Centre Tecnològic de Transferència de Calor
UNIVERSITAT POLITÈCNICA DE CATALUNYA

UNIVERSITÀ DEGLI STUDI DI PADOVA

Compressibility effects on decaying and forced homogeneous isotropic turbulence

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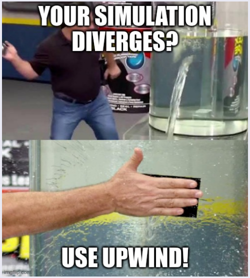

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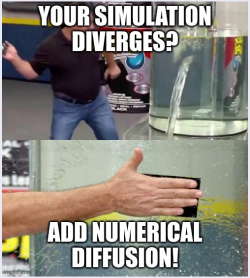

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- For transonic flows, do we need to add **numerical viscosity**, i.e. upwinding, for simulations **stability**?

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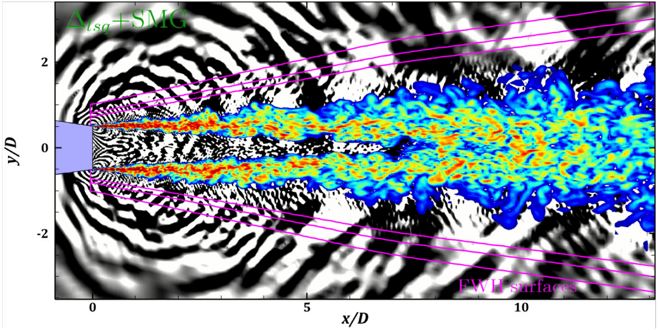

Instantaneous flow field in a jet plume ($Re_D = 1.1e6$, $Ma = 0.9$)[1]

[1] A.P. Duben, J. Ruano, A.V. Gorobets, J. Rigola, and F.X. Trias. (2023). Evaluation of enhanced gray area mitigation approaches based on jet aeroacoustics. AIAA journal, 61(2), 612-625.

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Research question #3:

- Can we increase dissipation at **0 cost**?
  - Upwinding is a cheap option, so if there is an alternative it should be cheap too...

Navier-Stokes equations

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Incompressible formulation

Navier-Stokes equations

Incompressible formulation

$$\nabla \cdot u = 0$$
$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla \cdot \tau \qquad (1)$$

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Compressible formulation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$$
$$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u u) = -\nabla p + \nabla \cdot \tau$$
$$\frac{\partial p}{\partial t} + \nabla \cdot (p u) = -(\gamma - 1) p \nabla \cdot u + (\gamma - 1)\nabla \cdot (\kappa \nabla T) + (\gamma - 1)\Phi \qquad (2)$$

Navier-Stokes equations

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$$\Phi = 2\mu S^{(d)} : S^{(d)} + \mu_B (\nabla \cdot u)^2 \tag{4}$$

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Stokes hypotheses for **incompressible flow** is true ($\nabla \cdot u = 0$).

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And for compressible flow??

Case: Decay/Forced Homogeneous Isotropic Turbulence

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- $(\mu_B/\mu)\langle(\nabla \cdot u)^2\rangle/(2S^d_{ij}S^d_{ij})$ (*Bulk to Shear dissipation ratio*).
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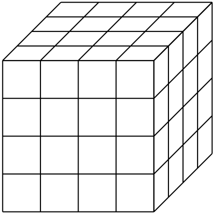

Numerical setup

Box sized domain: $L_x = L_y = L_z = 1$.
Spectral resolution: $N_x = N_y = N_z =$ 128 dealiased modes.
Algorithm: Pseudospectral with Logarithmic variables.
Re_λ (initial): ~ 100.
M_t (initial): 0.1-0.5-0.8.
Pr: 0.7

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Using two viscosities, implies the existance of another *Re* number (Re_{μ_B}).

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Decaying vs Forcing

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- We avoid the temporal mismatch between viscosities.
- We can analyse the average effect of omitting Stokes hypotheses.

Logarithmic variables

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$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\gamma} e^{\sigma} \nabla(\lambda + \sigma) + e^{-\lambda} \nabla \cdot \tau \tag{6}$$

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$$\begin{aligned}\frac{\partial \sigma}{\partial t} + u \cdot \nabla \sigma = &- (\gamma - 1) \nabla \cdot u \\ &+ \kappa e^{-\lambda} \left(\nabla^2 \sigma + (\nabla \sigma)^2 \right) \\ &+ \gamma(\gamma - 1) e^{-\lambda - \sigma} \phi\end{aligned} \tag{7}$$

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Now, ρ and T are completely bounded!

