COMPRESSIBILITY EFFECTS ON DECAYING AND FORCED HOMOGENEOUS ISOTROPIC TURBULENCE

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INTRODUCTION

The constitutive model that links the viscous stresses and the strain rate, for a Newtonian fluid, is expressed by the equation:

$$\tau = \mu \left(\nabla u + (\nabla u)^T - \frac{2}{3}(\nabla \cdot u)\mathrm{I} \right) + \xi(\nabla \cdot u)\mathrm{I}, \qquad (1)$$

where, following the usual notation, u is the velocity vector, ∇u its gradient, $\nabla \cdot u$ the divergence of the velocity field, μ indicates the dynamic viscosity, ξ indicates the second, bulk or volumetric viscosity, and I is the identity matrix.

Following Stokes' hypotheses [1], the last term of the equation is usually omitted, i.e. $\xi(\nabla \cdot u)\mathrm{I} = 0$. In a compressible case, where $\nabla \cdot u$ cannot be considered zero, this is achieved by assuming that the volumetric viscosity is equal to zero. Nonetheless, only monoatomic gases have a null volumetric viscosity, whereas polyatomic gases, such as O_2 or N_2, the main components of air, have volumetric viscosity values of the same order of magnitude as the dynamic viscosity. For CO_2, however, the volumetric viscosity is approximately 3 orders of magnitude higher than the dynamic one.

Previous work from Pan and Johnsen [2] concluded that the inclusion of bulk viscosity increases the decay rate of turbulent kinetic energy. Nonetheless, they also noted that, for a low bulk-to-kinematic viscosity ratio, neglecting the volumetric viscosity would not affect the numerical results. On the other hand, in cases with a high bulk-to-kinematic viscosity ratio, such as CO_2, assuming Stokes' hypothesis can lead to inaccurate results.

The present work aims to develop a platform for assessing the effect of introducing the bulk viscosity into the compressible Navier-Stokes equations. The main objective is to assess how the bulk viscosity can affect both the accuracy of the obtained numerical solutions as well as the stability of the whole numerical method.

EQUATIONS AND LOGARITHMIC VARIABLES

When solving the Compressible Navier-Stokes equations numerically, it is possible to obtain solutions without any physical sense. This means, for example, that even though density, temperature, and pressure can only take on positive values, a numerical simulation can assign negative values to these variables; when this happens, the simulation usually crashes. To overcome this problem, we use logarithmic variables [3]: $\rho = e^{\lambda}$ and $T = e^{\sigma}$.

With this modification, the variables λ and σ, which replace ρ and T respectively, can have negative values during the simulation. At the same time, with just this change of variables, the primitive variables can no longer be negative as $e^x > 0; \forall x \in \mathbb{R}$. This change modifies the usual formulation of the Navier-Stokes equations in non-dimensional form to [3]:

$$\frac{\partial \lambda}{\partial t} + u \cdot \nabla \lambda = -\nabla \cdot u \qquad (2)$$

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\gamma} e^{\sigma} \nabla(\lambda + \sigma) + e^{-\lambda} \nabla \cdot \tau \qquad (3)$$

$$\frac{\partial \sigma}{\partial t} + u \cdot \nabla \sigma = -(\gamma - 1)\nabla \cdot u + \kappa e^{-\lambda} \left(\nabla^2 \sigma + (\nabla \sigma)^2 \right) + \gamma(\gamma - 1) e^{-\lambda - \sigma} \phi \qquad (4)$$

where all the variables, i.e. λ, u, and σ, are non-dimensional, γ is the heat capacity ratio, κ is the non-dimensional heat conductivity, and ϕ is the viscous dissipation rate.

NUMERICAL METHOD

Solving the Navier-Stokes equations, whether in compressible or incompressible form, requires the use of numerical methods.

The most common approaches, i.e. FVM, FEM or FDM, rely on numerically discretizing the derivatives in order to generate an algebraic system of equations to solve. However, as the derivatives are "discretized", we are introducing a numerical error in our system. This numerical error can, among other things, introduce additional dissipation into the simulation.

