p}_c; \quad \mathbf{M}_c \mathbf{u}_c = \mathbf{0}_c$$

In collocated meshes

- $p-\mathbf{u}_c$ coupling is cumbersome **X**
- $\mathbf{C}(\mathbf{u}_s)$ and $\mathbf{D}$ easy to discretize **✓**
- Cheaper, less memory, ... **✓**



Then, why collocated arrangements are so popular?

Everything is easy except the pressure-velocity coupling...

- STAR-CCM+



SIEMENS



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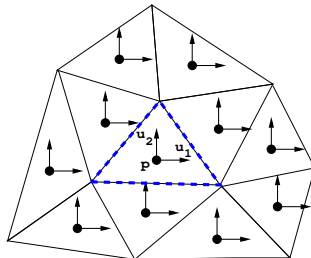
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In collocated meshes

- $p-\mathbf{u}_c$ coupling is cumbersome $\times$
- $\mathbf{C}(\mathbf{u}_s)$ and $\mathbf{D}$ easy to discretize $\checkmark$
- Cheaper, less memory, ... $\checkmark$



Pressure-velocity coupling on staggered grids

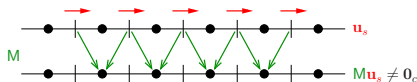
Works perfectly!



u_s

Pressure-velocity coupling on staggered grids

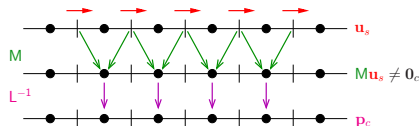
Works perfectly!



$$M u_s$$

Pressure-velocity coupling on staggered grids

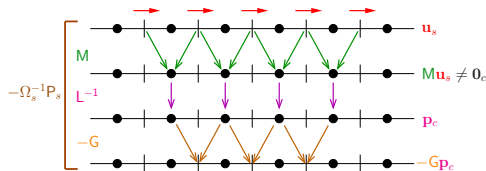
Works perfectly!



$$\underbrace{L^{-1} M u_s}_{p_c}$$

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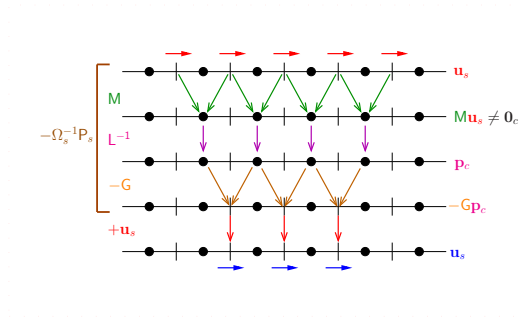
Works perfectly!



$$-\underbrace{G L^{-1} M}_{p_c} u_s$$

Pressure-velocity coupling on staggered grids

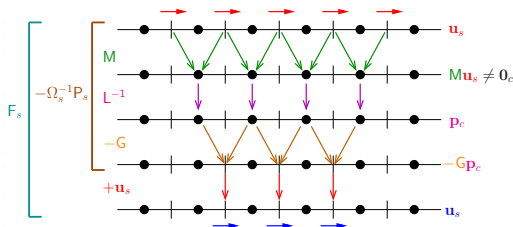
Works perfectly!



$$u_s = u_s - \underbrace{GL^{-1}Mu_s}_{p_c}$$

Pressure-velocity coupling on staggered grids

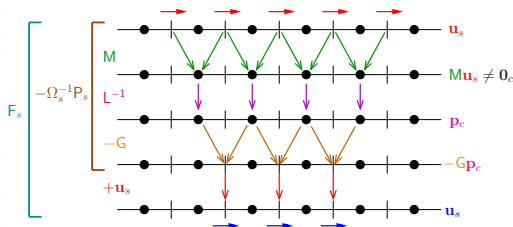
Works perfectly!



$$u_s = u_s - \underbrace{GL^{-1}Mu_s}_{p_c} = (I - \Omega_s^{-1}P_s)u_s = F_s u_s$$

Pressure-velocity coupling on staggered grids

Works perfectly!

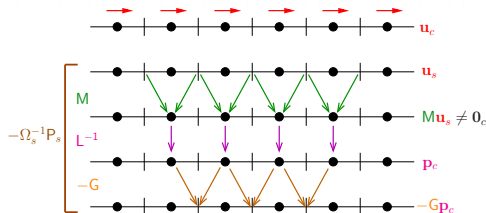


$$\mathbf{u}_s = \mathbf{u}_s - \underbrace{\mathbf{G} \mathbf{L}^{-1} \mathbf{M} \mathbf{u}_s}_{p_c} = (\mathbf{I} - \Omega_s^{-1} \mathbf{P}_s) \mathbf{u}_s = \mathbf{F}_s \mathbf{u}_s$$

$$\mathbf{M} \mathbf{u}_s = \mathbf{M} \mathbf{u}_s - \underbrace{\mathbf{M} \mathbf{G}}_{\mathbf{L}} \mathbf{L}^{-1} \mathbf{M} \mathbf{u}_s = \mathbf{0}_c$$

