Turbulent Differentially Heated Cavity of Aspect Ratio 5
DNS and regularization modeling

F.Xavier Trias⋆, Andrey Gorobets⋆,∗, Assensi Oliva⋆

⋆Heat and Mass Transfer Technological Center
Technical University of Catalonia
∗Keldysh Institute of Applied Mathematics of RAS, Russia

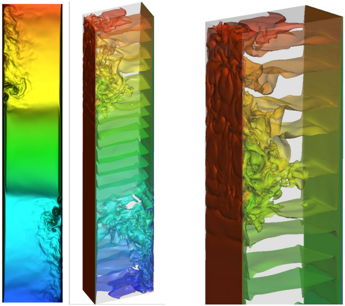
7th International Conference on Computational Heat and Mass Transfer
Istanbul, 18-22 July 2011

Contents

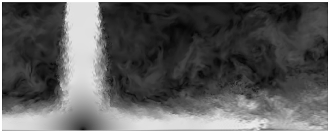
DNS of turbulent incompressible flows

Main features of the DNS code:

- Structured staggered grids
- High-order symmetry-preserving schemes
- Fully-explicit second-order time-integration method
- Poisson solver for 2.5D problems: FFT + PCG
- Hybrid MPI+OpenMP parallelization

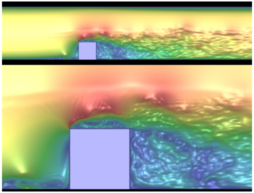


Air-filled differentially heated cavity at $Ra = 10^{11}$ (111M grid points)

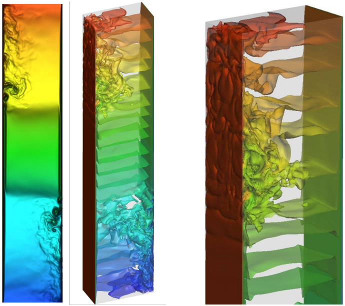


Plane impingement jet at $Re = 20000$ (102M grid points)

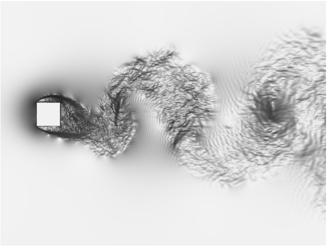
DNS of turbulent incompressible flows



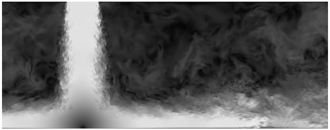
Wall-mounted cube at $Re = 7240$ (17M grid points)



Air-filled differentially heated cavity at $Ra = 10^{11}$ (111M grid points)

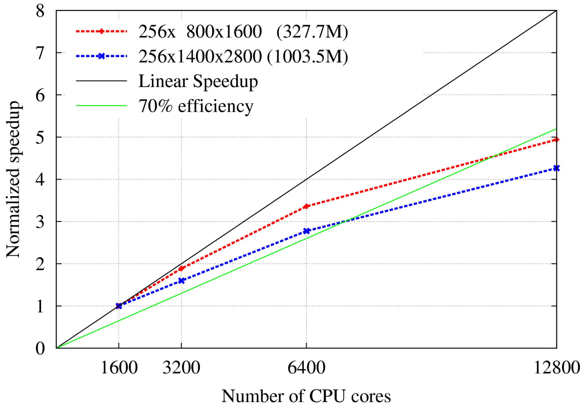


Square cylinder at $Re = 22000$ (75M grid points)



Plane impingement jet at $Re = 20000$ (102M grid points)

Scaling? Yes[1], we can... but never enough



[1] A. Gorobets *et al.* *"Hybrid MPI+OpenMP parallelization of an FFT-based 3D Poisson solver with one periodic direction"* **Computers&Fluids**, (available online).

Governing equations

Incompressible Navier-Stokes equations:

$$\begin{aligned} \nabla \cdot u &= 0 \\ \partial_t u + \mathcal{C}(u, u) &= \mathcal{D}(u) - \nabla p \end{aligned}$$

where the **nonlinear convective term** is given by

$$\mathcal{C}(u, \phi) = (u \cdot \nabla)\phi$$

and the linear dissipative term is given by

$$\mathcal{D}(\phi) = \nu \Delta \phi$$

Regularization modeling

As the full energy spectrum cannot be computed, a **dynamically less complex mathematical formulation** is sought. We consider smooth approximations (regularizations) of the nonlinearity,

$$\partial_t u_\epsilon + \widetilde{\mathcal{C}}(u_\epsilon, u_\epsilon) \;=\; \mathcal{D}(u_\epsilon) - \nabla p_\epsilon$$

