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Turbulent flow around a wall-mounted cube: DNS and regularization modelling

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Presentation outline

1. **Introduction**
   - Problem definition: Wall-mounted cube in a channel flow
   - Parallel fully-3D Poisson solver
   - DNS results for $Re_h = 7235$
   - Governing equations
2. **Regularization models for the simulation of turbulence**
   - Existing regularization: Leray and Navier-Stokes-α models
   - Symmetry-preserving regularization models
   - Mathematical foundation
   - Discretizing the $\mathcal{C}_n$ regularization modelling
   - Previous experience with $\mathcal{C}_4$ regularization modelling
3. **Results for a Turbulent flow around a Wall-mounted Cube**
   - Description of cases
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4. **Conclusions and Future Research**

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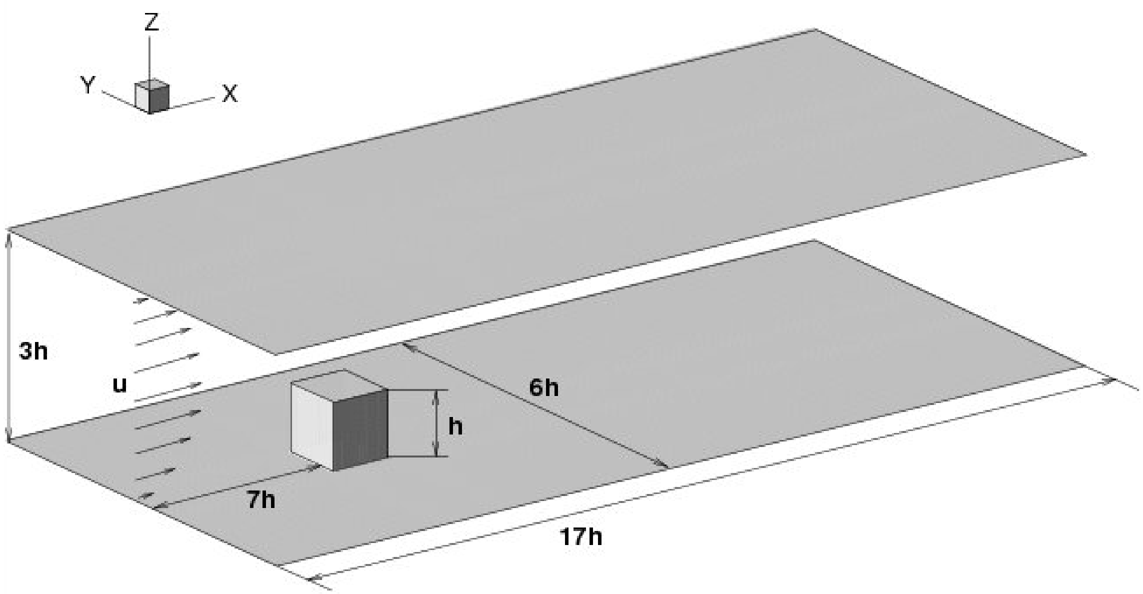
Problem definition: Wall-mounted cube in a channel flow



Dimensionless governing numbers:

- $Re_h = U_{bulk}h/\nu = 7235$ ...
- ...or $Re_\tau = 590$

Boundary conditions:

- Spanwise: Periodic BC
- Streamwise: prescribed **inlet profile** $\longrightarrow$
- **Non-slip BC** on the channel walls and one the surface of the cube (**No IB method is used!!**)

$$U/u_\tau = min(y^+, klny^+ + B)$$

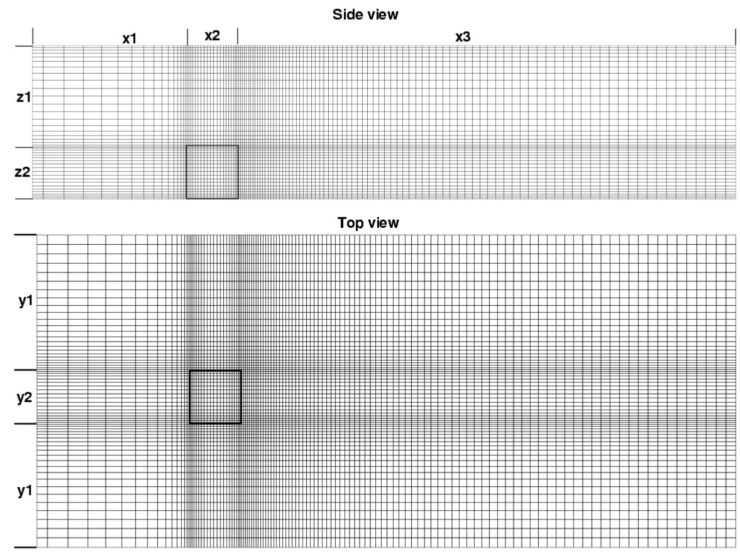
where $y^+ = (y/H)Re_\tau$, $u_\tau = Re_\tau \nu/H$ and $H = 3h/2$, $k = 0.25$, $B = 5.0$.

Parallel fully-3D Poisson solver

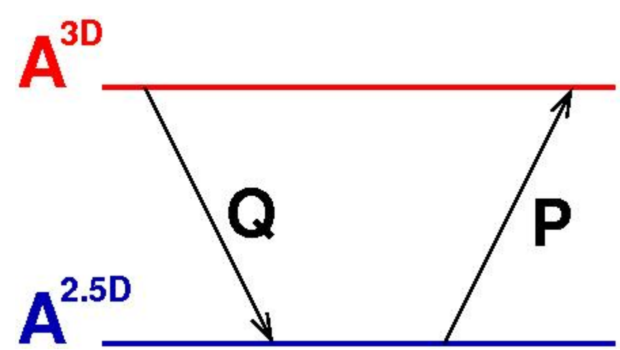
A two-level Multigrid method

Algorithm on the i-th iteration:

1. Smoother: $\mathbf{x}_i^{3D} \approx \left(\mathbf{A}^{3D}\right)^{-1} \mathbf{b}^{3D}$
   - **Locally preconditioned CG.**
   - Initial guess $\mathbf{x}_{i-1}^{3D}$
2. $\mathbf{r}_i^{3D} = \mathbf{A}^{3D}\mathbf{x}_i^{3D} - \mathbf{b}^{3D}$
3. $\mathbf{r}_i^{2.5D} = \mathbf{Q}\mathbf{r}_i^{3D}$
4. Error equation: $\mathbf{z}_i^{2.5D} \approx \left(\mathbf{A}^{2.5D}\right)^{-1} \mathbf{r}_i^{2.5D}$
   A **2.5D parallel Poisson solver** is used
   - FFT decompose $\mathbf{A}^{2.5D}$ into a set of 2D systems: $\hat{\mathbf{A}}_k^{2D}\hat{\mathbf{z}}_k^{2D} = \hat{\mathbf{r}}_k^{2D}$
   - $\hat{\mathbf{z}}_k^{2D} \approx \left(\hat{\mathbf{A}}_k^{2D}\right)^{-1} \hat{\mathbf{r}}_k^{2D}$ with $k = 1, \cdots, N_x$
     A 2D parallel solver is used.
   - Inverse FFT of $\hat{\mathbf{z}}_k^{2D}$ gives $\mathbf{z}_i^{2.5D}$.
5. $\mathbf{z}_i^{3D} = \mathbf{P}\mathbf{z}_i^{2.5D}$
6. $\mathbf{x}_{i+1}^{3D} = \mathbf{x}_i^{3D} + \mathbf{z}_i^{3D}$



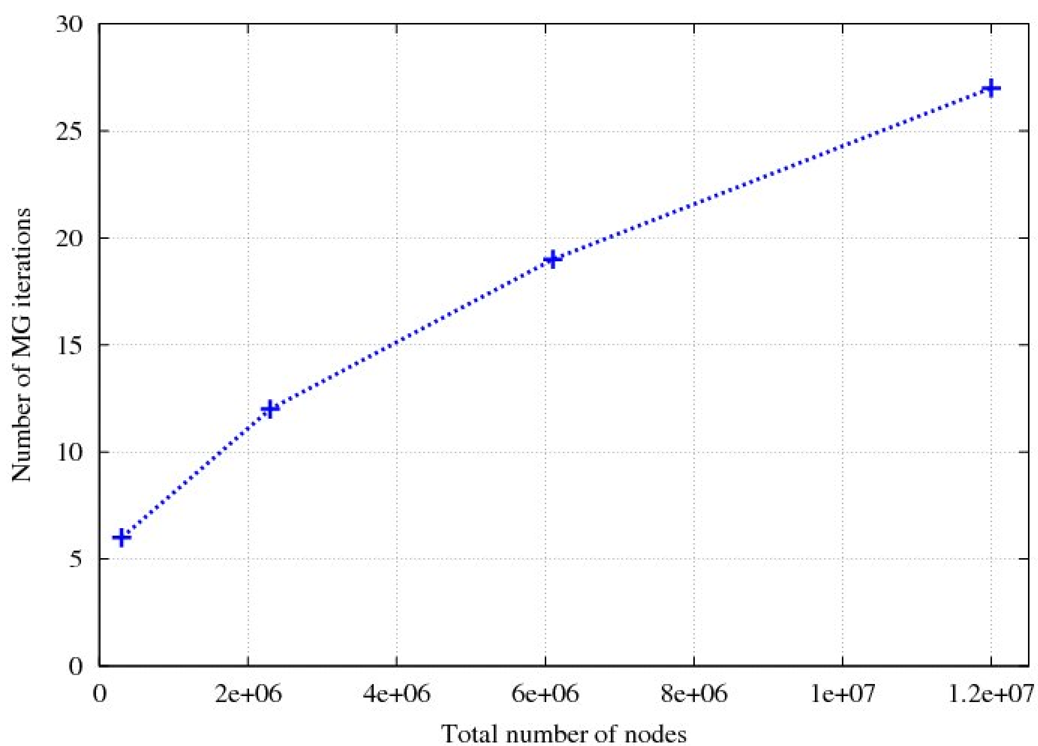
Cartesian grid **stretched in the 3 directions!!!**