Pressure-velocity coupling on collocated grids

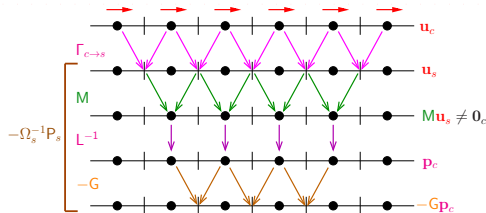
A vicious circle that cannot be broken...



u_c

Pressure-velocity coupling on collocated grids

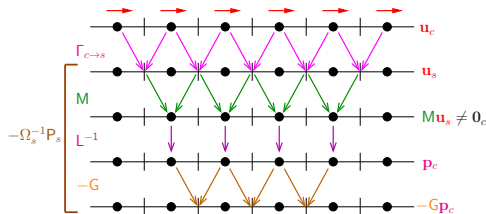
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$$\Gamma_{c \rightarrow s} u_c$$

Pressure-velocity coupling on collocated grids

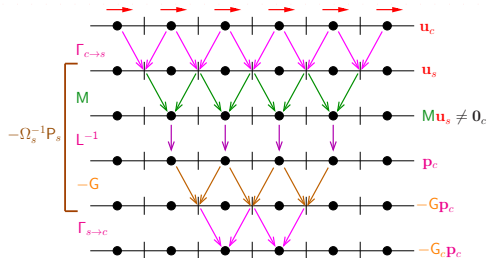
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$$GL^{-1}M\Gamma_{c \rightarrow s}u_c$$

Pressure-velocity coupling on collocated grids

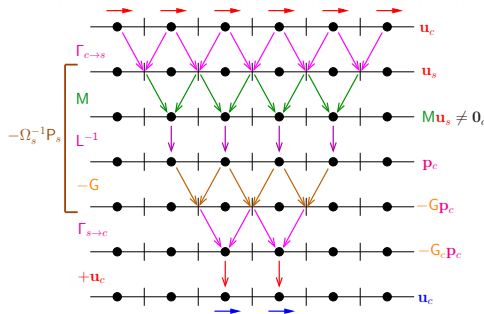
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$$-\Gamma_{s \rightarrow c} G L^{-1} M \Gamma_{c \rightarrow s} u_c$$

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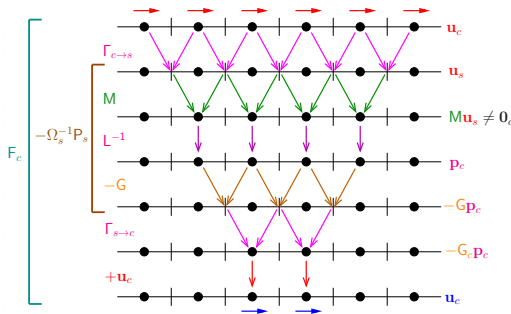
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$$u_c = u_c - \Gamma_{s \rightarrow c} G L^{-1} M \Gamma_{c \rightarrow s} u_c$$

Pressure-velocity coupling on collocated grids

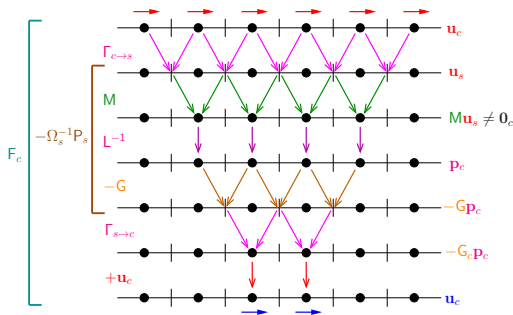
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$$u_c = u_c - \Gamma_{s \rightarrow c} G L^{-1} M \Gamma_{c \rightarrow s} u_c = F_c u_c$$

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...



$$\mathbf{u}_c = \mathbf{u}_c - \Gamma_{s \rightarrow c} \mathbf{G} \mathbf{L}^{-1} \mathbf{M} \Gamma_{c \rightarrow s} \mathbf{u}_c = \mathbf{F}_c \mathbf{u}_c$$

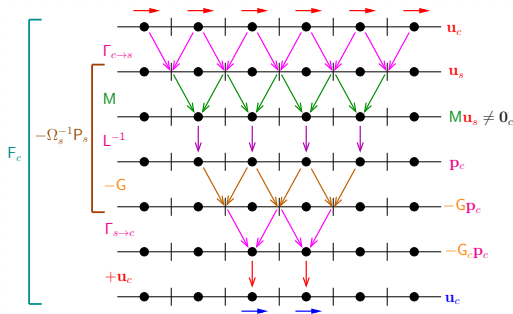
To preserve symmetry we impose $\Gamma_{s \rightarrow c} = \Omega_c^{-1} \Gamma_{c \rightarrow s}^T \Omega_s$. This leads to

$$\mathbf{M} \Gamma_{c \rightarrow s} \mathbf{u}_c = \mathbf{M} \Gamma_{c \rightarrow s} \mathbf{u}_c - \mathbf{L}_c \mathbf{L}^{-1} \mathbf{M} \Gamma_{c \rightarrow s} \mathbf{u}_c \approx \mathbf{0}_c \quad \times$$

where $\mathbf{L}_c = -\mathbf{M} \Gamma_{c \rightarrow s} \Omega_c^{-1} \Gamma_{c \rightarrow s}^T \mathbf{M}$ (wide-stencil discrete Laplacian).