such approximations may fall in the **Large-Eddy Simulation** (LES) concept,

$$\partial_t \overline{u}_\epsilon + \mathcal{C}(\overline{u}_\epsilon, \overline{u}_\epsilon) \;=\; \mathcal{D}(\overline{u}_\epsilon) - \nabla \overline{p}_\epsilon + \mathcal{M}_1(\overline{u}_\epsilon, \overline{u}_\epsilon)$$

if the filter is invertible:

$$\mathcal{M}_1(\overline{u}_\epsilon, \overline{u}_\epsilon) \;=\; \mathcal{C}(\overline{u}_\epsilon, \overline{u}_\epsilon) - \overline{\widetilde{\mathcal{C}}(u_\epsilon, u_\epsilon)}$$

Symmetry-preserving regularization models (1/2)

In order to conserve the following inviscid invariants

- Kinetic energy : (u, u)
- Enstrophy (in 2D) : (ω, ω)
- Helicity (in 3D) : (ω, u)

where $(a, b) = \int_\Omega a \cdot b d\Omega$ and $\omega = \nabla \times u$; the **approximate convective operator** must be **skew-symmetric**:

$$\left(\widetilde{\mathcal{C}}(u, \phi_1), \phi_2\right) = -\left(\widetilde{\mathcal{C}}(u, \phi_2), \phi_1\right)$$

Symmetry-preserving regularization models (2/2)

This criterion yields the following class of approximations[2],

$$\partial_t u_\epsilon + \mathcal{C}_n(u_\epsilon, u_\epsilon) = \mathcal{D}(u_\epsilon) - \nabla p_\epsilon$$

in which the convective term in smoothed according to:

$$\begin{aligned}
\mathcal{C}_2(u,\phi) &= \overline{\mathcal{C}(\bar{u},\bar{\phi})} \\
\mathcal{C}_4(u,\phi) &= \mathcal{C}(\bar{u},\bar{\phi}) + \overline{\mathcal{C}(\bar{u},\phi')} + \overline{\mathcal{C}(u',\bar{\phi})} \\
\mathcal{C}_6(u,\phi) &= \mathcal{C}(\bar{u},\bar{\phi}) + \mathcal{C}(\bar{u},\phi') + \mathcal{C}(u',\bar{\phi}) + \overline{\mathcal{C}(u',\phi')}
\end{aligned}$$

where $u' = u - \bar{u}$ and $\mathcal{C}_n(u,\phi) = \mathcal{C}(u,\phi) + \mathcal{O}(\epsilon^n)$ for **any symmetric filter**.

High-frequencies need to be effectively damped.

But how much?

[2] Roel Verstappen, **Computers & Fluids**, 37 (7): 887-897, 2008

Stopping the vortex-stretching[3]

Taking the curl of momentum equation the vorticity transport equation follows

$$\partial_t \omega + \mathcal{C}(u, \omega) = \mathcal{C}(\omega, u) + \mathcal{D}(\omega)$$

Let us now consider an arbitrary part of the flow domain, Ω, with periodic boundary conditions. Then, taking the L^2 innerproduct with $\omega = \nabla \times u$ leads to the enstrophy equation

$$\frac{1}{2}\frac{d}{dt}(\omega, \omega) = (\omega, \mathcal{C}(\omega, u)) - \nu(\nabla\omega, \nabla\omega)$$

where $(a, b) = \int_\Omega a \cdot b d\Omega$. Unless, the grid is fine enough convection dominates diffusion

$$(\omega, \mathcal{C}(\omega, u)) \;>\; \nu(\nabla\omega, \nabla\omega)$$

[3] F.X. Trias *et al.* **Computers&Fluids**, 39:1815-1831, 2010

Stopping the vortex-stretching

The vortex-stretching term can be expressed in terms of the invariant $r = -1/3tr(S^3)$

$$(\omega, \mathcal{C}(\omega, u)) = 4 \int_\Omega r d\Omega \tag{1}$$

whereas the $L^2(\Omega)$-norm of ω in terms of the invariant $q = -1/2tr(S^2)$

$$(\omega, \omega) = -4 \int_\Omega q d\Omega$$

Then, the diffusive term can be bounded by

$$\nu (\nabla\omega, \nabla\omega) = -\nu (\omega, \Delta\omega) \leq -\nu\lambda_\Delta (\omega, \omega) = 4\nu\lambda_\Delta \int_\Omega q d\Omega \tag{2}$$

where $\lambda_\Delta < 0$ is the largest (smallest in absolute value) non-zero eigenvalue of Laplacian operator Δ on Ω. If we now consider that the domain is a periodic box of volume h, then $\lambda_\Delta = -(\pi/h)^2$.