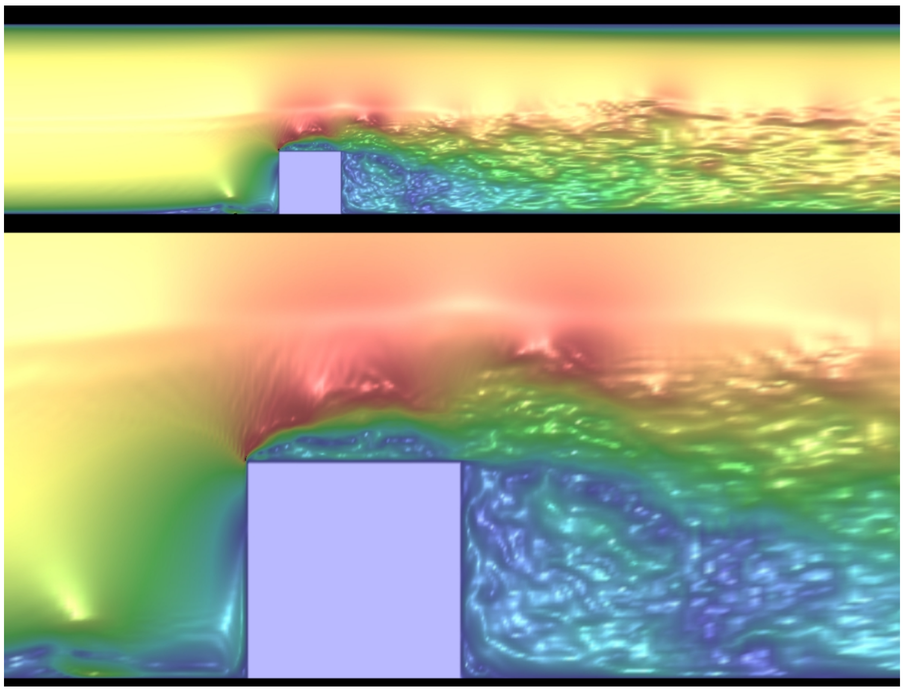
Parallel fully-3D Poisson solver

Parallel performance

- $Re_h = 5000$
- PCG: 15 iterations using Jacobi as preconditioner
- Most of the time is consumed solving $\mathbf{A}^{2.5D}\mathbf{z}_i^{2.5D} = \mathbf{r}_i^{2.5D}$ therefore number of MG iterations is a good measure of how many times slower is the 3D solver respect the 2.5D



DNS results at $Re_h = 7235$



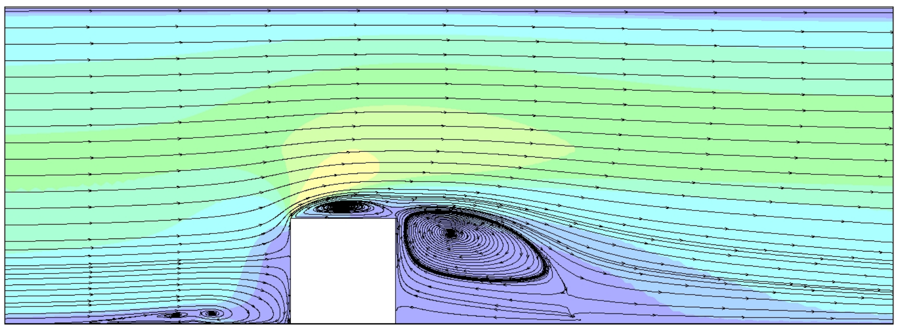
Some details about **DNS simulation**:

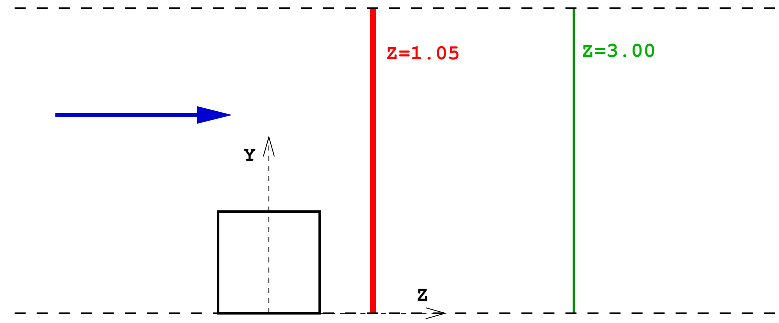
- Fully-3D Poisson solver
- Mesh size: $400 \times 200 \times 196 \approx 16M$
- Computing Time: ≈ 1 month - 300 CPUs on MareNostrum supercomputer
- Symmetry-preserving discretization

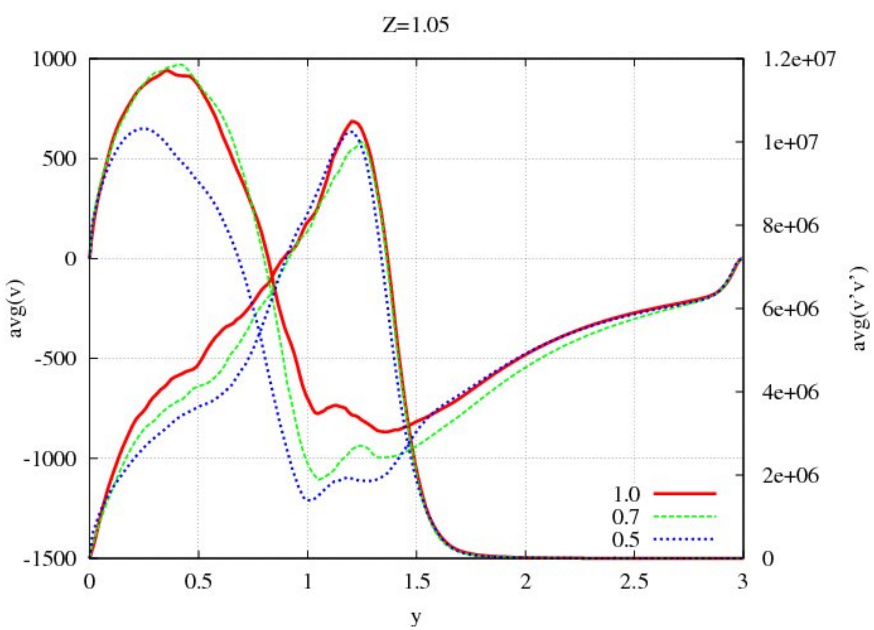
Complexity of the flow:

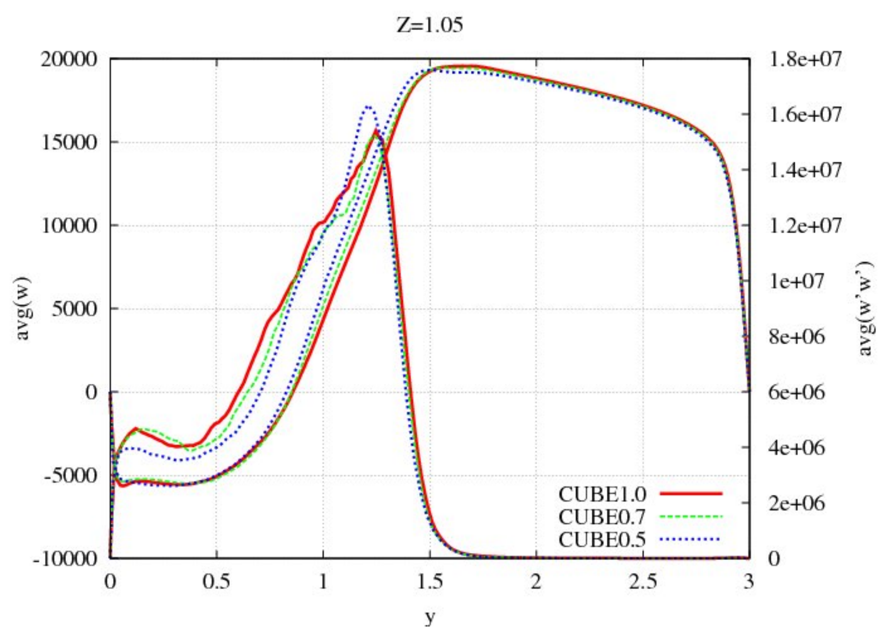
- Vortical structures:
  - ⋆ horseshoe-type at the upstream face
  - ⋆ arc-shaped in the wake
- Flow separation
- Vortex shedding