Pressure-velocity coupling on collocated grids

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Moreover, **contribution to kinetic enegy**: $p_c(L - L_c)p_c \neq 0$ **x**

Pressure-velocity coupling on collocated grids

A vicious circle that cannot be broken...

In summary⁴:

- Mass: $M\Gamma_{c \rightarrow s} \mathbf{u}_c = M\Gamma_{c \rightarrow s} \mathbf{u}_c - L_c L_c^{-1} M\Gamma_{c \rightarrow s} \mathbf{u}_c \approx \mathbf{0}_c \quad \mathbf{x}$
- Energy: $\mathbf{p}_c (L - L_c) \mathbf{p}_c \neq 0 \quad \mathbf{x}$

⁴F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen.
Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured grids, **Journal of Computational Physics**, 258 (1): 246-267, 2014.

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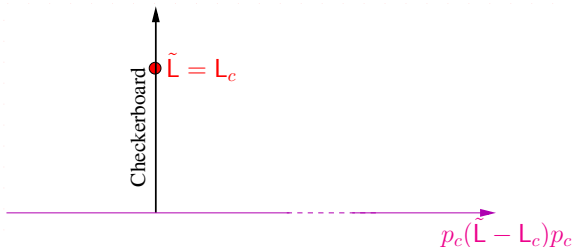
⁴F.X.Trias, O.Lehmkuhl, A.Oliva, C.D.Pérez-Segarra, R.W.C.P.Verstappen.
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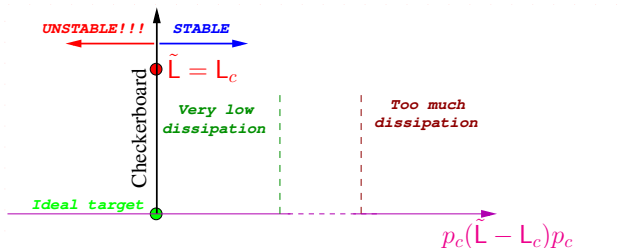
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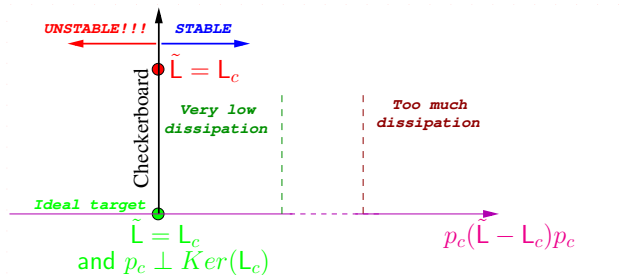
⁴D.Santos, J.Muela, N.Valle, F.X.Trias. *On the interpolation problem for the Poisson equation on collocated meshes*, **ECCOMAS2020**. Don't miss it!

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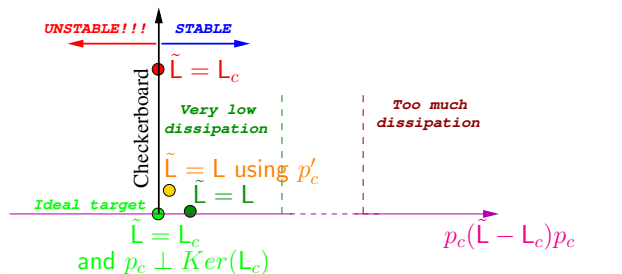
⁴Shashank, J.Larsson, G.laccarino. *A co-located incompressible Navier-Stokes solver with exact mass, momentum and kinetic energy conservation in the inviscid limit*, *Journal of Computational Physics*, 229: 4425-4430,2010.

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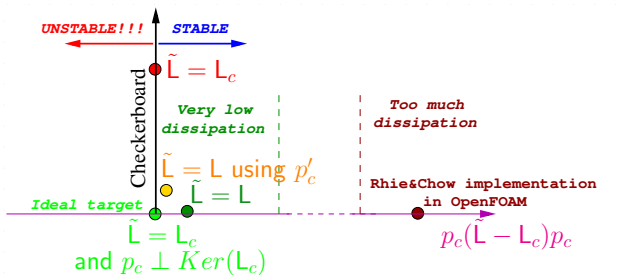
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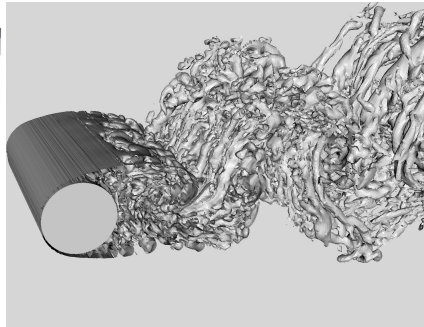
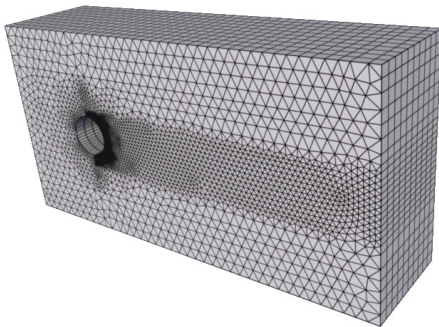


