ON THE INTERPOLATION PROBLEM FOR THE POISSON EQUATION ON COLLOCATED MESHES

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Finite-volume collocated discretizations on unstructured meshes is the solution adopted for most of the general-purpose CFD codes such as ANSYS-FLUENT, OpenFOAM, etc. Despite the intrinsic errors due to the improper pressure gradient formulation, this approach is usually preferred over a staggered one due to its simple form. In this context, a fully-conservative discretization method for general unstructured grids was proposed in [1]: it exactly preserves the symmetries of the underlying differential operators on a collocated mesh. Likewise other collocated codes, to suppress the well-known checkerboard problem the Poisson equation is solved using a compact stencil. Using the same notation than in [1], this reads

$$ L p_c = M u^p_c $$

(1)

where $L = -\Omega^{-1}_c M^T$ is the Laplacian operator, $p_c$ is the cell-centered pressure field, $u^p_c$ is a face-normal velocity and $\Omega_c$ is a diagonal matrix that contains the staggered control volumes. For staggered velocity fields, the projection onto a divergence-free space is a well-posed problem. This is not the case for collocated velocity fields. Namely, cell-centered velocity field, $u^p_c$, needs to be interpolated to the faces, $u^p_s = \Gamma_{c\rightarrow s} u^p_c$ using a cell-to-face interpolation, $\Gamma_{c\rightarrow s}$. Then, the staggered gradient, $G p_c = -\Omega^{-1}_c M^T$, of the pressure field obtained by solving Eq.(1) must be interpolated back to the cells. Namely, the overall procedure can be compactly written as follows

$$ u^{n+1}_c = (1 + \Omega^{-1}_c \Gamma_{c\rightarrow s}^T M^T L^{-1} \Gamma_{c\rightarrow s}) u^n_c. $$

(2)

The new cell-centered velocity field will not be exactly incompressible, $M p^{n+1}_c \approx 0$, and the overall procedure will inevitably introduce some artificial dissipation. Apart from this well-known drawbacks of using collocated formulations, instability issues may also appear for highly distorted meshes. Namely, let us consider that we recursively apply the pseudo-projection given in Eq.(2). Then, we obtain

$$ L p^{n+1}_c = M u^n_c + (L - L_c) p^n_c, $$

(3)

where $L_c \equiv -M \Gamma_{c\rightarrow s} \Omega^{-1}_c \Gamma_{c\rightarrow s}^T M^T$ is the non-compact Laplacian operator. This can be viewed like a stationary iterative solver. The stability of this process will depend on the eigenvalues of $(L - L_c)$, which subsequently depend on the interpolation operators. This will be carefully analysed and results for general unstructured grids will be presented.

REFERENCES