Verification of simulation









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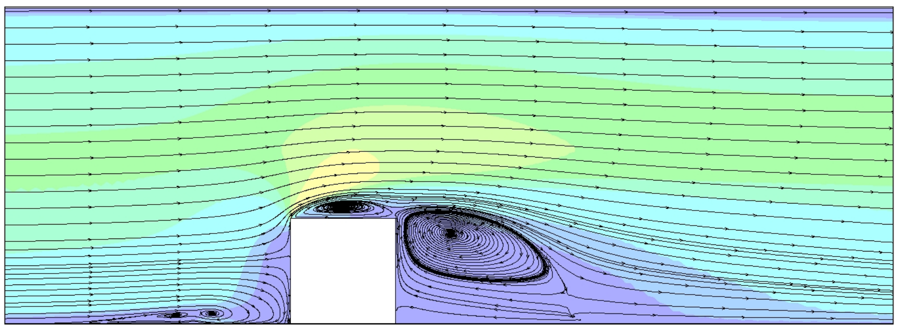
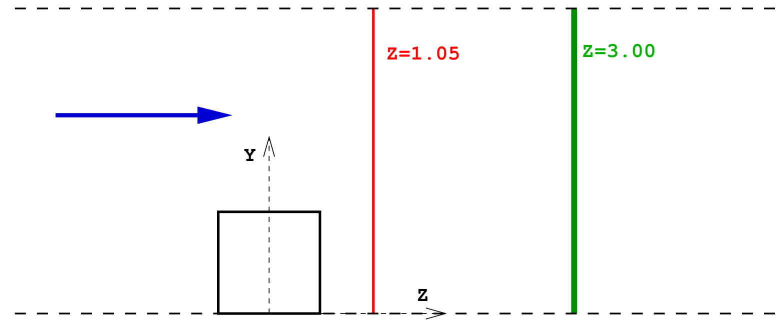
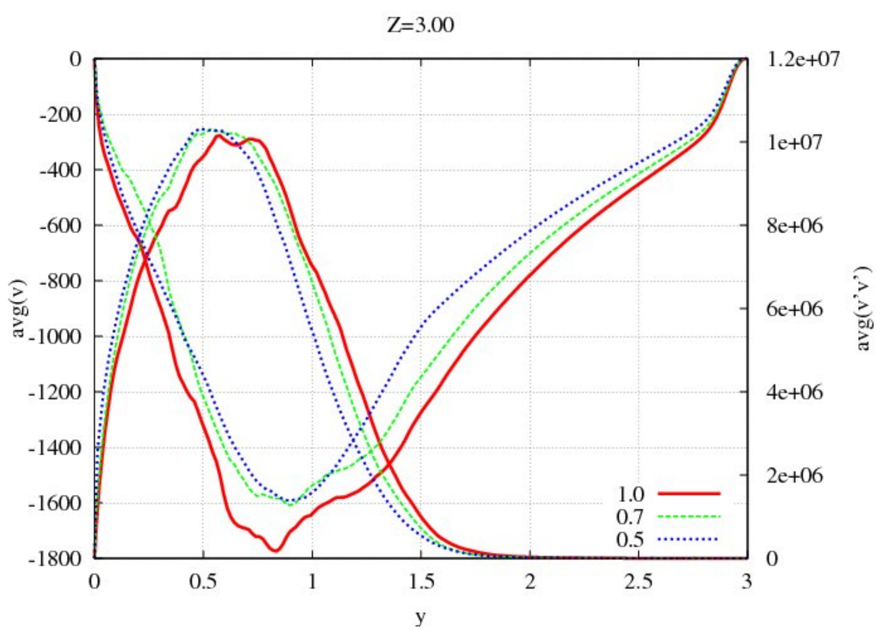
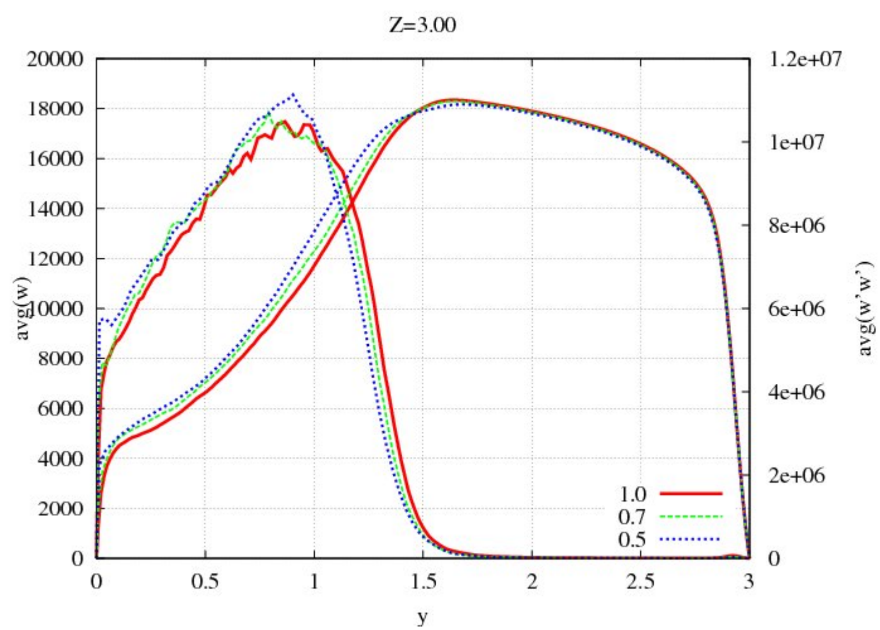
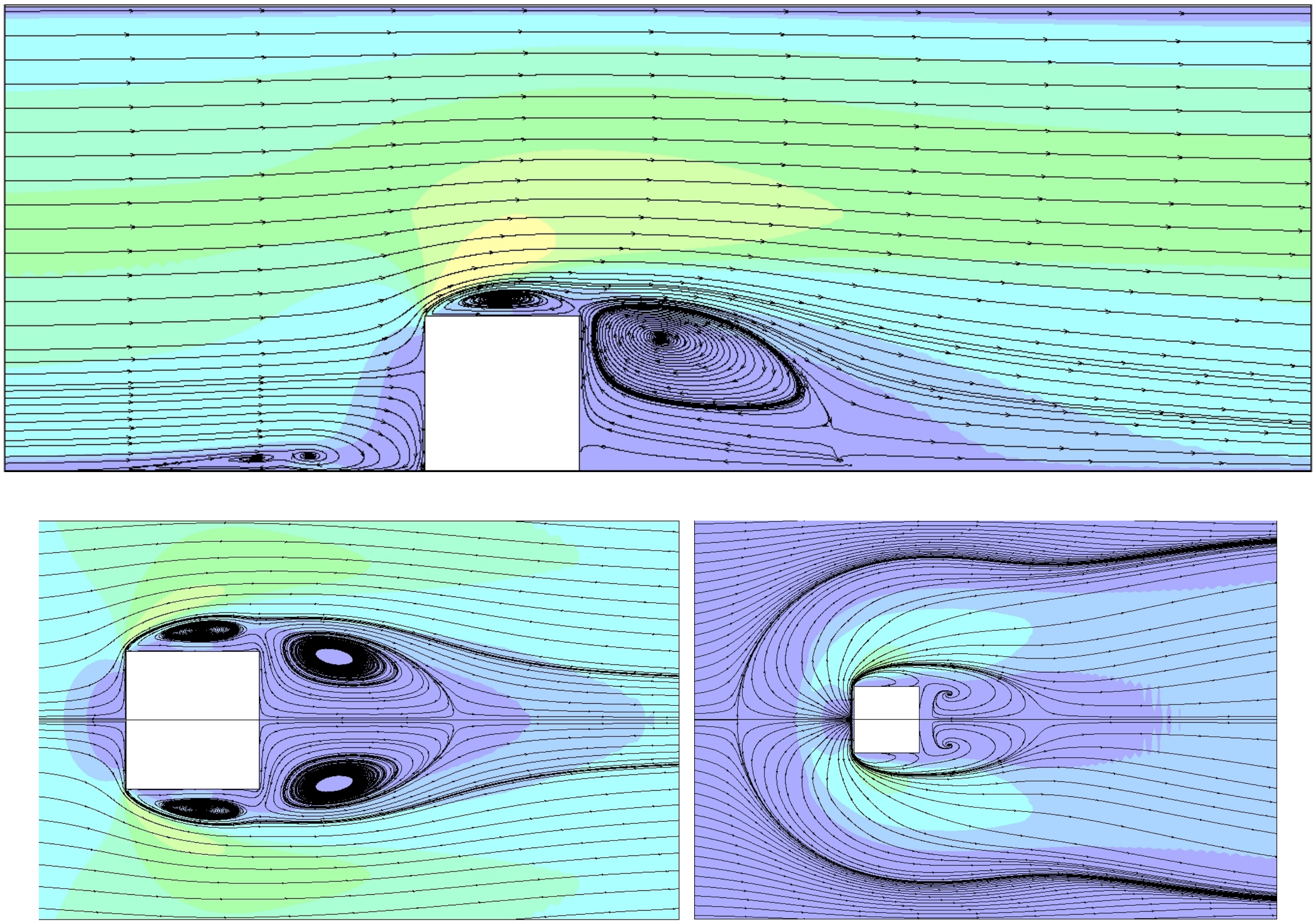








Streamlines



Governing equations

Incompressible Navier-Stokes equations

$$\begin{aligned} \partial_t u + \mathcal{C}(u,u) &= Pr\mathcal{D}(u) - \nabla p + f \\ \nabla \cdot u &= 0 \end{aligned}$$

where the **nonlinear convective term** is given by

$$\mathcal{C}(u,v) = (u \cdot \nabla) v$$

and the linear dissipative term is given by

$$\mathcal{D}(u) = \frac{1}{Re}\nabla^2 u$$

Regularization modelling

As the full energy spectrum cannot be computed, a **dynamically less complex mathematical formulation** is sought. We consider smooth approximations (regularizations) of the nonlinearity,

$$\partial_t u_\epsilon + \widetilde{\mathcal{C}}(u_\epsilon, u_\epsilon) \quad = \quad \mathcal{D}(u_\epsilon) - \nabla p_\epsilon + f$$

such approximations may fall in the **Large-Eddy Simulation** (LES) concept,

$$\partial_t \bar{u}_\epsilon + \mathcal{C}(\bar{u}_\epsilon, \bar{u}_\epsilon) \quad = \quad \mathcal{D}(\bar{u}_\epsilon) - \nabla \bar{p}_\epsilon + f + \mathcal{M}_1(\bar{u}_\epsilon, \bar{u}_\epsilon)$$

if the filter is invertible:

$$\mathcal{M}_1(\bar{u}_\epsilon, \bar{u}_\epsilon) \quad = \quad \mathcal{C}(\bar{u}_\epsilon, \bar{u}_\epsilon) - \overline{\widetilde{\mathcal{C}}(u_\epsilon, u_\epsilon)}$$

Previous regularization modellings

Leray and Navier-Stokes-α models

The regularization methods basically **alters the convective term** to **restrain the production of small scales** of motion.