⁴E.Komen, J.A.Hopman, E.M.A.Frederix, F.X.Trias, R.W.C.P.Verstappen.
Energy-preserving discretisation for LES/DNS with unstructured collocated grids in OpenFOAM, ECCOMAS2020. Don't miss it!

Pressure-velocity coupling on collocated grids

Examples of simulations

Despite these inherent limitations, symmetry-preserving collocated formulation has been successfully used for DNS/LES simulations⁵:

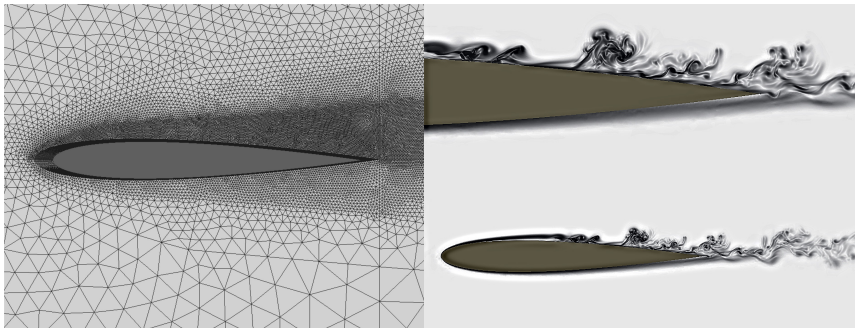


⁵R.Borrell, O.Lehmkuhl, F.X.Trias, A.Oliva. *Parallel Direct Poisson solver for discretizations with one Fourier diagonalizable direction*. **Journal of Computational Physics**, 230:4723-4741, 2011.

Pressure-velocity coupling on collocated grids

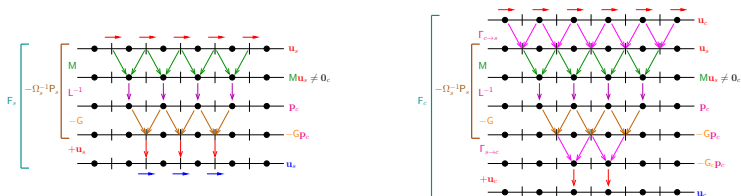
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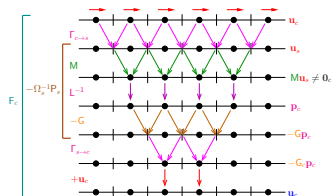
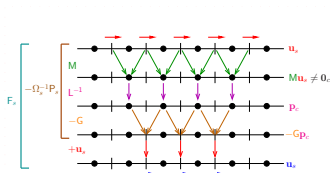
⁵F.X.Trias and O.Lehmkuhl. *A self-adaptive strategy for the time-integration of Navier-Stokes equations*. **Numerical Heat Transfer, part B**, 60(2):116-134, 2011.

Are staggered and collocated so different at the end?



$$\text{Collocated: } \underbrace{u_c^{n+1}}_{F_c} = \underbrace{(I_c - \Gamma_{s \rightarrow c} \Omega_s^{-1} P_s \Gamma_{c \rightarrow s})}_{T_c} \underbrace{\left[I_c + \partial_t^c \right] u_c^n}_{T_c} = \underbrace{F_c T_c}_{NS_c} u_c^n$$

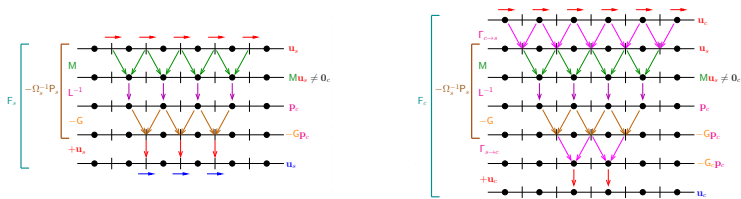
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$$\text{Naïve collocated: } \mathbf{u}_c^{n+1} = \underbrace{\Gamma_{s \rightarrow c} F_s \Gamma_{c \rightarrow s} T_c}_{\tilde{NS}_c} \mathbf{u}_c^n$$

