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DLES 15 – ERCOFTAC Workshop Direct & Large Eddy Simulation

Towards a numerical dissipation free symmetry-preserving collocated method for turbulent flows

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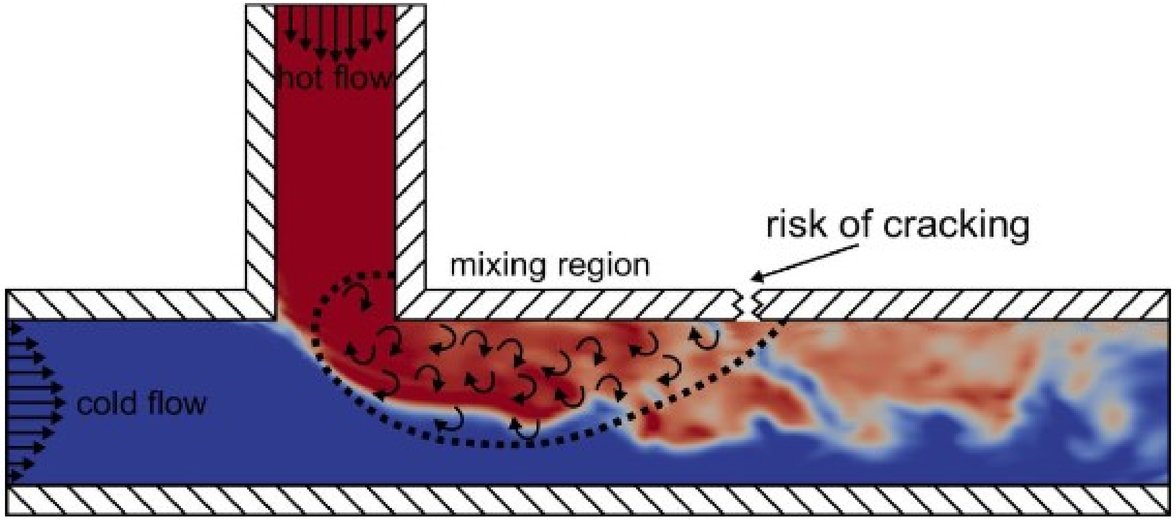
Content

- Definitions and motivation of the work
- Numerical tests
- Conclusions and next steps

Motivation of the work

- Flow mechanism:
  - Turbulent mixing of streams with differeint inlet temperature.
  - Temperature fluctuations near the pipe wall.

- Structural impact:
  - Thermal stresses induced by temperature fluctuations.
  - Frequency propagates the fatigue process.
  - Magnitude determines the intensity of thermal stresses.



Thermal Mixing

- Results for a T-Junction with Pr=1 and Pr=10 were almost identical: numerical dissipation suppresses turbulence.
- Identify all the artificial sources of numerical dissipation in FV collocated methods.
- Propose solutions to reduce or eliminate each of them if required.

Definitions and assumptions

$$E_k = \frac{1}{2} \|\boldsymbol{u_c}\|^2$$

Semi-Discretised incompressible N-S equations

$$\Omega \frac{\partial \boldsymbol{u_c}}{\partial t} + C(\boldsymbol{u_s})\boldsymbol{u_c} = D\boldsymbol{u_c} - \Omega G_c \boldsymbol{p_c}$$

$$M\boldsymbol{u_s} = \boldsymbol{0_c}$$

$u_c^T \cdot \rightarrow$

$$\frac{\partial E_k}{\partial t} = -\boldsymbol{u_c^T} C(\boldsymbol{u_s})\boldsymbol{u_c} + \boldsymbol{u_c^T} D\boldsymbol{u_c} - \boldsymbol{u_c^T} \Omega G_c \boldsymbol{p_c} + \left(\frac{\partial E_k}{\partial t} - \boldsymbol{u_c^T} \Omega \frac{\partial \boldsymbol{u_c}}{\partial t} \right)$$

Symmetry-preserving operators

$$C(\boldsymbol{u_s}) = -C(\boldsymbol{u_s}) \Rightarrow \boldsymbol{u_c^T} C(\boldsymbol{u_s})\boldsymbol{u_c} = 0$$

$$G = -\Omega_s M^T \Rightarrow L = MG$$ Negative definite

Use of a Compact Laplacian to build the Poisson eq.:

- Pros: Avoids checkerboarding
- Cons: $\boldsymbol{u_c^T} \Omega G_c \boldsymbol{p_c} \neq 0$

Definitions and assumptions

$$\frac{\partial E_k}{\partial t} = -\boldsymbol{u}_c^T C(\boldsymbol{u}_s)\boldsymbol{u}_c + \boldsymbol{u}_c^T D \boldsymbol{u}_c - \boldsymbol{u}_c^T \Omega G_c \boldsymbol{p}_c + \left(\frac{\partial E_k}{\partial t} - \boldsymbol{u}_c^T \Omega \frac{\partial \boldsymbol{u}_c}{\partial t}\right)$$

*Side note 1

$$RES = \Omega \frac{\partial \boldsymbol{u}_c}{\partial t} + C(\boldsymbol{u}_s)\boldsymbol{u}_c - D\boldsymbol{u}_c + \Omega G_c \boldsymbol{p}_c$$

$$\Rightarrow \epsilon_{U,res} = \boldsymbol{u}_c^T \cdot RES$$

*Side note 2

BC may contribute the coefficients. If so, its contribution needs to be substracted in order to obtain the numerical dissipation.