- Leray model:

$$\partial_t u_\epsilon + \mathcal{C}(\bar{u}_\epsilon, u_\epsilon) = \mathcal{D}(u_\epsilon) - \nabla p_\epsilon$$

- Navier-Stokes-α model:

$$\partial_t u_\epsilon + \mathcal{C}_r(u_\epsilon, \bar{u}_\epsilon) = \mathcal{D}(u_\epsilon) - \nabla \pi_\epsilon$$

where the $\pi = p + u^2/2$ and the convetive operator in rotational form is defined as

$$\mathcal{C}_r(u, v) = (\nabla \times u) \times v$$

However, in doing so some of the **inviscid invariants** (kinetic energy, enstrophy in 2D and helicity in 3D) are **not conserved**.

Symmetry-preserving regularization models (1/2)

In order to conserve the following inviscid invariants

- Kinetic energy
$$\int_\Omega \boldsymbol{u} \cdot \boldsymbol{u} d\Omega$$
- Enstrophy (in 2D)
$$\int_\Omega (\nabla \times \boldsymbol{u}) \cdot (\nabla \times \boldsymbol{u}) d\Omega$$
- Helicity (in 3D)
$$\int_\Omega (\nabla \times \boldsymbol{u}) \cdot \boldsymbol{u} d\Omega$$

the **approximate convective operator** has to be **skew-symmetric**:

$$\left(\widetilde{\mathcal{C}}(u, v), w\right) = -\left(\widetilde{\mathcal{C}}(u, w), v\right)$$

Symmetry-preserving regularization models (2/2)

This criterion yields the following class of approximations,

$$\partial_t u_\epsilon + \mathcal{C}_n(u_\epsilon, u_\epsilon) = \mathcal{D}(u_\epsilon) - \nabla p_\epsilon$$

in which the convective term in smoothened according to:

$$\begin{aligned}
\mathcal{C}_2(u,v) &= \overline{\mathcal{C}(\bar{u},\bar{v})} \\
\mathcal{C}_4(u,v) &= \mathcal{C}(\bar{u},\bar{v}) + \overline{\mathcal{C}(\bar{u},v')} + \overline{\mathcal{C}(u',\bar{v})} \\
\mathcal{C}_6(u,v) &= \mathcal{C}(\bar{u},\bar{v}) + \mathcal{C}(\bar{u},v') + \mathcal{C}(u',\bar{v}) + \overline{\mathcal{C}(u',v')}
\end{aligned}$$

where $u' = u - \bar{u}$ and $\mathcal{C}_n(u,v) = \mathcal{C}(u,v) + \mathcal{O}(\epsilon^n)$ for **any symmetric filter**.

Mathematical foundation

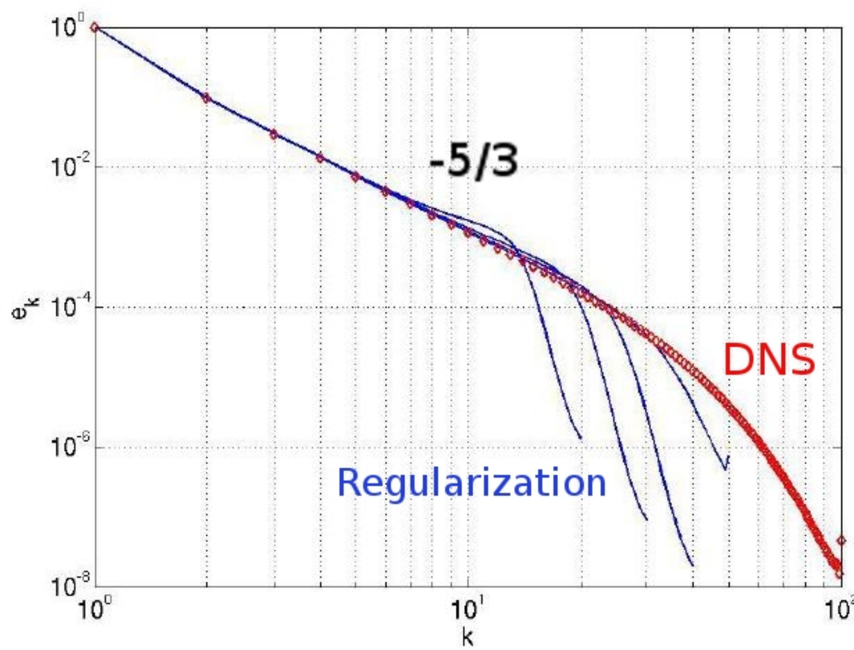
Energy flux equation for the symmetry-preserving regularization resembles the NS

$$\frac{1}{2}\frac{d}{dt}\,|u_{kk'}|^2 + \nu\,|\nabla u_{kk'}|^2 = \widetilde{T}_k - \widetilde{T}_{k'} \quad \longrightarrow \quad \nu < |\nabla u_{kk'}|^2 >=< \widetilde{T}_k > - < \widetilde{T}_{k'} >$$

$\Longrightarrow$ Following the same steps as Foias *et al.* (2001)

- $< \widetilde{T}_k >$ is a nonnegative, monotone decreasing function.
- $< \widetilde{T}_k >$ is approximately constant for $k_a < k < k_b$ (existence of inertial range).

$\Longrightarrow$ **$-5/3$ scaling !!!**



LES-interpretation of $\mathcal{C}_4$-regularization

$$\partial_t \bar{u}_\epsilon + \mathcal{C}(\bar{u}_\epsilon, \bar{u}_\epsilon) - \mathcal{D}(\bar{u}_\epsilon) + \nabla \bar{p}_\epsilon =$$

$$\mathcal{C}(\bar{u}_\epsilon, \bar{u}_\epsilon) - \overline{\mathcal{C}_4(u_\epsilon, u_\epsilon)} =$$

$$-\frac{\epsilon^2}{12}\nabla \cdot (\nabla \bar{u}_\epsilon : \nabla \bar{u}_\epsilon) + \mathcal{O}(\epsilon^4)$$

gradient model + stabilization

Discretizing the $\mathcal{C}_n$ regularization modelling

- The discretization is also a regularization. The **spatial discretization** method preserves the symmetry and conservation properties too

$$\Omega_s \frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}(\boldsymbol{u}_s)\boldsymbol{u}_s + \mathsf{D}\boldsymbol{u}_s + \Omega_s \mathsf{G}\boldsymbol{p}_c = \boldsymbol{0}_s \qquad \text{with} \quad \mathsf{C}(\boldsymbol{u}_s) = -\mathsf{C}^*(\boldsymbol{u}_s)$$

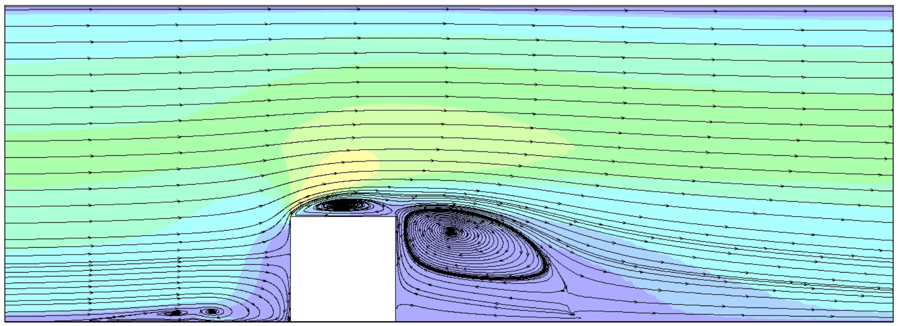
and is therefore well-suited to test the proposed regularization model.