Stopping the vortex stretching

$\Longrightarrow$ In the present work we **determine the filter width** ϵ from

$$(\omega, \mathcal{C}_4(\omega, u)) \quad \approx \quad f_4(\hat{g}_k(\epsilon))(\omega, \mathcal{C}(\omega, u)) \quad \leq \quad \nu\,(\nabla\omega, \nabla\omega)$$

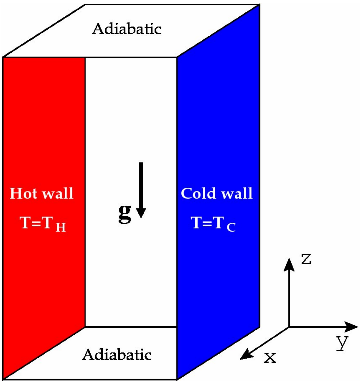
Then, recalling identity (1) and inequality (2), we propose to rewrite the previous inequality in terms of the invariants q and r

$$f_4(\hat{g}_k) = min\left\{\nu\lambda_\Delta \frac{q}{r^+}, 1\right\} \qquad with \quad r^+ = max(r, 0)$$

Notice that $q < 0$ (dissipation) whereas r can be either positive or negative.

- Switches off ($f_4 = 1$) for: laminar ($r \rightarrow 0$), 2D flows ($r = 0$) and for fine enough meshes, $|\nu\lambda_\Delta q/r| \geq 1$.
- Consistent near-wall behavior $r \propto y^3$ and $q \propto y^0$.
- Consistent with the preferential vorticity alignment with the intermediate eigenvector, λ_2 (experimentally observed)

Test-case: Differentially Heated Cavity



Boundary conditions:

- **Isothermal vertical walls**
- **Adiabatic horizontal walls**
- **Periodic** boundary conditions in the spanwise direction

Dimensionless governing numbers:

- $Ra = \beta \Delta T L_z^3 g / (\alpha \nu)$
- $Pr = \nu / \alpha$
- Height aspect ratio $A_z = L_z / L_y$
- Depth aspect ratio $A_x = L_x / L_y$

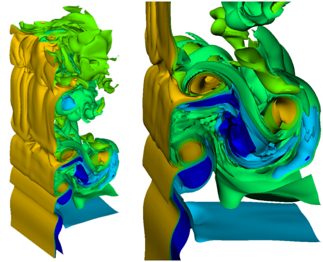
DNS results for $Ra = 4.5 \times 10^{10}$, $Pr = 0.7$

Some details about **DNS**:

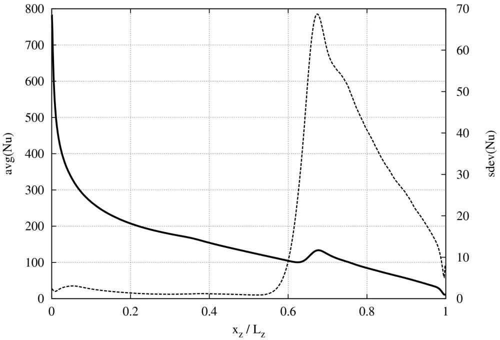
- Mesh size: $128 \times 318 \times 862$
- $\approx$ 1 months - 128 CPUs
- 4^{th}-order symmetry-preserving scheme
- $A_z = 5$

Complexity of the flow:

- Boundary layers
- Stratified cavity core
- Internal waves
- Recirculation areas



DNS results for $Ra = 4.5 \times 10^{10}$, $Pr = 0.7$



Transition point is located at $z/L_z \approx \mathbf{0.67}$ whereas experiments (also some LES and RANS simulations) predicted much more upstream positions ($z/L_z \approx \mathbf{0.2}$). This discrepant behavior was also observed for a DHC[4,5] of $A_z = 4$, $Ra \leq 10^{11}$.

[4] F. X. Trias *et al.* Int. Journal of Heat and Mass Transfer, 53:665-673, 2010
[5] F. X. Trias *et al.* Int. Journal of Heat and Mass Transfer, 53:674-683, 2010

Results for differentially heated cavity at $Ra = 4.5 \times 10^{10}$

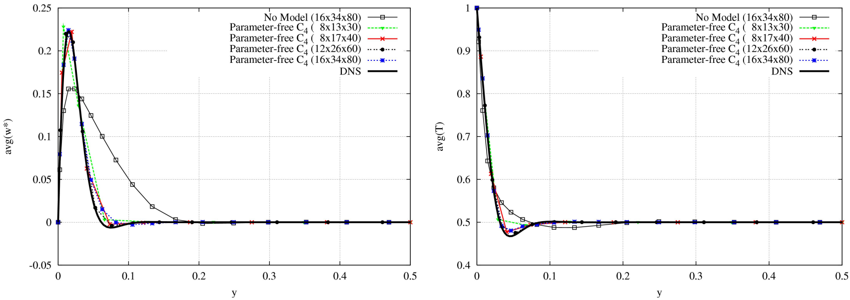
- Regularization model $\mathcal{C}_4$ is tested.
- Two coarse meshes are considered