Overall dissipation coefficients

$$\epsilon_{U,v} = \boldsymbol{u}_c^T D \boldsymbol{u}_c < 0$$

$$\epsilon_{U,C} = -\boldsymbol{u}_c^T C(\boldsymbol{u}_s)\boldsymbol{u}_c \rightarrow R_{U,C} = \frac{\epsilon_{U,C}}{\epsilon_{U,v}}$$

$$\epsilon_{U,p} = -\boldsymbol{u}_c^T \Omega G_c \boldsymbol{p}_c \rightarrow R_{U,p} = \frac{\epsilon_{U,p}}{\epsilon_{U,v}}$$

$$\epsilon_{U,t} = \frac{\partial E_k}{\partial t} - \boldsymbol{u}_c^T \Omega \frac{\partial \boldsymbol{u}_c}{\partial t} \rightarrow R_{U,t} = \frac{\epsilon_{U,t}}{\epsilon_{U,v}}$$

$$\epsilon_{U,res} = \frac{\partial E_k}{\partial t} - (\epsilon_{U,v} + \epsilon_{U,C} + \epsilon_{U,p} + \epsilon_{U,t})$$

$$\rightarrow R_{U,res} = \frac{\epsilon_{U,res}}{\epsilon_{U,v}}$$

Numerical dissipation

Overall dissipation coefficients of the scalar equation

$$E_p = \frac{1}{2}\|T\|^2$$

$$\frac{\partial E_p}{\partial t} = -\boldsymbol{T}_c^T C(\boldsymbol{u}_s)\boldsymbol{T}_c + \boldsymbol{T}_c^T D_T \boldsymbol{T}_c + \left(\frac{\partial E_p}{\partial t} - \boldsymbol{T}_c^T \Omega \frac{\partial \boldsymbol{T}_c}{\partial t}\right)$$

$$\epsilon_{T,\alpha} = \boldsymbol{T}_c^T D_T \boldsymbol{T}_c < 0$$

$$\epsilon_{T,C} = -\boldsymbol{T}_c^T C(\boldsymbol{u}_s)\boldsymbol{T}_c \rightarrow R_{T,C} = \frac{\epsilon_{T,C}}{\epsilon_{T,\alpha}}$$

$$\epsilon_{T,t} = \frac{\partial E_p}{\partial t} - \boldsymbol{T}_c^T \Omega \frac{\partial \boldsymbol{T}_c}{\partial t} \rightarrow R_{T,t} = \frac{\epsilon_{T,t}}{\epsilon_{T,\alpha}}$$

$$\epsilon_{T,res} = \frac{\partial E_p}{\partial t} - (\epsilon_{T,\alpha} + \epsilon_{T,C} + \epsilon_{T,t})$$

$$\rightarrow R_{T,res} = \frac{\epsilon_{T,res}}{\epsilon_{T,\alpha}}$$

Numerical dissipation

*Side note 1

$$RES = \Omega \frac{\partial \boldsymbol{T}_c}{\partial t} + C(\boldsymbol{u}_s)\boldsymbol{T}_c - D_T \boldsymbol{T}_c$$

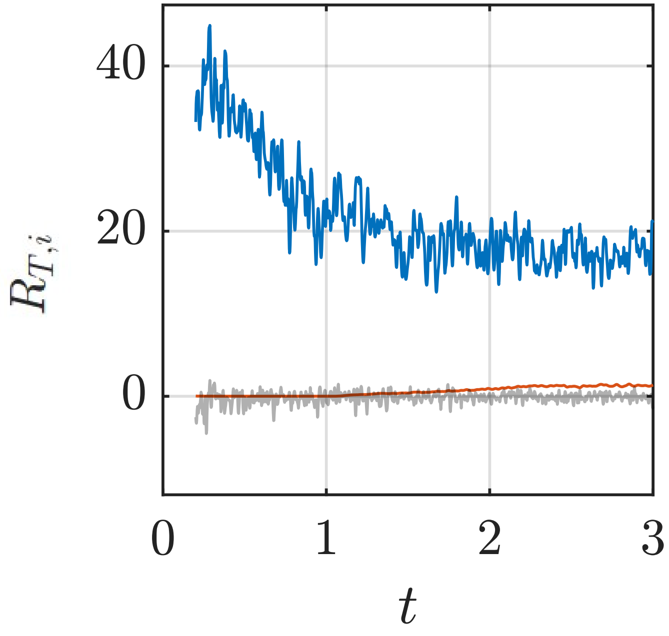
$$\Rightarrow \epsilon_{T,res} = \boldsymbol{T}_c^T \cdot RES$$

*Side note 2

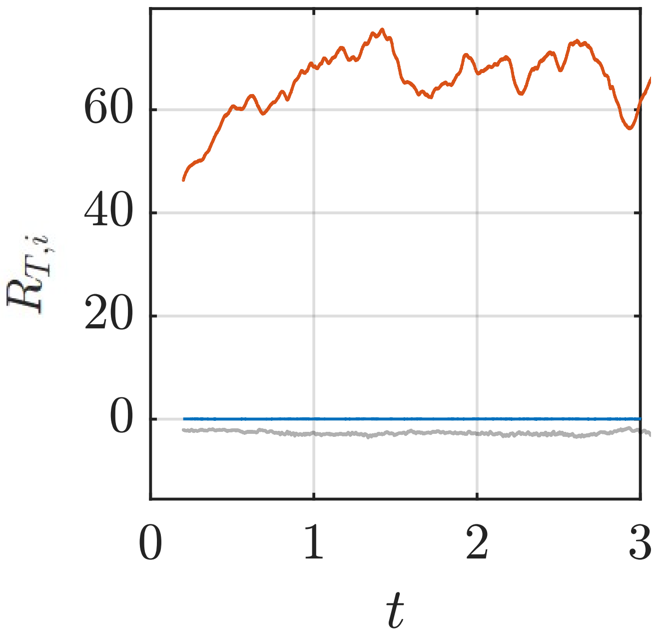
BC may contribute the coefficients. If so, its contribution needs to be substracted in order to obtain the numerical dissipation.