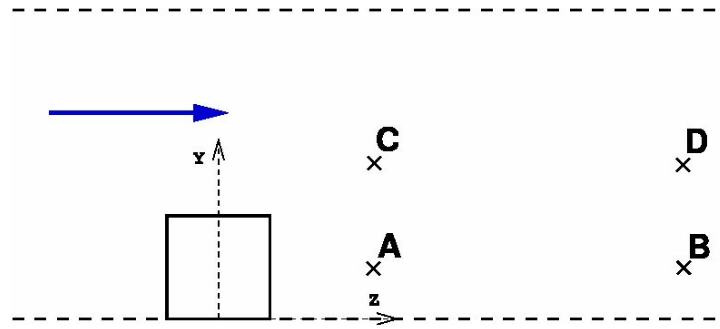
- A normalized self-adjoint **filter** has been chosen. In 1D it becomes

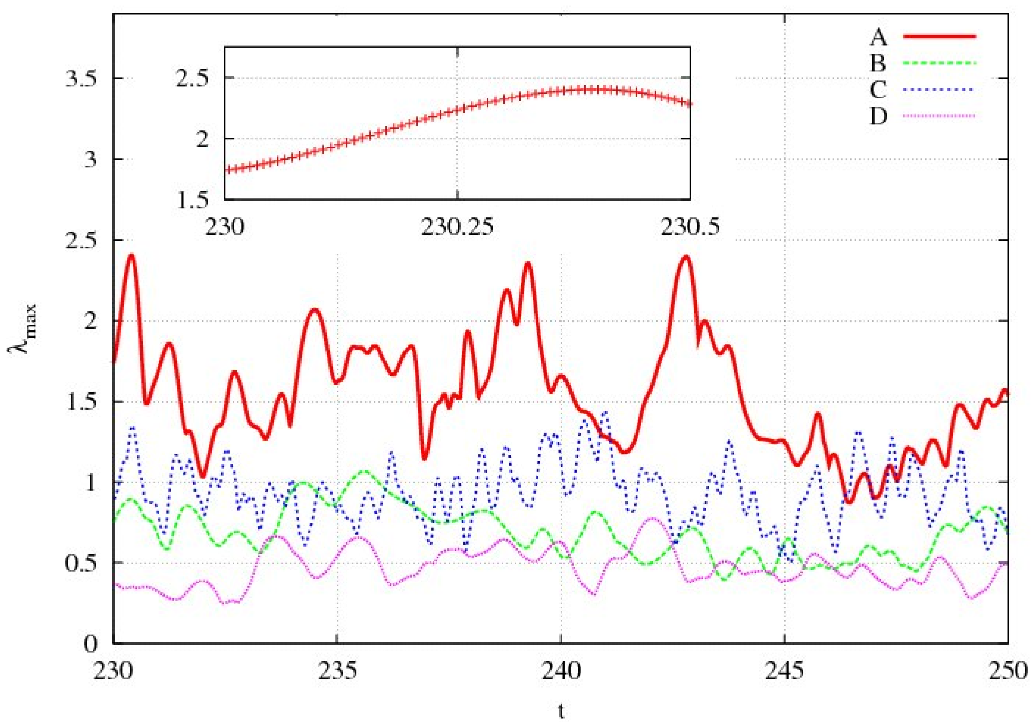
$$\overline{\phi}_i = \frac{\epsilon^4 - 4\epsilon^2}{1152}(\phi_{i+2} + \phi_{i-2}) + \frac{16\epsilon^2 - \epsilon^4}{288}(\phi_{i+1} + \phi_{i-1}) + \frac{\epsilon^4 - 20\epsilon^2 + 192}{192}\phi_i$$

Thus, **only one parameter** needs to be **prescribed**: the **local filter length** ϵ **!!!**

Parameter-free approach







Parameter-free approach

The **vortex-stretching** and **dissipation term** contributions to $(1/|\omega|^2)\partial_t|\omega|^2$ are given by

$$\frac{\omega \cdot \mathcal{C}(\omega, u)}{\omega \cdot \omega} = \frac{\omega \cdot \mathcal{S}(u)\,\omega}{\omega \cdot \omega} \quad \text{and} \quad \frac{1}{Re}\frac{\nabla\omega : \nabla\omega}{\omega \cdot \omega}$$

At the **smallest grid scale**, $k = \pi/h$, **convection** may **dominate** diffusion

$$\frac{\omega_k \cdot \mathcal{C}(\omega, u)_k}{\omega_k \cdot \omega_k} > \frac{1}{Re}k^2$$

$\Longrightarrow$ In the present work we **determine the filter width** ϵ from

$$\frac{\omega_k \cdot \mathcal{C}_4(\omega, u)_k}{\omega_k \cdot \omega_k} \approx \frac{1}{Re}k^2$$

Parameter-free approach

Note that $\mathcal{C}_4(u, v)$ depends on the filter length ϵ. For the **smallest scale** this dependence becomes

$$\frac{\omega_k \cdot \mathcal{C}_4(\omega, u)_k}{\omega_k \cdot \omega_k} \approx f_4(\hat{g}_k(\epsilon)) \frac{\omega_k \cdot \mathcal{S}(u)\,\omega_k}{\omega_k \cdot \omega_k} \leq f_4(\hat{g}_k(\epsilon))\,\lambda_{max}(\mathcal{S})$$

where $0 < \hat{g}_k(\epsilon) \leq 1$ is the transfer function of the filter and the damping function $0 < f_4 \leq 1$.

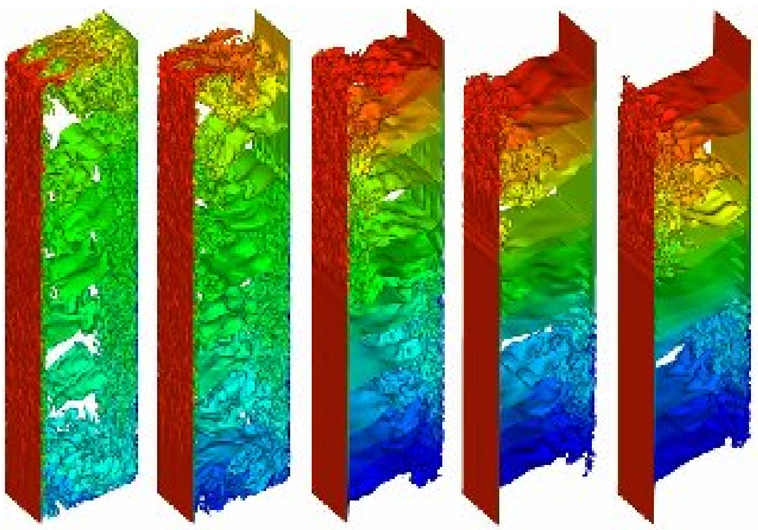
$\Longrightarrow$ Therefore, it suffices that following inequality be **locally** hold

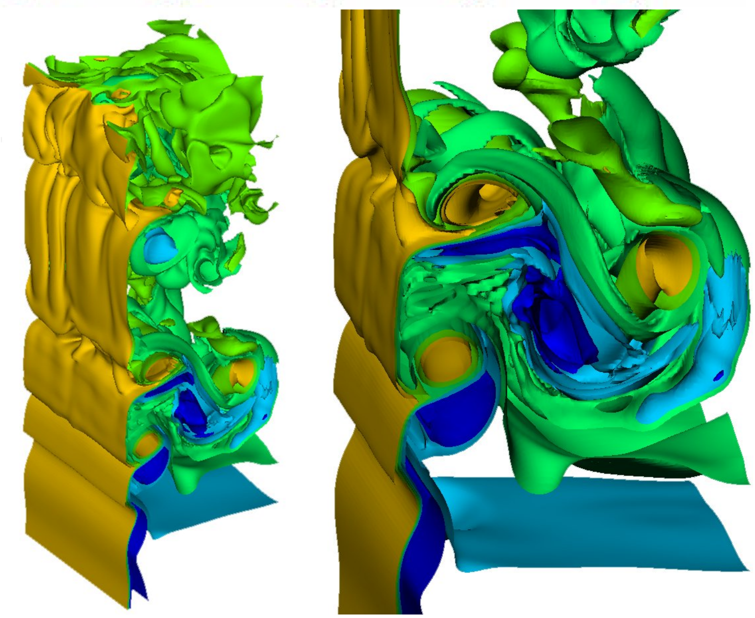
$$f_4(\hat{g}_k(\epsilon)) \leq \frac{1}{Re}\frac{k^2}{\lambda_{max}(\mathcal{S})} \qquad \longrightarrow \qquad \epsilon$$

to guarantee that the **production of smaller scales of motion be stopped at the smallest scale** set by the mesh.

Previous experience with the parameter-free $\mathcal{C}_4$-regularization

Turbulence flow in a differentially heated cavity





Some details about **DNS simulations**:

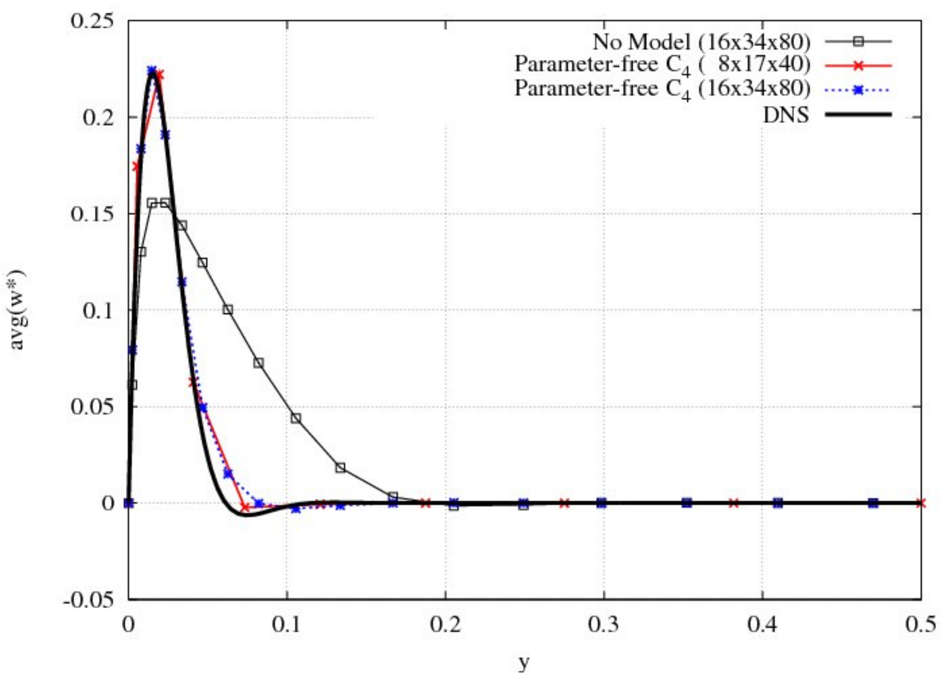
- Mesh size: $128 \times 682 \times 1278$
- Computing Time: ≈ 3 months - 256 CPUs
- 4^{th}-order symmetry-preserving discretization
- $A_z = 4$