	DNS	RM1		RM2	
Mesh	$128 \times 318 \times 862$	$8 \times 20 \times 54$		$8 \times 14 \times 38$	
		No model	$\mathcal{C}_4$	No model	$\mathcal{C}_4$
Nu	154.5	194.1	**157.5**	210.5	**159.4**
Nu_{max}	781.5	535.2	682.6	558.2	711.8
Nu_{min}	10.5	86.9	18.1	93.7	15.1

- The **discrete linear filter**[6] is based on polynomial functions of the discrete diffusive operator, D

[6] F.X. Trias and R.W.C.P. Verstappen, **Computers & Fluids**, 40:139-148, 2011

How does the parameter-free $\widetilde{\mathcal{C}}_4$ regularization modeling behave for other grids and *Ra*-numbers?

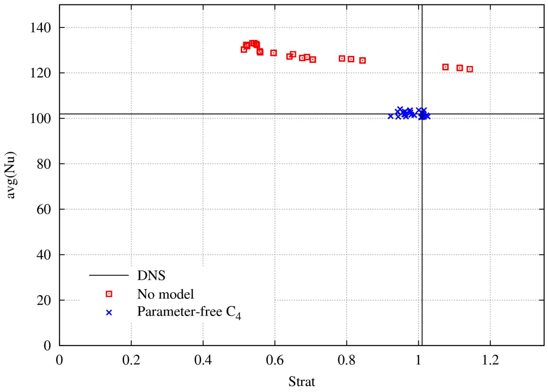


Averaged vertical velocity and temperature profiles at the horizontal mid-height plane at $Ra = 10^{10}$.

Even for a **very coarse** $8 \times 13 \times 30$ grid **reasonable results** are obtained!

$\Longrightarrow$ Results for different grids show the **robustness** of the method.

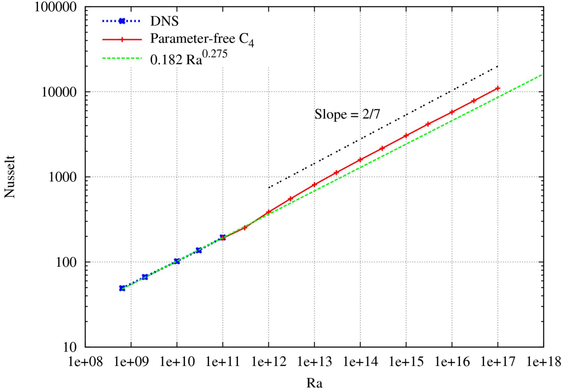
Challenging $\mathcal{C}_4$: mesh independence analysis



The overall Nusselt number and the centerline stratification for 50 randomly generated coarse grids with fixed stretching at $Ra = 10^{10}$.

$8 \leq N_x \leq 16$, $17 \leq N_y \leq 34$, and $40 \leq N_z \leq 80$.

Performance at very high Rayleigh numbers



Meshes have been generated with the criteria of keeping the same number of points in the BL than for $Ra = 10^{10}$.

Conclusions and Future Research

The results shown illustrate the potential of the conservative parameter-free $\widetilde{\mathcal{C}}_4$ smoothing as a new simulation shortcut.

The main advantages with respect exiting LES models can be summarized:

- **Robustnest**. As the smoothed governing equations preserve the symmetry properties of the original NS equations the solution cannot blow up (in the energy-norm, in 2D also: enstrophy-norm). It seems that even for very coarse meshes reasonably results can be obtained.
- **Universality**. No *ad hoc* phenomenological arguments that cannot be formally derived for the NS equations are used.
- The proposed method constitutes a **parameter-free turbulence model**.

Thank you for you attention

Further reading about $\mathcal{C}_4$ regularization

- Roel Verstappen, *"On restraining the production of small scales of motion in a turbulent channel flow"*, Computers & Fluids, 37 (7): 887-897, 2008
- F. X. Trias *et al.*, *"Parameter-free symmetry-preserving regularization modeling of a turbulent differentially heated cavity"*, Computers & Fluids, 39:1815-1831, 2010.
- F. X. Trias and R.W.C.P. Verstappen, *"On the construction of discrete filters for symmetry-preserving regularization models"*, Computers & Fluids, 40:139-148, 2011.