Overall coefficients for the T-Junction

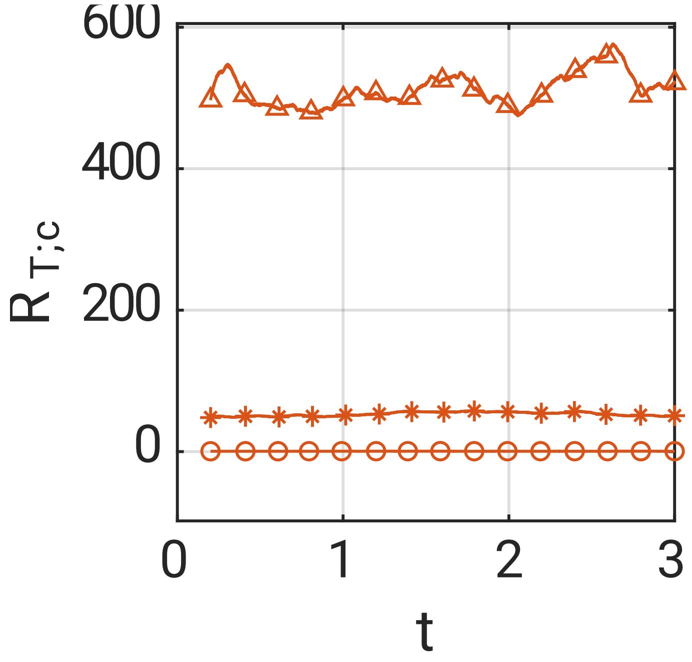
Backward + Co 2



midpoint



midpoint/van Leer blend



Dissipation ratios for different Prandtl numbers. —o— Pr = 0.01, —✱— Pr = 1, —△— Pr = 10.

— $R_{U,t}, R_{T,t}$; — $R_{U,c}, R_{T,c}$; — $R_{U,p}$; — $R_{U,b}$; — $R_{U,res}, R_{T,res}$.

Numerical test 1: Taylor-Green-Vortex at Re=1600

- Solver: OpenFOAM FV PISO-based solver with symmetry-preserving operators.
- 32x32x32 mesh (very coarse).
- Initial conditions: $U_X = U_0 \cos(x) \sin(y) \sin(z)$

$$U_Y = -U_0 \sin(x) \cos(y) \sin(z)$$

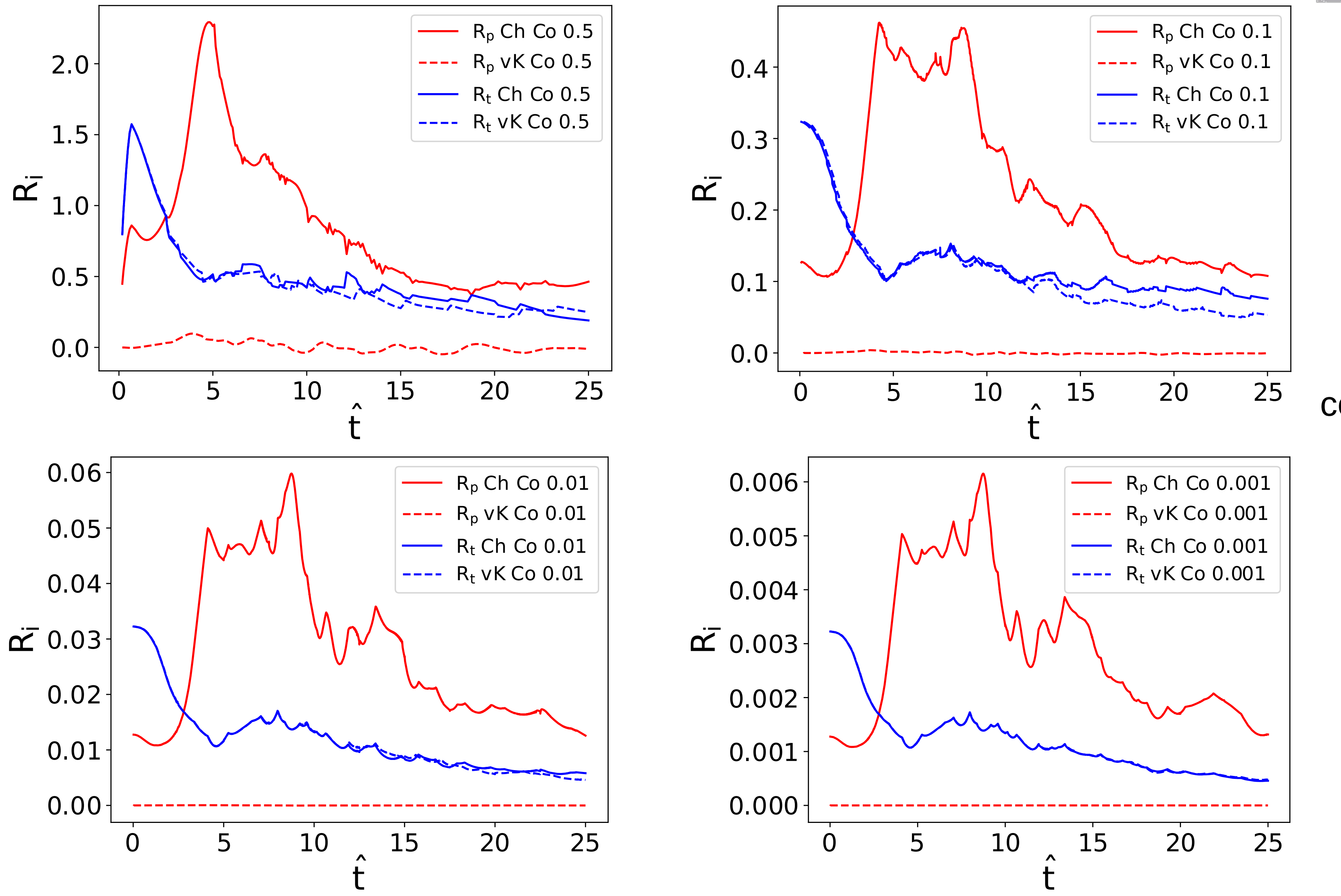
$$U_Z = 0$$

- Periodic boundary conditions
- Two projections for the pressure: $\boldsymbol{p}_c^{n+1} = \boldsymbol{p}_c^p + \boldsymbol{p}'_c$