For the present case, i.e. a periodic domain in the 3 dimensions, the pseudospectral method becomes a possibility. It allows for the exact computation of derivatives in Fourier space rather than approximating them in physical space.

Therefore, all the derivatives are evaluated in the Fourier space as:

$$\hat{\phi}'(k) = k\hat{\phi}(k), \qquad (5)$$

and then transformed in the physical space via the inverse Fourier transform:

$$\phi' = \sum_{k<N/2} \hat{\phi}' e^{-ikx}. \qquad (6)$$

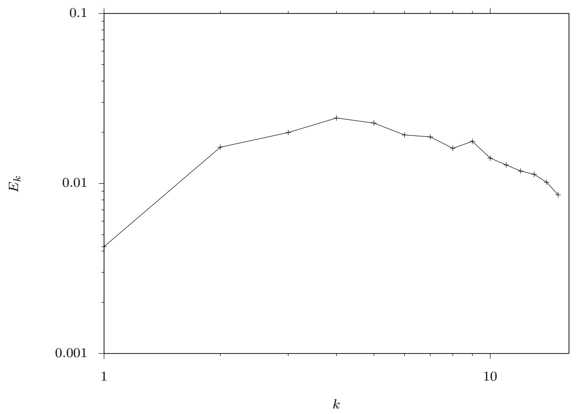

Figure 1: Initial Kinetic energy spectrum for all simulations

Once in the physical space, the products between the derivatives and the rest of the variables, such as $u \cdot \nabla\lambda$, are performed.

CASE SETUP

We define a box-sized domain discretized in M_x x M_y x M_z grid points in physical space, and N_x x N_y x N_z in Fourier space. The relation between the number of points per direction, i.e. M_i, in the physical space is 3/2 higher than the number of modes for the same direction. The initial velocity is imposed such that it follows an initial kinetic energy distribution (see Figure 1).

The viscosity is set so that a given Re_λ is achieved; the value selected for the presented results is $Re_\lambda = 295$. The heat capacity is obtained by fixing the Pr number (0.7 for all the presented cases). And finally, an initial M_t is fixed by setting the initial Temperature field such that the relation: $M_t = \sqrt{\frac{<u^2>}{<T>}}$ is fulfilled. The tools used to perform the Fast Fourier Transform between domains, as well as the functions to post-process the obtained results, have been stored in an online repository [4].

NUMERICAL RESULTS

Here, we include the preliminary results of this investigation, focusing on the decay of homogeneous isotropic turbulence. The mesh used in these preliminary results, a 64x64x64 mesh in Fourier space, is unable to compute all the energy-cascade mechanisms due to a lack of enough oscillation modes. However, the objective of this preliminary investigation is to qualitatively assess whether the inclusion of the second viscosity affects the results rather than obtaining a perfect result.

Effect of the turbulent Mach number

In this subsection, we analyze the effect that the initial turbulent Mach number has on the temporal evolution of the Taylor-based Reynolds number, as well as its own temporal evolution. Five different turbulent Mach numbers have been tested: $M_t = (0.05, 0.1, 0.2, 0.5, 1)$. For all these simulations, the ratio between the volumetric viscosity and the dynamic one has been set equal to one.

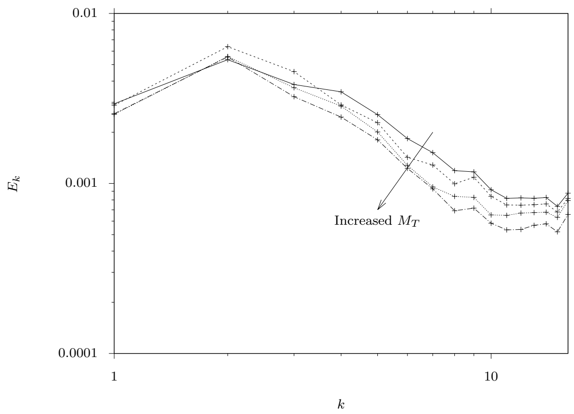

Figure 2: Final Kinetic energy spectrum for different initial turbulent Mach numbers

As previously commented, the initial Reynolds number, based on the Taylor microscale, is kept constant at 295 throughout all simulations. We have included the kinetic energy spectrum at the end of the simulation in Figure 2

M_T	E_k	Re_λ	M_T
0	0.071	92.261	0.024
0.1	0.044	75.576	0.04707
0.2	0.043	75.42	0.09388
0.5	0.038	69.237	0.21899
1	0.035	67.132	0.39203

Table 1: Final values (non-dimensional time = 5) for different initial turbulent Mach numbers.