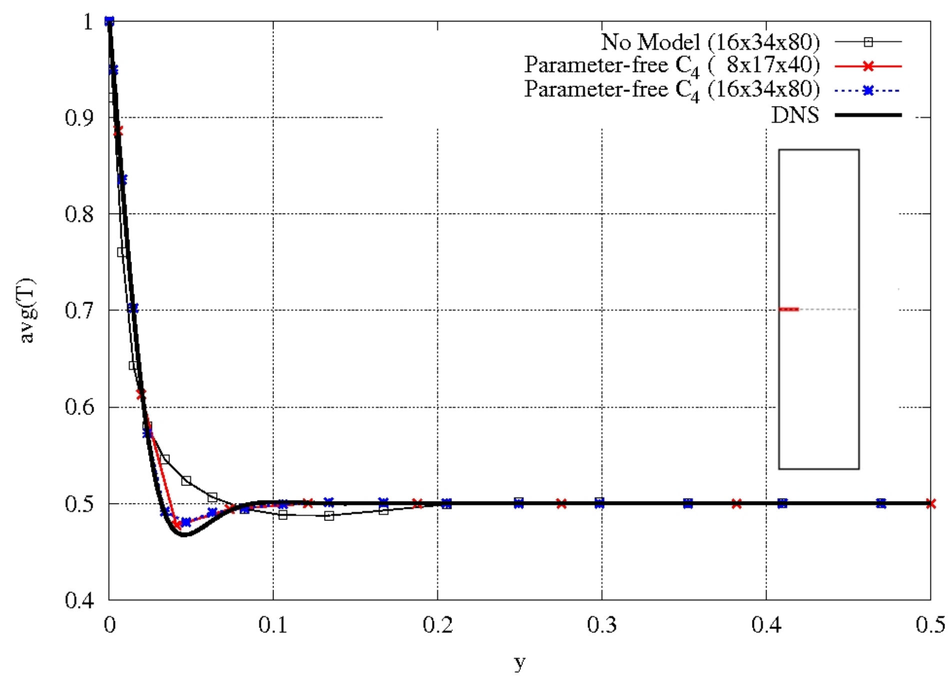
Complexity of the flow:

- Boundary layers
- Stratified cavity core
- Internal waves
- Recirculation areas

Previous experience with the parameter-free $\mathcal{C}_4$-regularization

Turbulence flow in a differentially heated cavity at $Ra = 10^{10}$

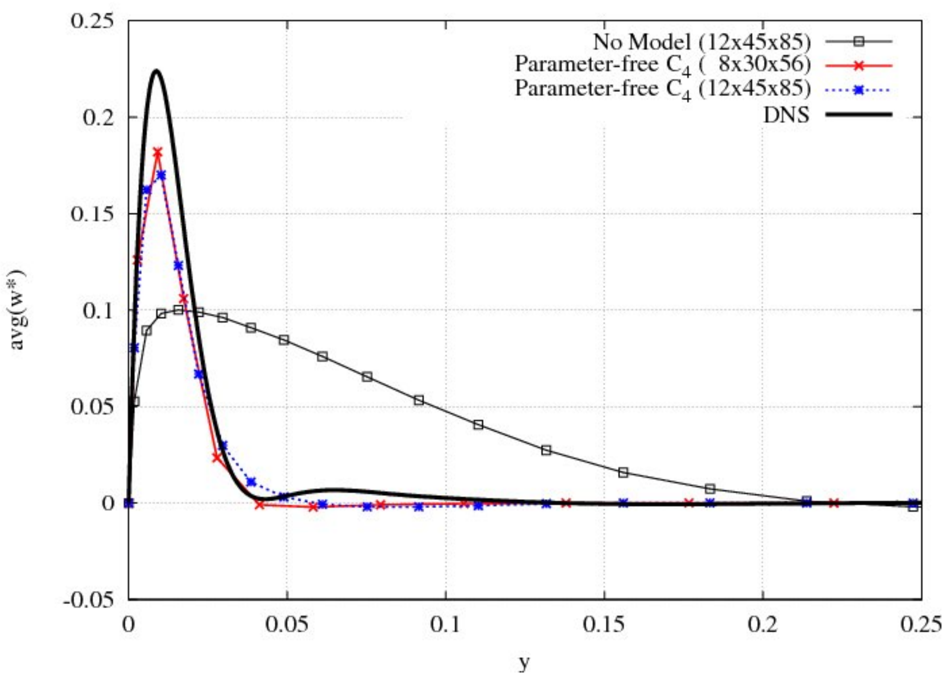


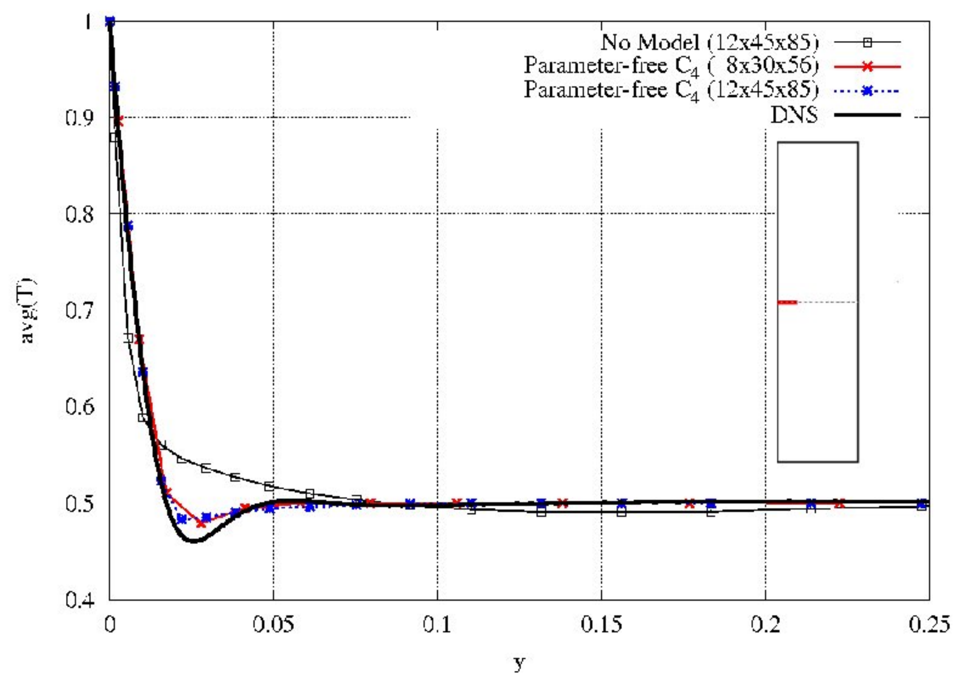


Averaged vertical velocity and temperature profiles at the horizontal mid-height plane.

Previous experience with the parameter-free $\mathcal{C}_4$-regularization

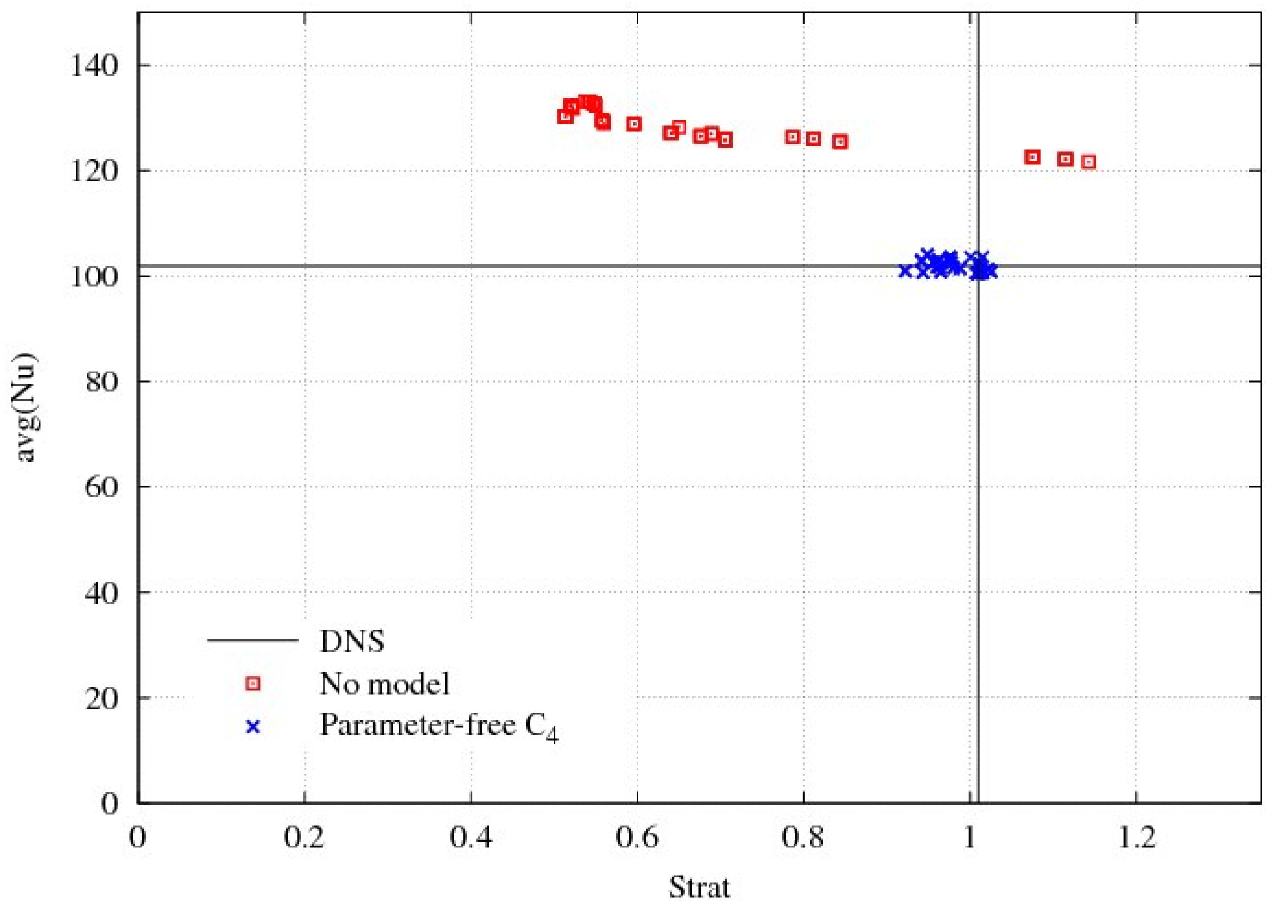
Turbulence flow in a differentially heated cavity at $Ra = 10^{11}$