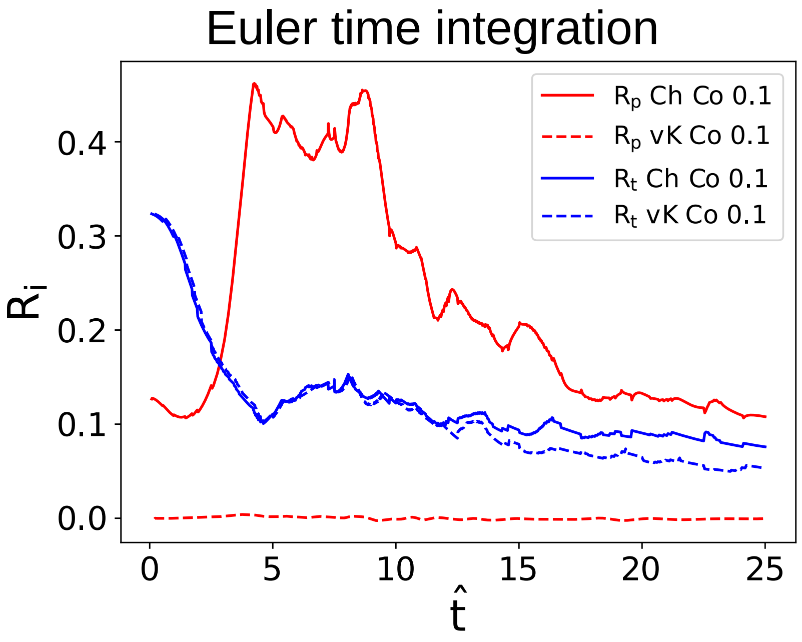
$$\boldsymbol{p}_c^p = \boldsymbol{0}_c \quad \text{Chorin}$$

$$\boldsymbol{p}_c^p = \boldsymbol{p}_c^n \quad \text{van Kan}$$

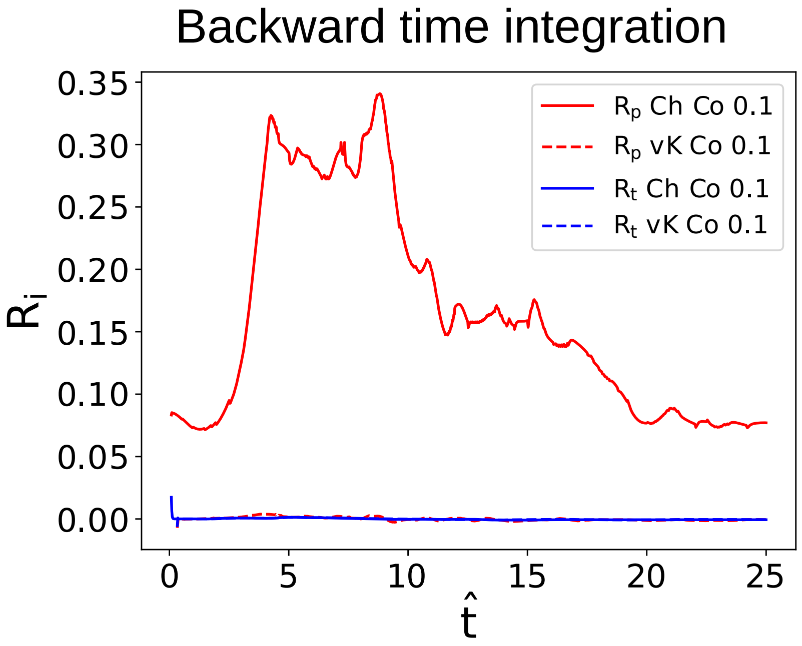
Comparison for different Courant numbers

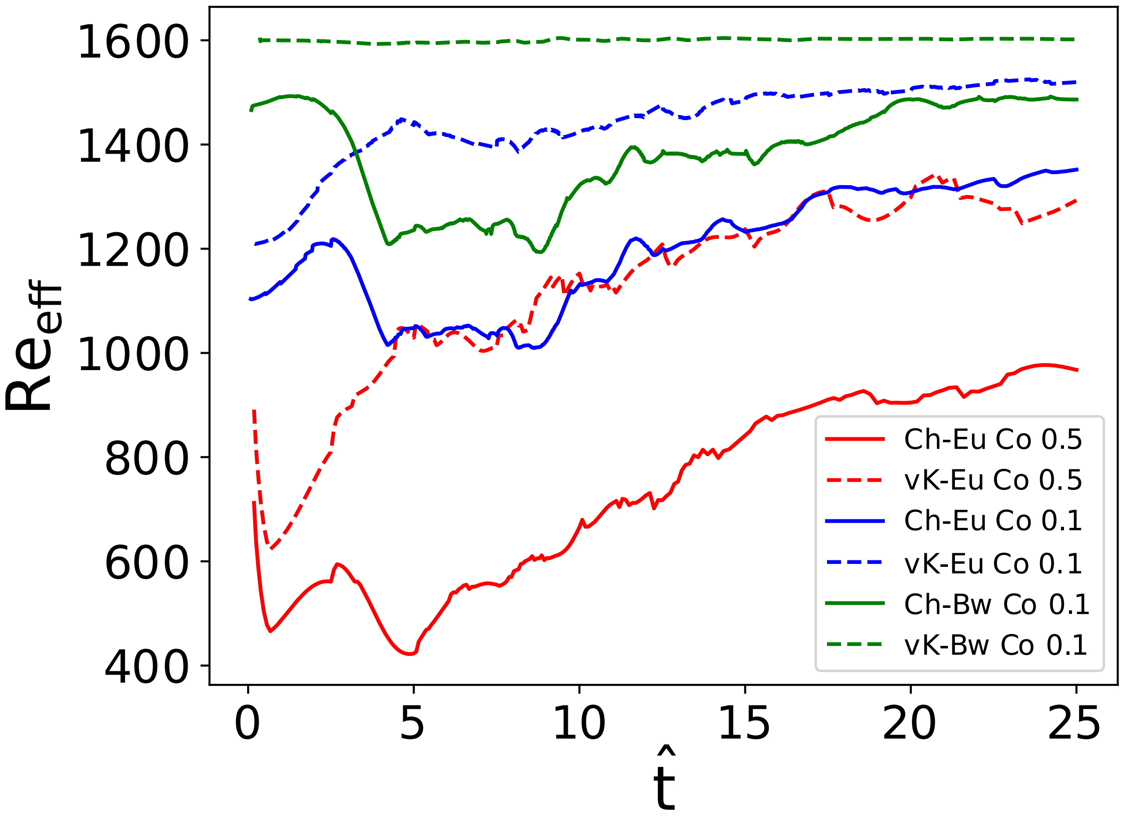


Overall dissipation coefficients using symmetry-preserving operators and Euler time integration