DHIT results: Dissipation ratios

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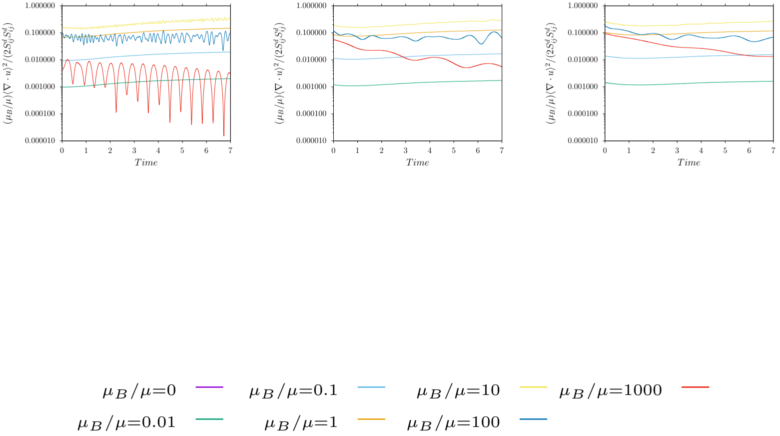

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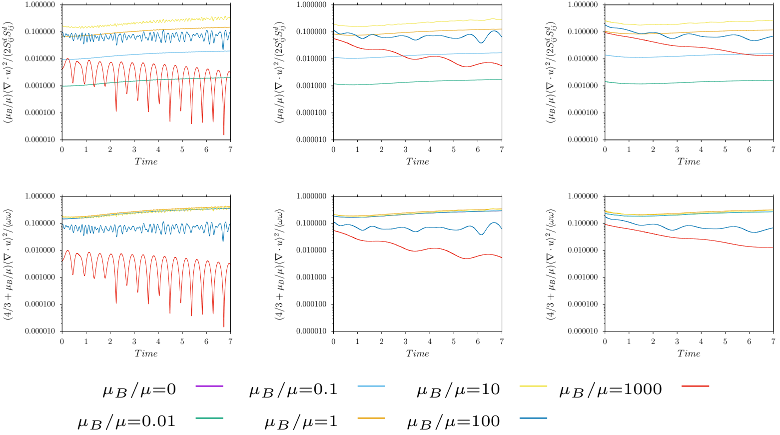

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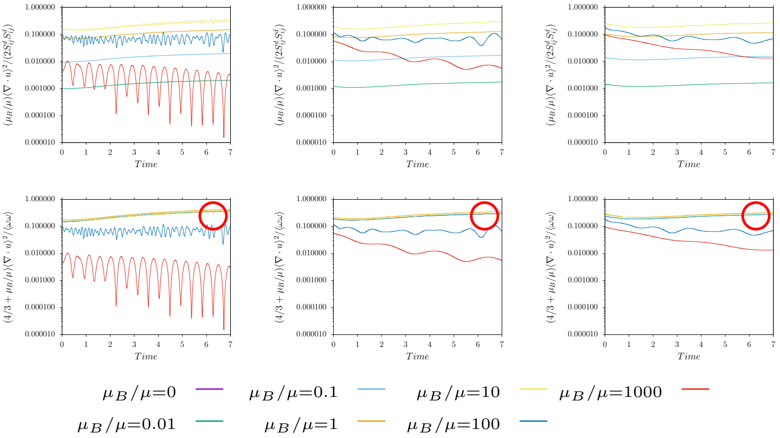

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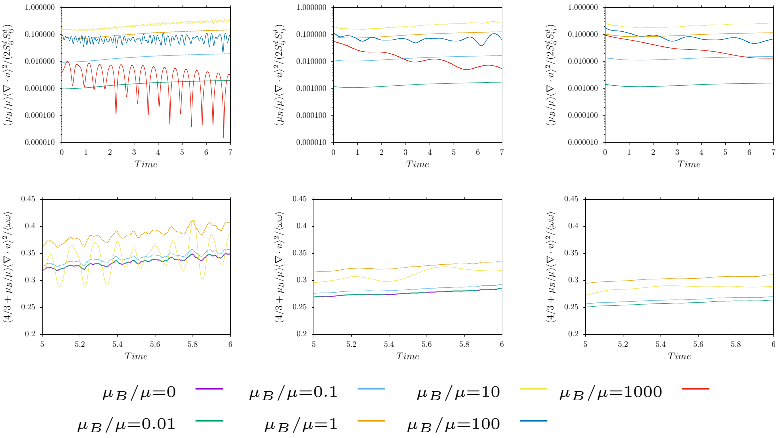