Averaged vertical velocity and temperature profiles at the horizontal mid-height plane.

A challenging test: mesh independence analysis at very coarse grids



The overall Nusselt number and the centerline stratification for 50 randomly generated coarse grids for the DHC problem at $Ra = 10^{10}$. $8 \leq N_x \leq 16$, $17 \leq N_y \leq 34$, and $40 \leq N_z \leq 80$.

Results for the flow around the wall-mounted cube at $Re_h = 7235$

- Parameter-free $\mathcal{C}_4$-regularization model is tested.
- Two coarse meshes are considered

	DNS	MeshA	MeshB
Nx	400	40	50
Ny	196	20	26
Nz	200	20	25

- Coarse meshes MeshA and MeshB keep the same grid points distribution of the DNS but with **much less spatial resolutions**.

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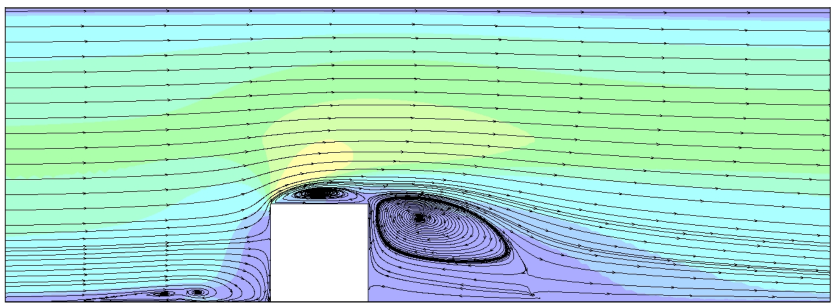
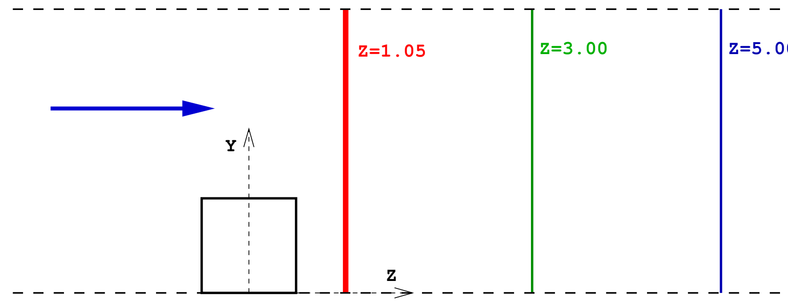
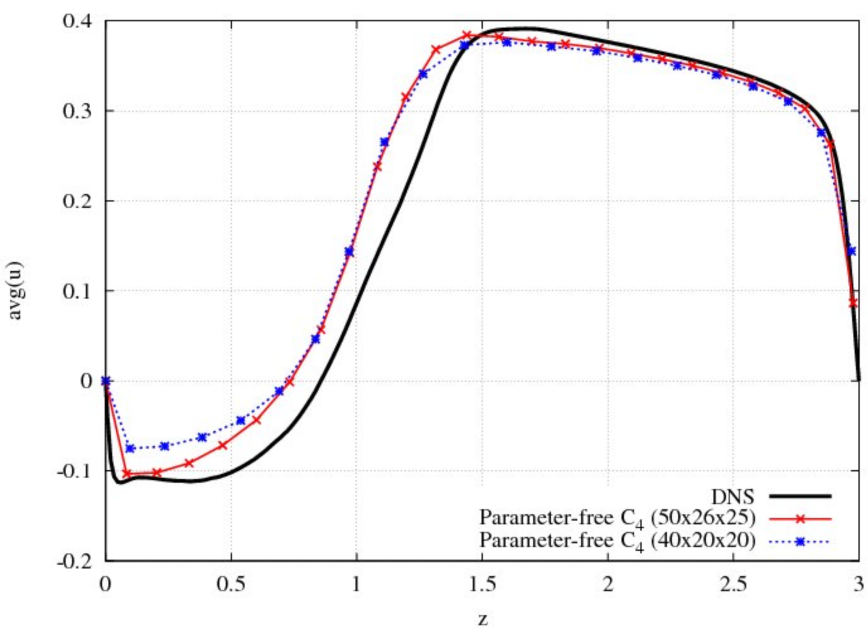
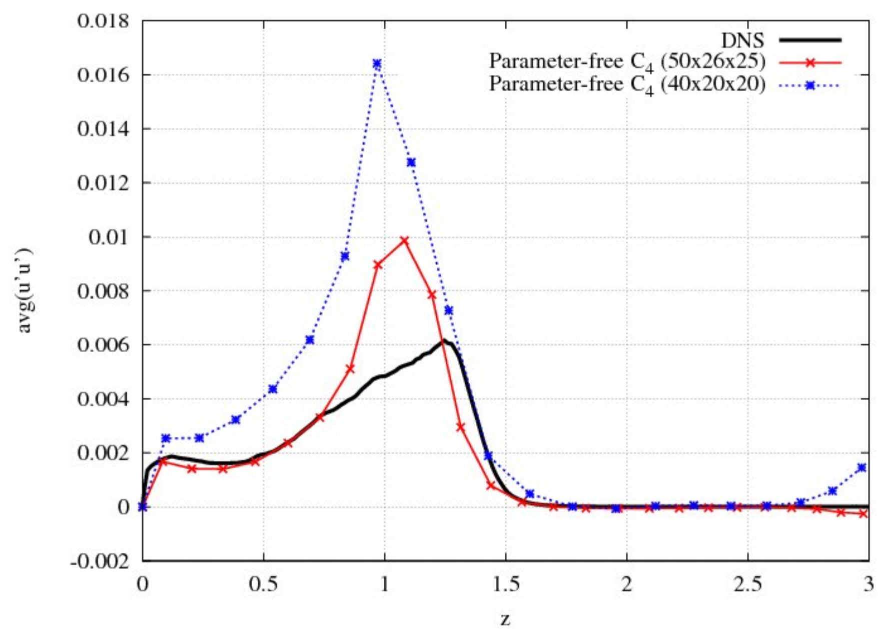
Results for the flow around the wall-mounted cube at $Re_h = 7235$