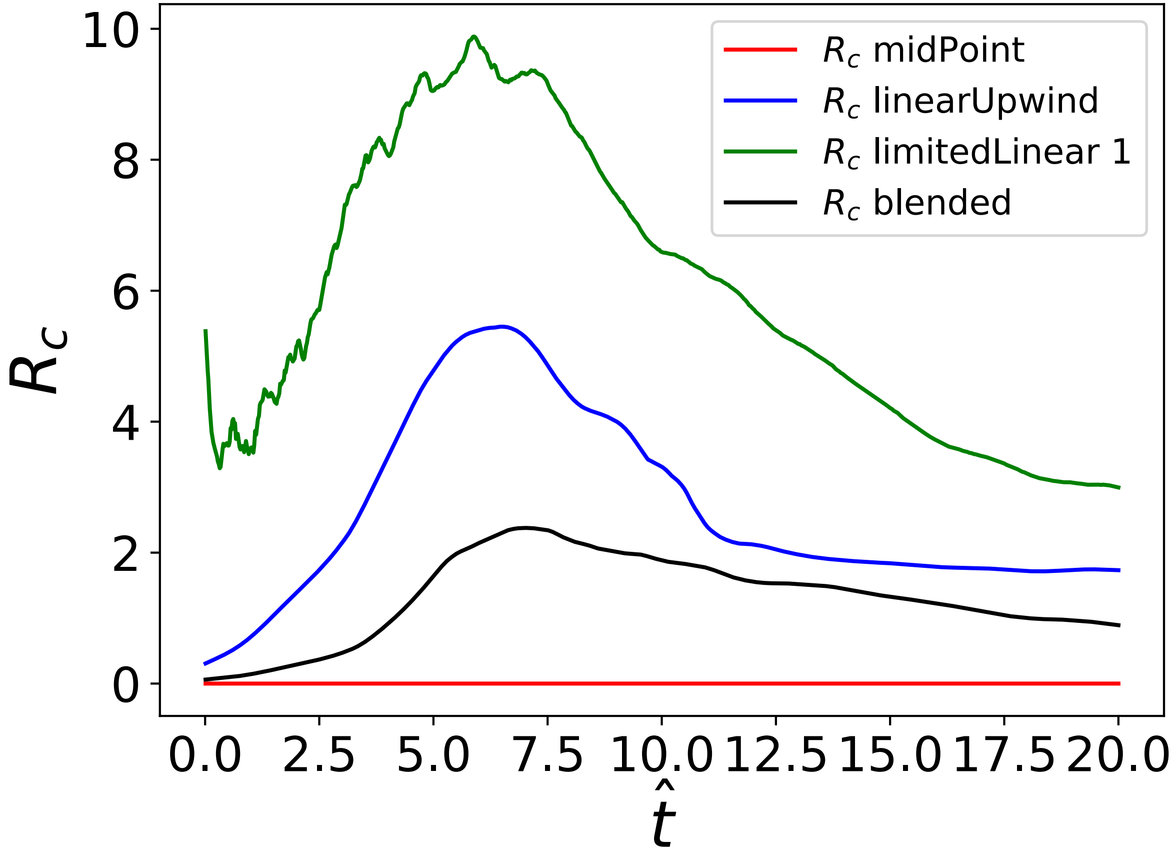
$$Re_{eff} = \frac{U_0 L}{\nu + \nu_{num}} = \frac{1}{1 + \sum R_i} Re$$





Comparison for different convective interpolators

Backward time integration + Co 0.1



- Blended: 0.8 midPoint + 0.2 linearUpwind
  - The amount of dissipation coming from the convective term can be adjusted by changing the coefficients of the blending.
- The symmetry-preserving operator (midPoint) is not introducing dissipation (up to machine precision).
- LimitedLinear 1 introduces a peak dissipation of about 10 times larger than the physical one.

Numerical test 2: stratified TGV at Re=1600

- Solver: OpenFOAM FV PISO-based solver with symmetry-preserving operators.
- 32x32x32 mesh (very coarse)
- Initial conditions:

$$U_X = U_0 \cos(x)\sin(y)\sin(z)$$
$$U_Y = -U_0 \sin(x)\cos(y)\sin(z)$$
$$U_Z = 0$$
$$T = 0$$

- Periodic boundary conditions + Boussinesq approximation

$$T^* = T_b(z) + T$$ Fluctuations of the scalar

$$Fr = \frac{U}{N\mathscr{L}} = 1$$

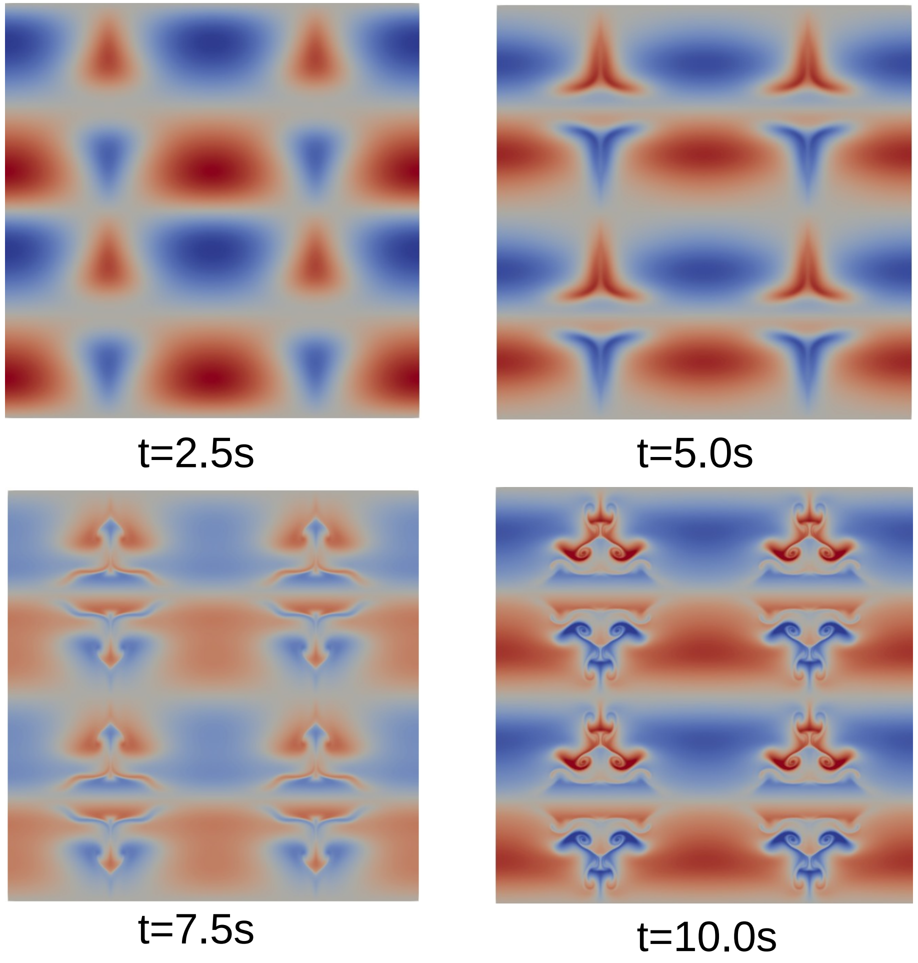
$$T_b(z) = T_0 + \Gamma z$$ prescribed background density profile