Effect of the ratio between volumetric and dynamic viscosities

In this second subsection, we analyze the effect of the ratio between volumetric and dynamic viscosities. Five different ratios have been tested: $\xi/\mu = (0, 1, 10, 100, 1000)$. The turbulent Mach number has been kept constant at 0.5 for these simulations. As in the previous subsection, the initial Reynolds number based on the Taylor microscale is kept again at 295 throughout all simulations. We have included the kinetic energy spectrum at the end of the simulation in Figure 3

ξ/μ	E_k	Re_λ	M_T
0	0.04	69.045	0.22357
1	0.038	69.237	0.21899
10	0.036	67.875	0.21069
100	0.031	59.667	0.19892
1000	0.029	56.644	0.19253

Table 2: Final values (non-dimensional time = 5) for different viscosity ratios.

CONCLUSIONS

Here we have presented the preliminary results of ongoing work. Initial results show that the inclusion of the second viscosity seems to affect the transient evolution of both the turbulent Mach number and the Taylor-based Reynolds number.

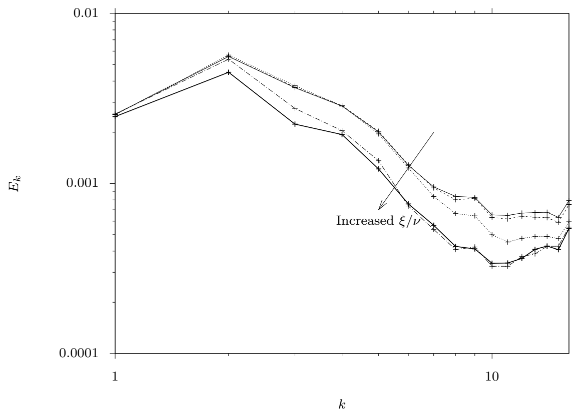

Figure 3: Final Kinetic energy spectrum for different viscosity ratios

The results presented show the effect of volumetric viscosity on the evolution of kinetic energy and the turbulent Mach number over time. Figure 3 and Table 2 show how an increased bulk-to-dynamic viscosity ratio effectively reduces the kinetic energy spectra.

The used mesh, i.e., 64^3 modes, does not have sufficient resolution, as the obtained spectra indicate an energy pile-up at higher modes. Nonetheless, future work includes conducting a mesh-convergence study to determine the minimum number of modes required to accurately reproduce the energy cascade.

We are studying how the conclusions extracted on the first stage of this investigation can be correctly extrapolated to a case in which energy is continuously added to the system, rather than letting an initial stage evolve over time without any additional input. Most of the current CFD simulations deal with problems in which the total amount of energy of a system is kept constant rather than decaying over time. Problems such as channel flows or bluff bodies immersed within a fluid in motion, are constantly injecting energy via fixing the velocities at the inlets or by fixing a driving force. Therefore, by analyzing how a forced rather than decaying homogeneous isotropic turbulence dissipates energy per time-step, up to achieving a constant rate of dissipation, we could examine how the effect of including the volumetric viscosity affects the overall results.

Finally, and as a further work, one of the objectives of the present study is to establish whether the numerical diffusion added by hybrid schemes usually used within Compressible Navier-Stokes simulations, like the Flux Limiting family [5] or the ENO-WENO schemes [6], can be totally substituted by the inclusion of a more physically meaningful parameter.

ACKNOWLEDGMENTS

J.R. and F.X.T. are supported by SIMEX project (PID2022-142174OB-I00) of the Ministerio de Ciencia e Innovacion MCIN/AEI/ 10.13039/501100011033 and the European Union Next GenerationEU. A.A.B. was financially supported by the EU's Horizon Europe programme under the Marie Sk lodowska-Curie grant agreement No. 101208388

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