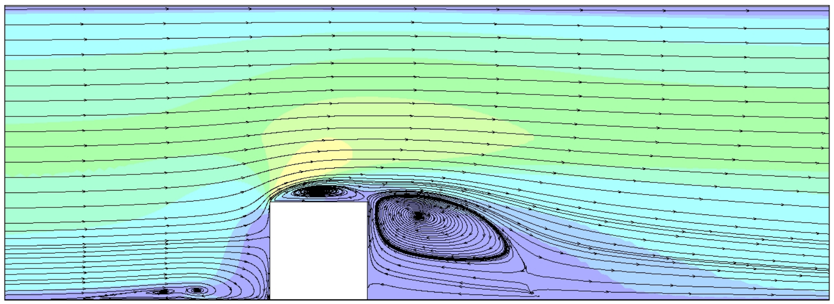


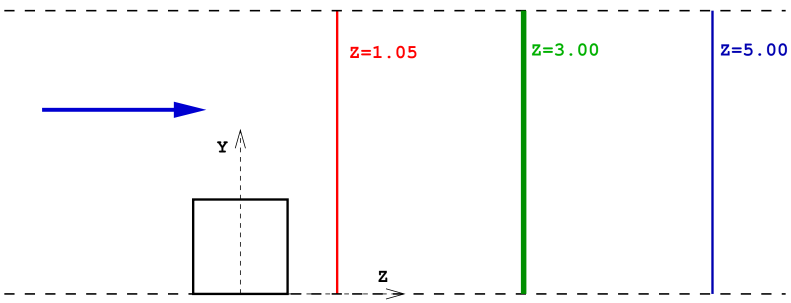


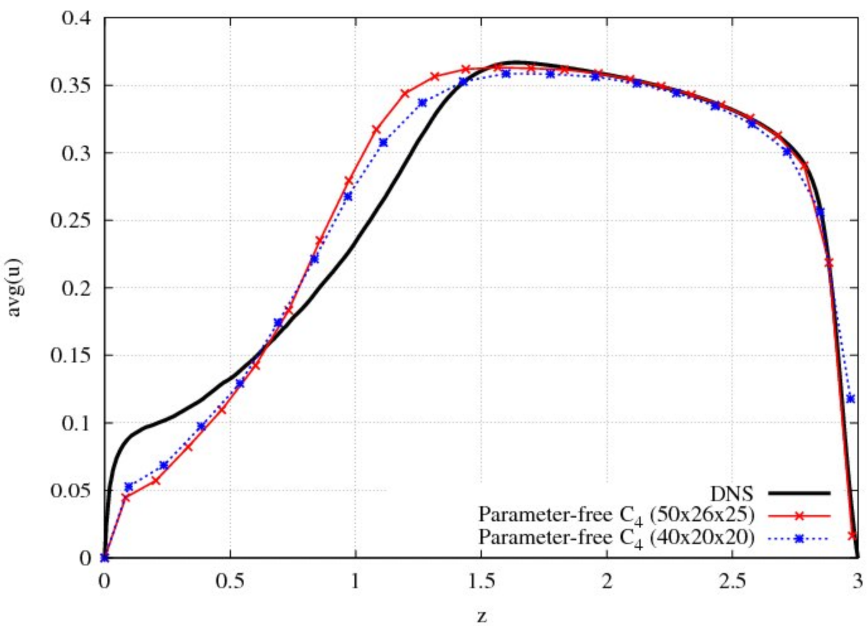


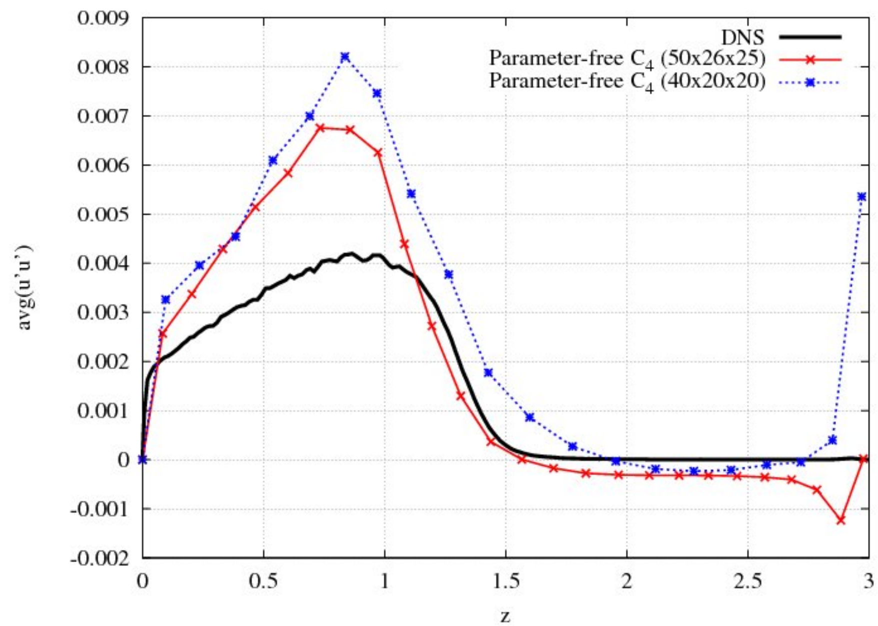


Results for the flow around the wall-mounted cube at $Re_h = 7235$

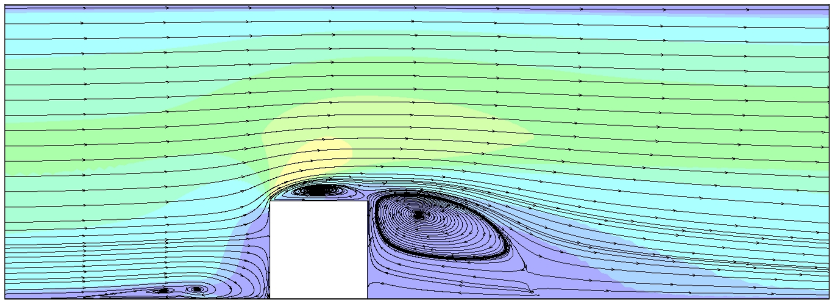


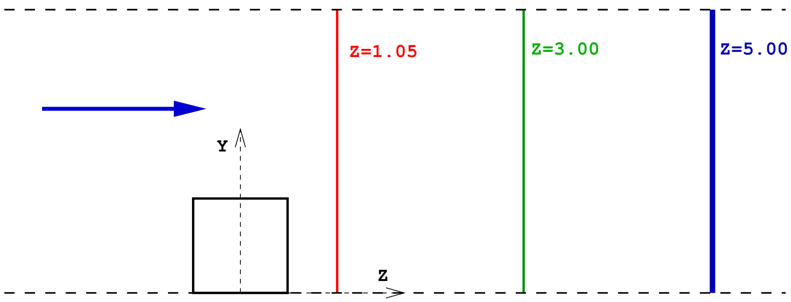


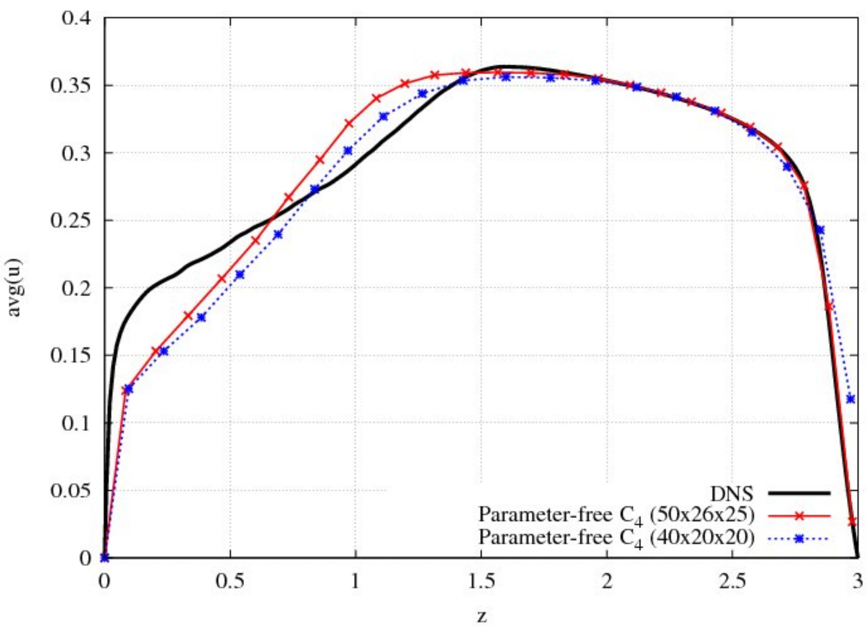


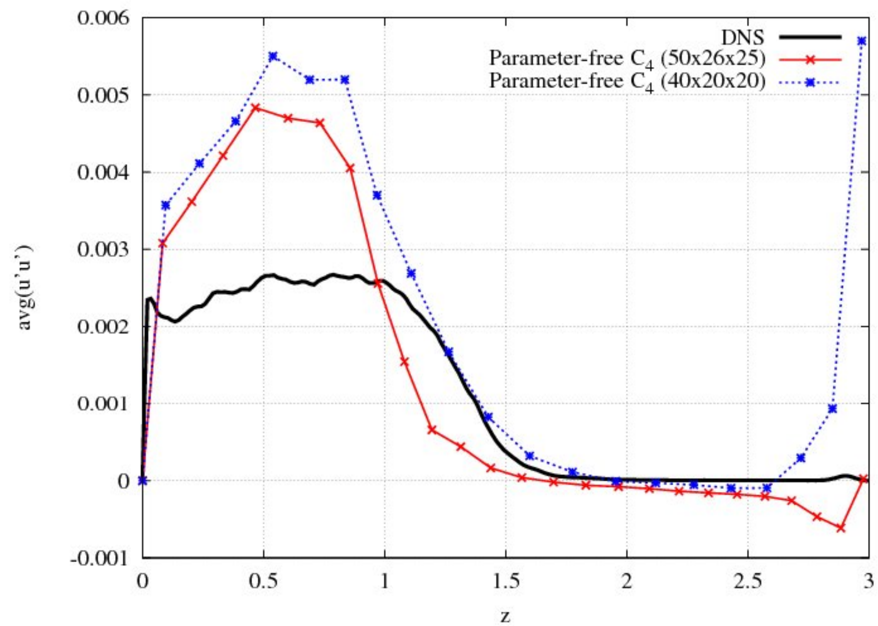


Results for the flow around the wall-mounted cube at $Re_h = 7235$









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Conclusions and Future Research

The results shown illustrate the potential of the conservative parameter-free $\widetilde{\mathcal{C}}_4$ smoothing as a new simulation shortcut.

The main advantages with respect exiting LES models can be summarized:

- **Robustnest**. As the smoothed governing equations preserve the symmetry properties of the original Navier-Stokes equations the solution cannot blow up (in the energy-norm, in 2D also: enstrophy-norm). It seems that even for very coarse meshes reasonably results can be obtained.
- **Universality**. No *ad hoc* phenomenological arguments that can not be formally derived for the Navier-Stokes equations are used.
- The proposed method constitutes a **parameter-free turbulence model**.

Since now, the method has been **successfully tested** on completely different turbulent configurations such as:

- Channel flow.
- Differentially heated cavity at different Ra-numbers.
- A plane impinging jet.
- **Flow around a wall-mounted cube.**

Thank you for you attention