$$\Gamma = N^2 / g\beta$$

$$\Omega \frac{\partial \boldsymbol{T_c}}{\partial t} + C(\boldsymbol{u_s})\boldsymbol{T_c} = D_T \boldsymbol{T_c} - \Gamma w$$

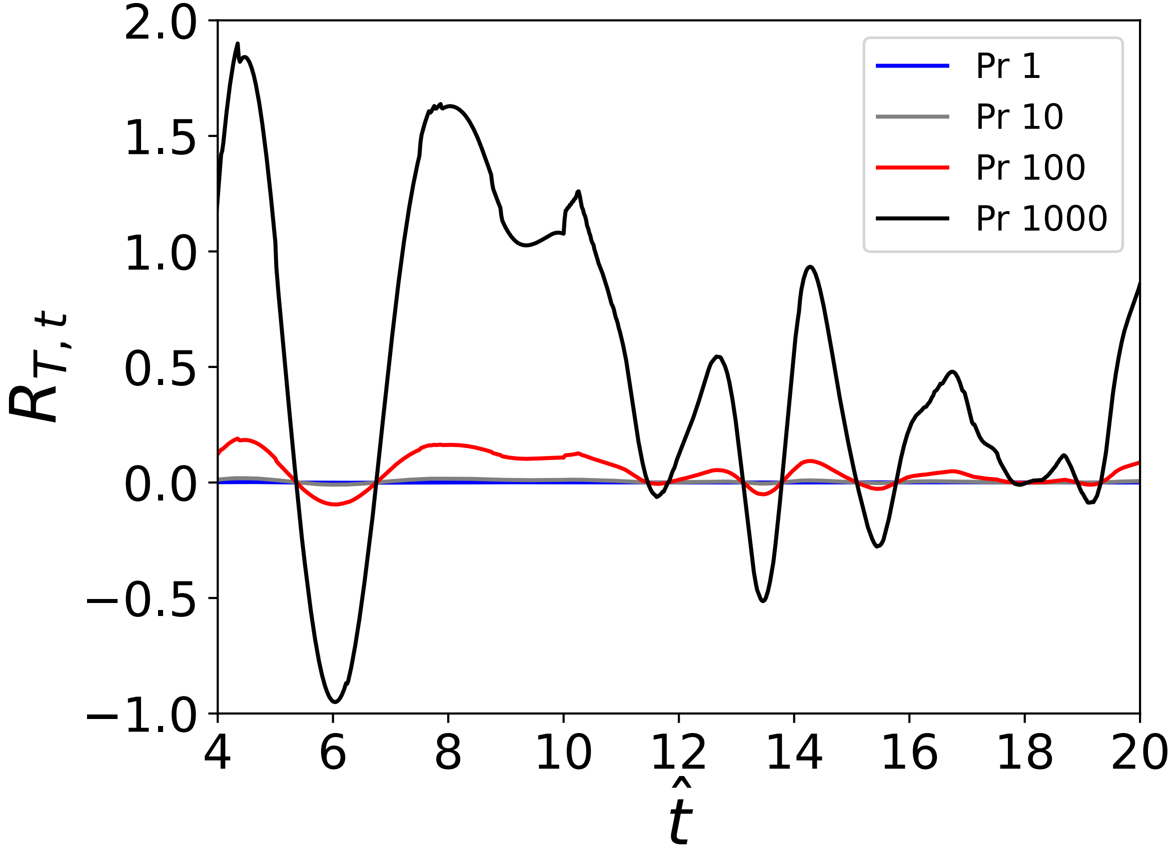
Source term

Fluctuation visualization at Y center plane with a 700x700x700 mesh



$$\epsilon_{T,t} = \frac{\partial E_p}{\partial t} - \boldsymbol{T}_c^T \Omega \frac{\partial \boldsymbol{T}_c}{\partial t} \rightarrow R_{T,t} = \frac{\epsilon_{T,t}}{\epsilon_{T,\alpha}}$$

Backward time integration + Co 0.1

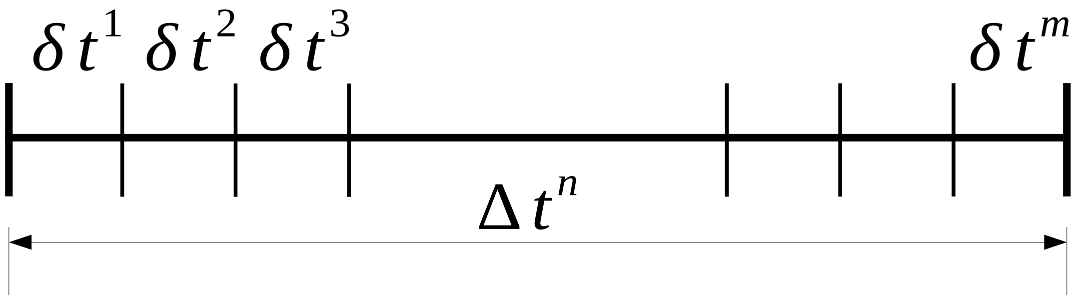


- We increase $Pr = \frac{\nu}{\alpha}$ by diminishing α.
- $\epsilon_{T,t}$ remains similar, however $\epsilon_{T,\alpha} \sim \alpha$

$$\Rightarrow R_{T,t} \sim \frac{1}{\alpha} \sim Pr$$

- If we want to reduce $R_{T,t}$, we will need to reduce Δt.