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$$\frac{(\mu_B/\mu)\langle(\nabla\cdot u)^2\rangle}{(2S^d_{ij}S^d_{ij})}$$

M_t	μ_B/μ	0	0.01	0.1	1	10	100	1000
0.1		0	0.002	0.019	0.147	0.303	0.1	0.003
0.5		0	0.002	0.017	0.127	0.290	0.063	0.005
0.8		0	0.002	0.016	0.118	0.270	0.07	0.013

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$$\frac{(4/3+\mu_B/\mu)\langle(\nabla\cdot u)^2\rangle}{\langle\omega\omega\rangle}$$

M_t	μ_B/μ	0	0.01	0.1	1	10	100	1000
0.1		0.367	0.368	0.377	0.428	0.356	0.101	0.003
0.5		0.301	0.302	0.309	0.357	0.342	0.064	0.005
0.8		0.275	0.276	0.282	0.327	0.317	0.071	0.013

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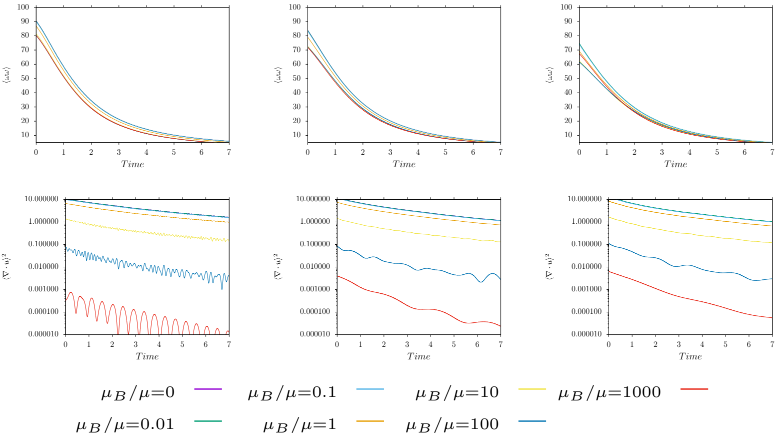

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$< \omega\omega >$

M_t	μ_B/μ	0	0.01	0.1	1	10	100	1000
0.1		5.934	5.924	5.834	5.265	4.432	4.280	4.282
0.5		5.252	5.246	5.188	4.822	4.290	4.326	4.284
0.8		5.10	5.095	5.047	4.727	4.303	4.337	4.200

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$\langle(\nabla \cdot u)^2\rangle$

M_t	μ_B/μ	0	0.01	0.1	1	10	100	1000
0.1		1.633	1.622	1.533	0.965	0.140	0.004	1e-5
0.5		1.185	1.180	1.120	0.737	0.130	0.003	2e-5
0.8		1.05	1.05	0.995	0.662	0.120	0.003	5e-5

FHIT results: Energy Spectra

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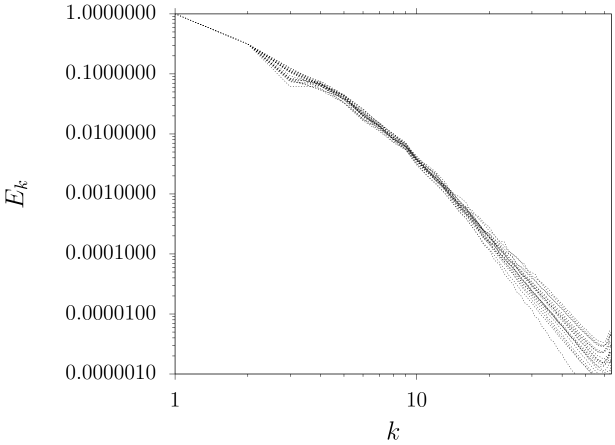