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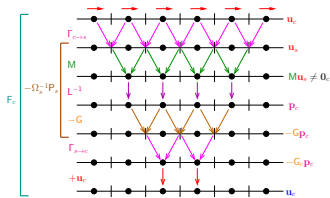
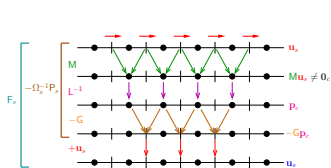


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$$\text{Naïve staggered: } \mathbf{u}_s^{n+1} = \underbrace{F_s \Gamma_{c \rightarrow s} T_c \Gamma_{s \rightarrow c}}_{\widetilde{NS}_s} \mathbf{u}_s^n$$

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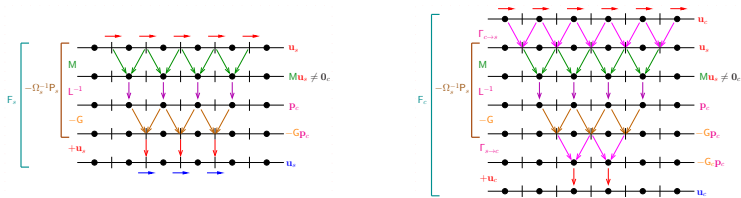
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$$\Gamma_{s \rightarrow c} (\widetilde{NS}_s)^n = (\widetilde{NS}_c)^n \Gamma_{c \rightarrow s}$$

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$$\text{Staggered: } \mathbf{u}_s^{n+1} = \underbrace{(I_s - \Omega_s^{-1} P_s)}_{F_s} \underbrace{[I_s + \Gamma_{c \rightarrow s} \partial_t^c \Gamma_{s \rightarrow c}]}_{T_s} \mathbf{u}_s^n$$

Can we have a staggered formulation based only on collocated operators?

Then, it could be easily implemented in existing collocated codes such as OpenFOAM

$$\text{Staggered: } \mathbf{u}_s^{n+1} = \underbrace{(\mathbf{I}_s - \underbrace{\Omega_s^{-1}}_{\mathbf{F}_s} \mathbf{P}_s)}_{\mathbf{F}_s} \underbrace{\left[\mathbf{I}_s + \underbrace{\Gamma_{c \rightarrow s}}_{\mathbf{T}_s} \overbrace{\partial_t^c \Gamma_{s \rightarrow c}}^{\mathbf{u}_s^p} \right]}_{\mathbf{T}_s} \mathbf{u}_s$$

Similar approaches have been proposed in the literature before^{6,7,8,9,10}.

⁶ B.Perot. *Conservative properties of unstructured staggered meshes schemes*. **Journal of Comp. Physics**, 159: 58-89, 2000

⁷ X.Zhang, D.Schmidt, B.Perot. *Accuracy and conservation properties of a three-dimensional unstructured staggered mesh scheme for fluid dynamics*. **Journal of Computational Physics**, 175: 764-791, 2002.

⁸ K.Mahesh, G.Constantinescu, P.Moin. *A numerical method for large-eddy simulation in complex geometries*. **Journal of Computational Physics**, 197: 215-240, 2004.

⁹ J.E.Hicken, F.E.Ham, J.Militzer, M.Koksal. *A shift transformation for fully conservative methods: turbulence simulation on complex, unstructured grids*. **Journal of Computational Physics**, 208:704-734, 2005.

¹⁰ L.Jofre, O.Lehmkuhl, J.Ventosa, F.X.Trias, A.Oliva. *Conservation properties of unstructured finite-volume mesh schemes for the Navier-Stokes equations*. **Numerical Heat Transfer, Part B**, 65:1-27, 2014.

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Similar approaches have been proposed in the literature before^{6,7,8,9,10}.

Research question: then, why at the end collocated approach seems to be the winner?

- STAR-CCM+



SIEMENS

- ANSYS-FLUENT



- Code-Saturne



- OpenFOAM

OpenFOAM®



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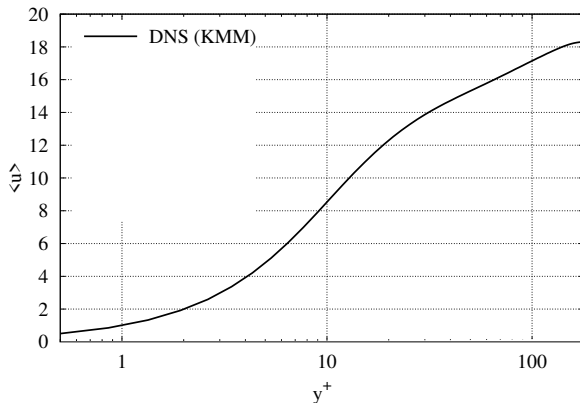
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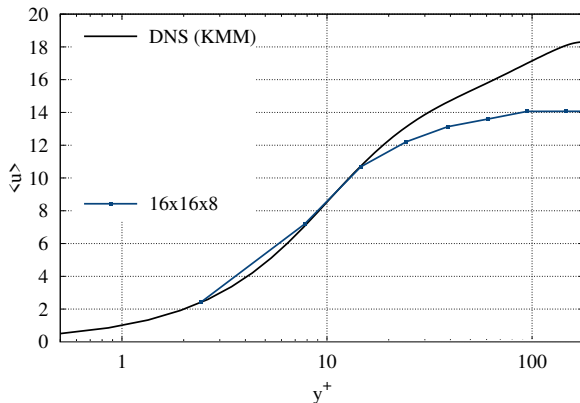
Results for a turbulent channel flow at $Re_\tau = 180$