Subcycling strategy



$$\delta t^i = \frac{\Delta t^n}{\text{n subcycles}}$$

$$R_{T,k} = \frac{\frac{1}{\Delta t^n} \sum \epsilon^i_{T,k} \cdot \delta t^i}{\frac{1}{\Delta t^n} \sum \epsilon^i_{T,\alpha} \cdot \delta t^i} = \frac{\sum \epsilon^i_{T,k}}{\sum \epsilon^i_{T,\alpha}}$$

- **If no subcycling:**

$\epsilon^n_{T,k}$ Dissipation at interval n [energy/s]

$\epsilon^n_{T,k} \cdot \Delta t^n$ Energy dissipated at interval n [energy]

- **If m subcycles:**

$\epsilon^i_{T,k}$ Dissipation at subinterval i [energy/s]

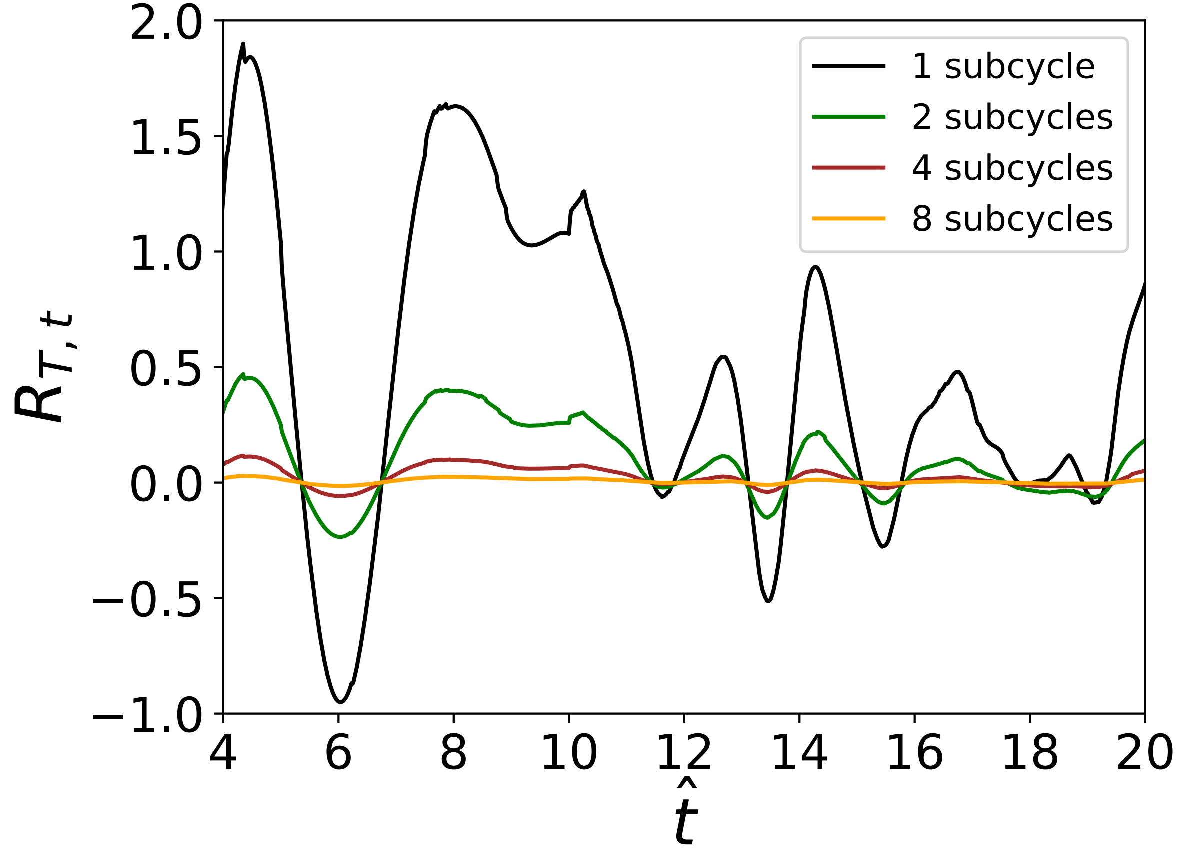
$\epsilon^i_{T,k} \cdot \delta t^i$ Energy dissipated at subinterval i [energy]