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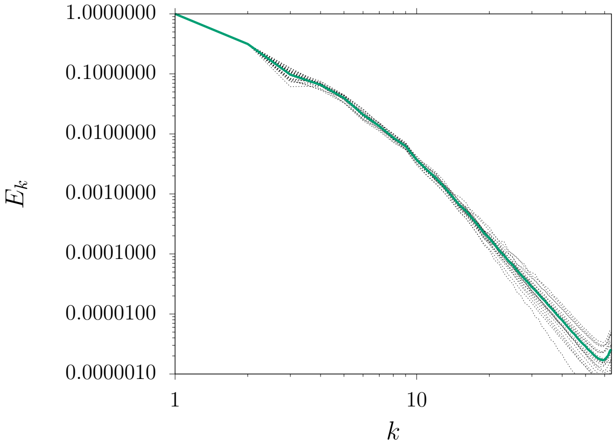

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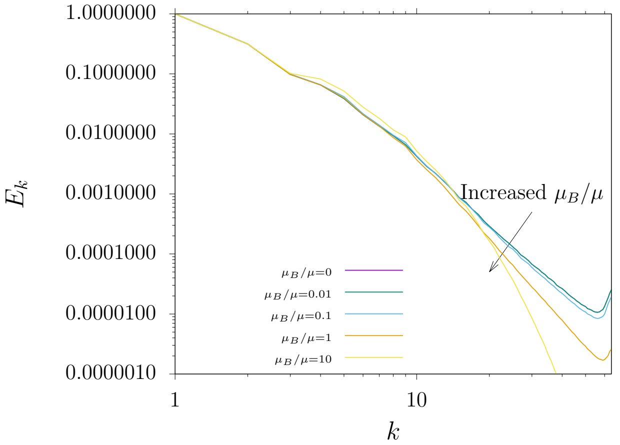

FHIT results: Average values

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$\langle(\nabla\cdot u)^2\rangle$	11.796	11.154	10.82	9.07	2.10
$\langle\omega\omega\rangle$	22.435	22.231	21.982	21.596	20.509
$\langle p\nabla\cdot u\rangle$	-0.0029	0.093	0.044	-0.064	0.019

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$\langle p\nabla \cdot u\rangle$	-0.0029	0.093	0.044	-0.064	0.019
$(\mu_B/\mu)\langle(\nabla \cdot u)^2\rangle/(2S^d_{ij}S^d_{ij})$	0	0.003	0.031	0.267	0.718

FHIT results: Average values

μ_B/μ	0	0.01	0.1	1	10
Re_λ	92.93	91.57	92.37	94.63	89.09
M_t	0.09	0.091	0.092	0.09	0.085
$\langle(\nabla\cdot u)^2\rangle$	11.796	11.154	10.82	9.07	2.10
$\langle\omega\omega\rangle$	22.435	22.231	21.982	21.596	20.509
$\langle p\nabla\cdot u\rangle$	-0.0029	0.093	0.044	-0.064	0.019
$(\mu_B/\mu)\langle(\nabla\cdot u)^2\rangle/(2S^d_{ij}S^d_{ij})$	0	0.003	0.031	0.267	0.718
$(4/3+\mu_B/\mu)\langle(\nabla\cdot u)^2\rangle/\langle\omega\omega\rangle$	0.649	0.716	0.823	0.972	0.902

Concluding remarks

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DHIT and FHIT results

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  - A maximum bulk to shear dissipation rate around $\mu_B/\mu \sim$1-10.
  - A maximum dilatational to solenoidal dissipation rate around $\mu_B/\mu \sim$1.
- A slight reduction of the final enstrophy of the system when μ_B/μ increases.
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- Dissipation is increased, at maximum, **around 15-20%**.
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General conclusions

- Dissipation is increased, at maximum, **around 15-20%**.
  - But at 0 cost; $\nabla \cdot u$ is already being computed.
- Dilatation is **extremely sensitive** to the bulk viscosity ratio; fluid becomes incompressible.
  - It may help to stabilize the simulation.

Further work

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High μ_B/μ and M_t results

We want to compute FHIT at the highest μ_B/μ ratios to assess extracted conclusions. Additionally, we want to compute results at $M_t =$ 0.5 and 0.8

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Simulations at higher Re_λ are planned in the future

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Maximum dissipation regime

We want to determine the mechanisms that trigger the maximum dissipation regime.

Thanks for your attention