Staggered $C_s^{0'}$:
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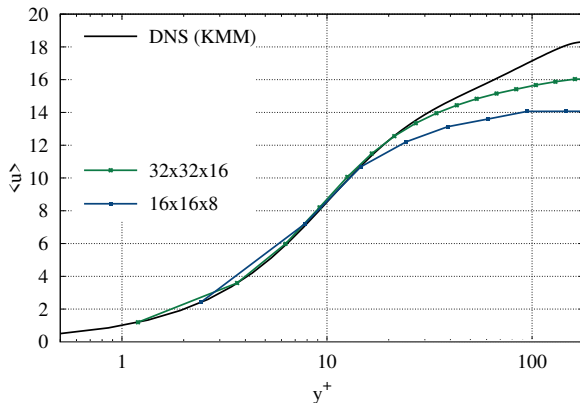
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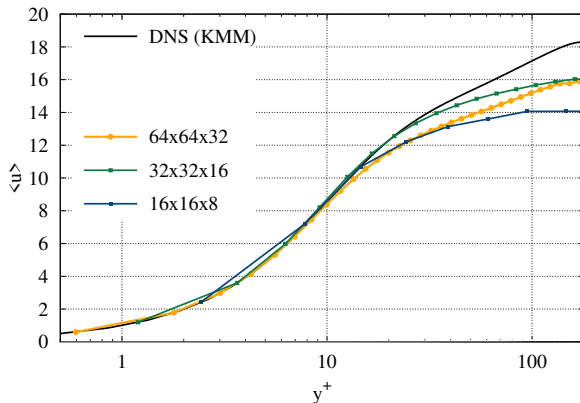
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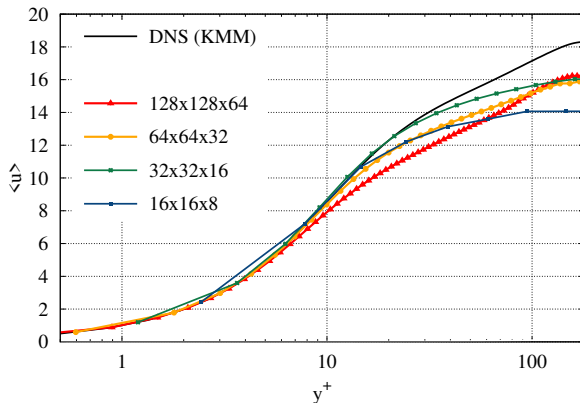
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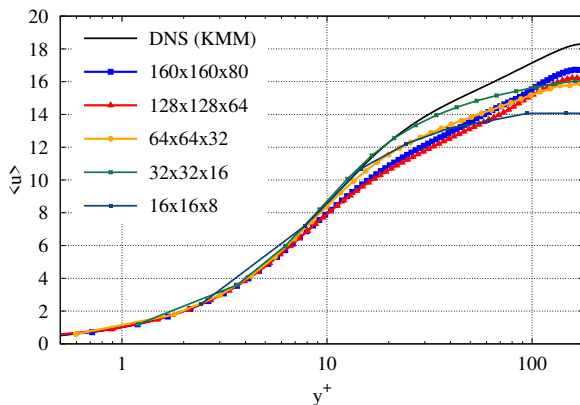
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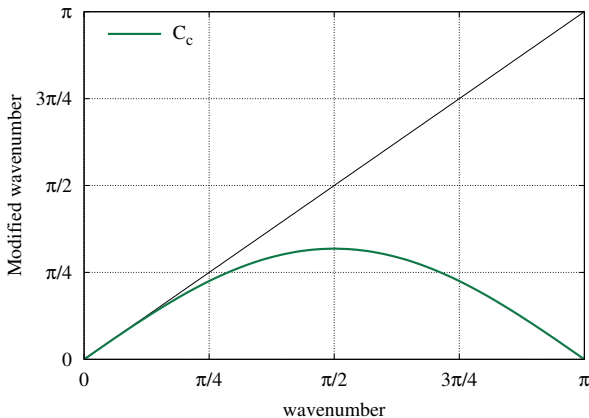
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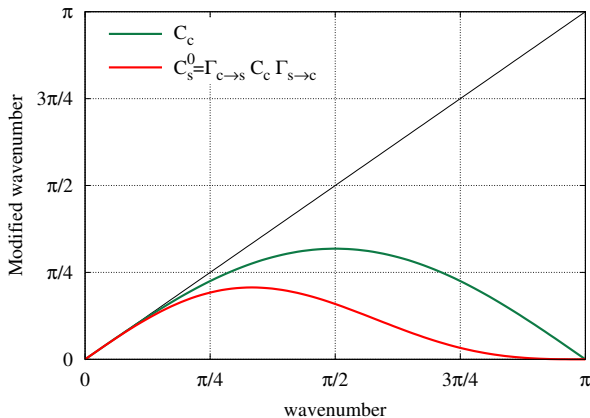
Dispersion errors analysis