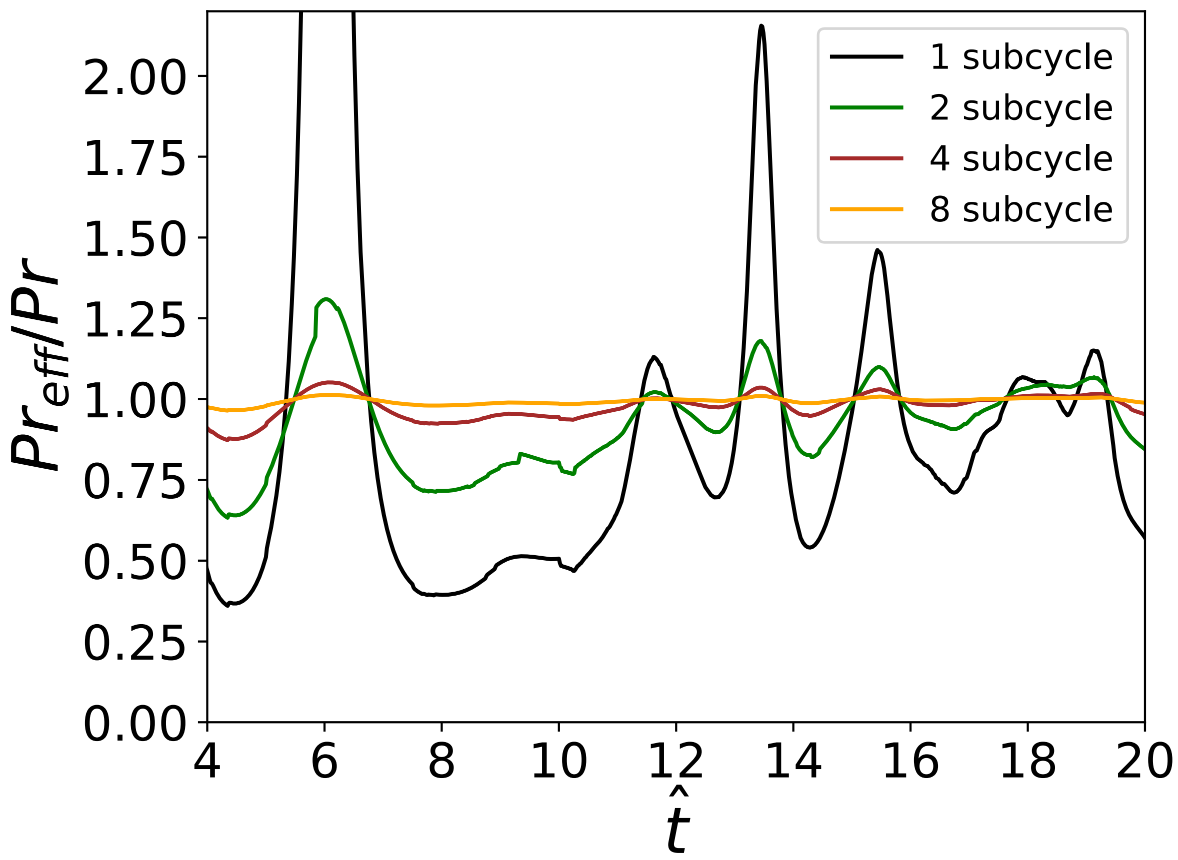
$\sum \epsilon^i_{T,k} \cdot \delta t^i$ Energy dissipated at interval n [energy]

$\frac{1}{\Delta t^n} \sum \epsilon^i_{T,k} \cdot \delta t^i$ Equivalent dissipation at interval n [energy/s]

$$\mathrm{Pr}_{eff} = \frac{\nu + \nu_{num}}{\alpha + \alpha_{num}} = \frac{1 + \sum R_{U,i}}{1 + \sum R_{T,i}} \mathrm{Pr}$$

Pr 1000 Backward time integration + Co 0.1





Conclusions

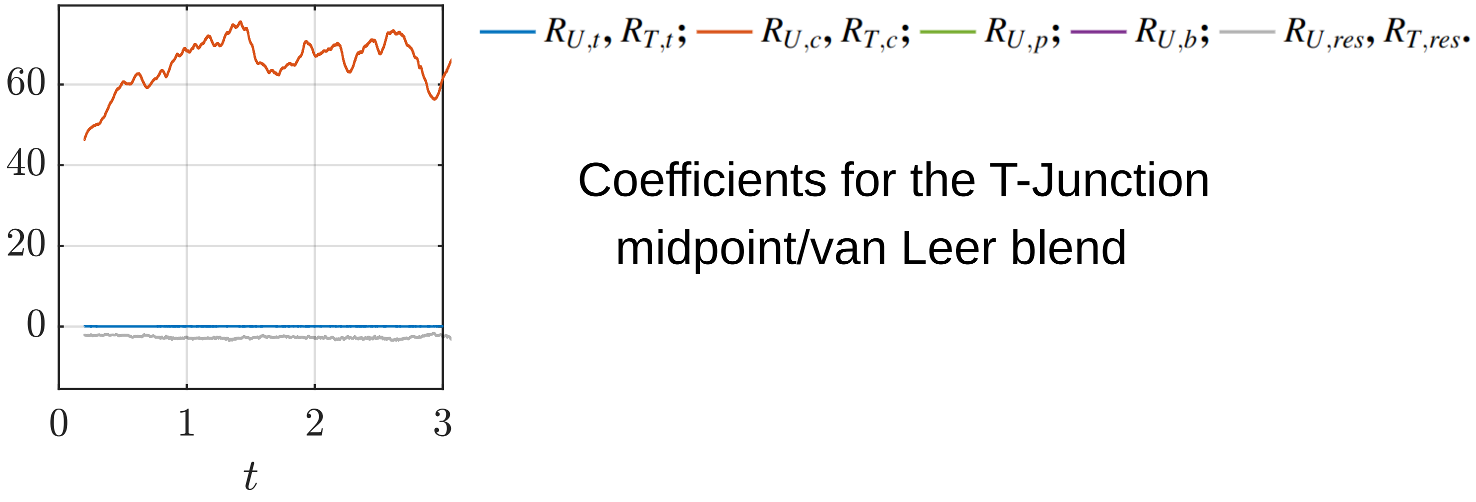
- Numerical dissipation can suppress turbulence and disturb the effective Reynolds and Prandtl numbers.

Kinetic energy numerical dissipation		Scalar invariance numerical dissipation
• = 0 if midPoint used for convection. • Reduced by blending.	R_C	• = 0 if midPoint used for convection. • Reduced by blending.
• Significantly reduced by van Kan projection.	R_p	
• Significantly reduced with high order time schemes. • Reduces with the time step.	R_t	• Significantly reduced with high order time schemes. • Reduces with the time step. • Increases with the Pr number (if thermal diffusion is reduced). • Reduces drastically by subcycling
• Already very small when the solution is well converged (low tolerances and sufficient corrector loops.)	R_{res}	• Already very small when the solution is well converged (low tolerances and sufficient corrector loops.).

- This can prevent collocated LES from being affected by numerical dissipation.

Conclusions

- In practice, R_C cannot always be fully suppressed, which may lead to instabilities and blow up…
- And in some cases it is the dominant term:



Coefficients for the T-Junction midpoint/van Leer blend

- Takeaway message: if you need to add dissipation, check the effective Re or Pr as a sanity check.

Next steps for the T-Junction case

- Use the subcycle strategy when the Pr number is high (it allows to use slightly higher Courant numbers).
- LES modelling strategy.

Thank you very much!