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Dispersion errors analysis

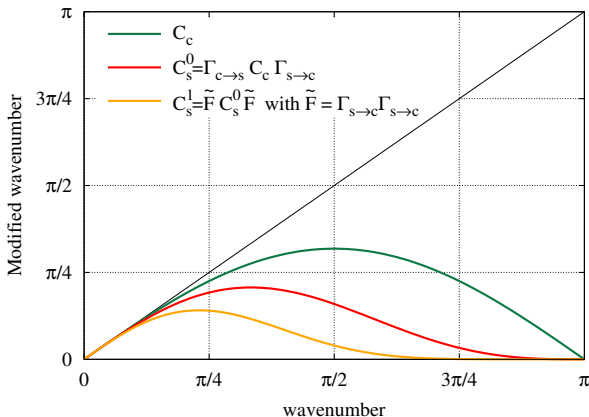
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Dispersion errors analysis

$$\text{Staggered } C_s^1: \quad \underbrace{u_s^{n+1}}_{F_s} = \underbrace{(I_s - \Omega_s^{-1} P_s)}_{F_s} \underbrace{\left[I_s + \tilde{\Gamma} \Gamma_{c \rightarrow s} \partial_t^c \Gamma_{s \rightarrow c} \tilde{\Gamma} \right]}_{T_s} \underbrace{u_s^p}_{u_s^p}$$

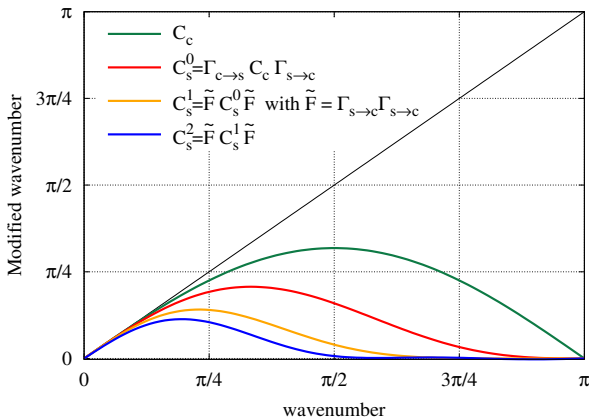
Filter: $\tilde{\Gamma} = \Gamma_{s \rightarrow c} \Gamma_{c \rightarrow s}$ ($\tilde{\Gamma} = \tilde{\Gamma}^T$ and positive semi-definite)



Dispersion errors analysis

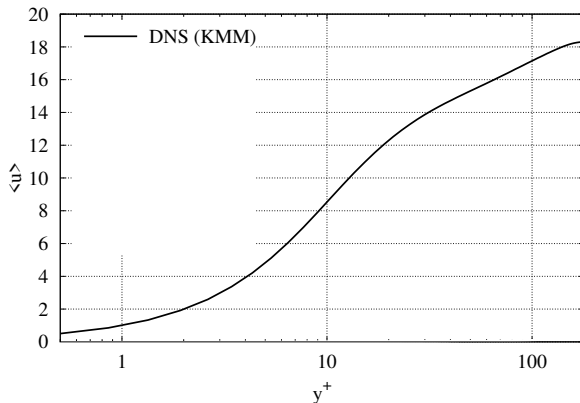
$$\text{Staggered } C_s^2: \quad \underbrace{u_s^{n+1}}_{F_s} = \underbrace{(I_s - \Omega_s^{-1} P_s)}_{F_s} \underbrace{\left[I_s + \tilde{F} \tilde{F} \Gamma_{c \rightarrow s} \partial_t^c \Gamma_{s \rightarrow c} \tilde{F} \tilde{F} \right]}_{T_s} \underbrace{u_s^p}_{u_s^p}$$

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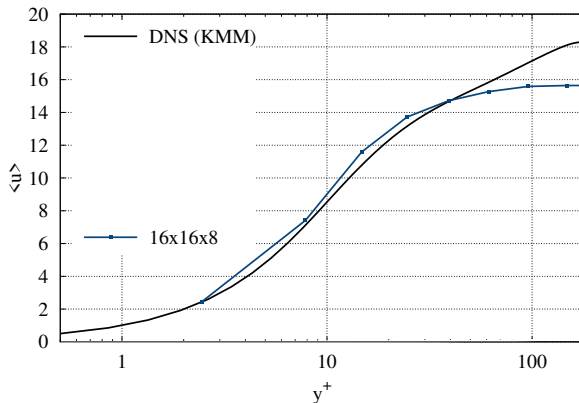
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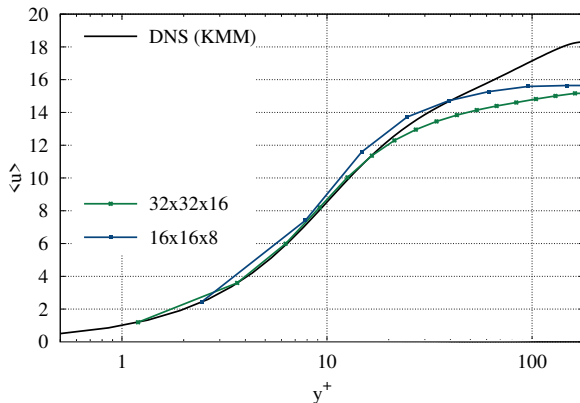
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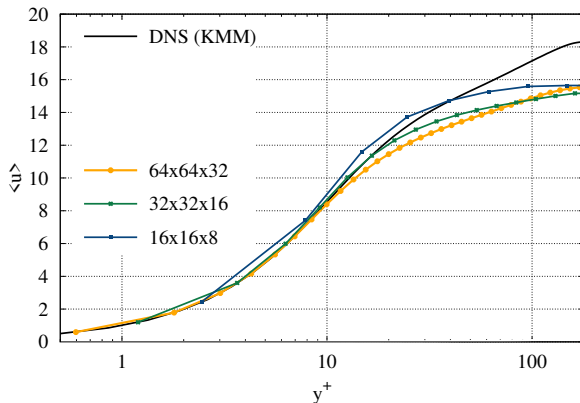
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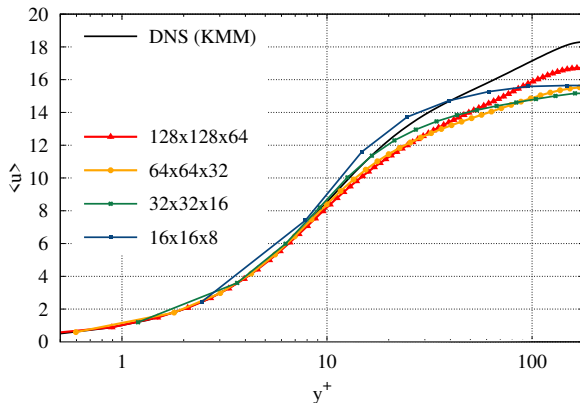
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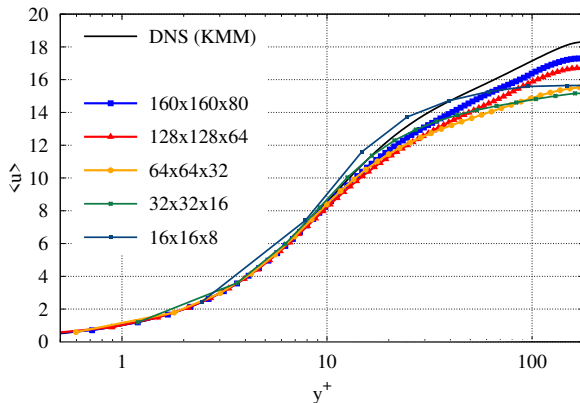
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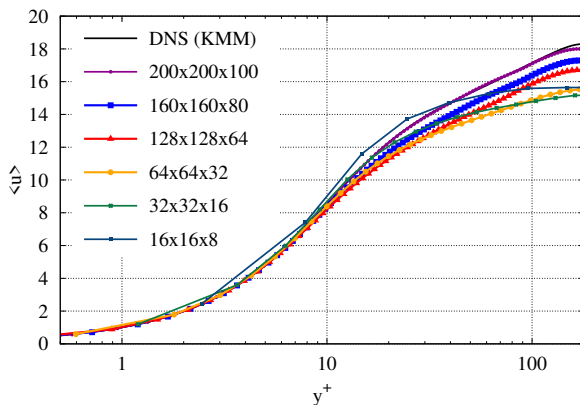
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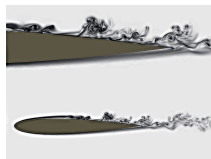
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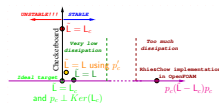
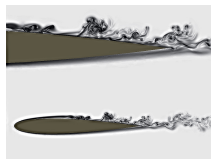
Concluding remarks

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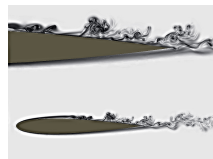
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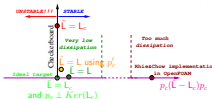
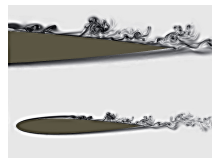
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Future research:

- Complete the analysis for higher Re_τ (running now...)
- Test for complex geometries using unstructured meshes (on-going)

Thank you for your virtual attendance

