Spectrally-consistent regularization of turbulent Rayleigh-Bénard convection

F. Dabbagh[1] F. X. Trias[1] A. Gorobets[1,2] and A. Oliva[1]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia, ETSEIAT, Terrassa, Spain
[2]Keldysh Institute of Applied Mathematics, 4A, Miusskaya Sq., Moscow 125047, Russia

6th European Conference on Computational Fluid Dynamics (ECFD VI)
21st July 2014

Centre Tecnològic de Transferència de Calor
Laboratori de Termotècnia i Energètica
UNIVERSITAT POLITÈCNICA DE CATALUNYA

Outline

Rayleigh-Bénard convection (RBC)

- Definition: a convective cell heated from below and cooled from above.
- Flow dynamics are characterized by:

Rayleigh number:

$$Ra = \frac{g\alpha H^3 \Delta T}{\nu\kappa}$$

Prandtl number:

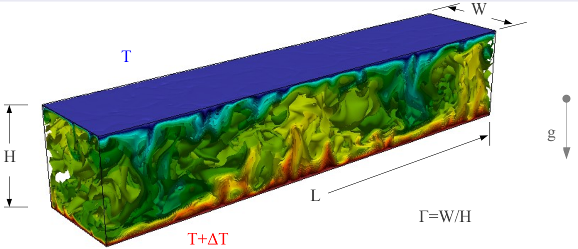
$$Pr = \frac{\nu}{\kappa}$$

cell aspect ratio Γ.

- system respond

$$Nu = \frac{wT - \kappa \partial T/\partial z}{\kappa \Delta T/H}$$

- Different flow behaviour between the near-isothermal walls and the bulk that is connected by the thermal plumes [Chillá & Schumacher, Eur. J. Phys. 2012]



Non-dimensional governing equations:

$$\begin{aligned} \nabla \cdot \boldsymbol{u} &= \boldsymbol{0} \,, \\ \partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u}, \boldsymbol{u}) &= -\nabla p + \mathcal{D}\boldsymbol{u} + \boldsymbol{f} \,, \\ \partial_t T + \mathcal{C}(\boldsymbol{u}, T) &= Pr^{-1} \mathcal{D} T \end{aligned}$$

- $\mathcal{C}(\boldsymbol{u}, \boldsymbol{u}) = (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$ is the convective term.
- $\mathcal{D}\boldsymbol{u} = (Pr/Ra)^{1/2} \nabla^2 \boldsymbol{u}$ is the diffusive term.
- $\boldsymbol{f} = (0, 0, T)$ is the body forces term.

- 4^{th}-order symmetry-preserving spatial discretizations [Verstappen & Veldman, J. Comp. Phys. 2003].

Parameters

- $Ra = 10^8$, $Pr = 0.7$, rectangular cell of $\Gamma = 1$ and $L = \pi H$.
- Periodic boundaries at L direction.
- Adiabatic side walls.

Non-dimensional governing equations:

$$\begin{aligned} \nabla \cdot \boldsymbol{u} &= \boldsymbol{0}\,, \\ \partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u},\boldsymbol{u}) &= -\nabla p + \mathcal{D}\boldsymbol{u} + \boldsymbol{f}\,, \\ \partial_t T + \mathcal{C}(\boldsymbol{u},T) &= Pr^{-1}\mathcal{D}T \end{aligned}$$

- $\mathcal{C}(\boldsymbol{u},\boldsymbol{u}) = (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}$ is the convective term.
- $\mathcal{D}\boldsymbol{u} = (Pr/Ra)^{1/2}\nabla^2\boldsymbol{u}$ is the diffusive term.
- $\boldsymbol{f} = (0,0,T)$ is the body forces term.
- 4^{th}-order symmetry-preserving spatial discretizations [Verstappen & Veldman, J. Comp. Phys. 2003].

Global heat flux $Nu_V = 1 + (RaPr)^{1/2}\langle wT\rangle_{V,t}$
ref. ratio$=\Delta l_{i,max}/\eta_{Gr\ddot{o}}$.

Parameters

- $Ra = 10^8$, $Pr = 0.7$, rectangular cell of $\Gamma = 1$ and $L = \pi H$.
- Periodic boundaries at L direction.
- Adiabatic side walls.

Grötzbach criterion [Grötzbach, J. Comp. Phys. 1983]

maximum cell size

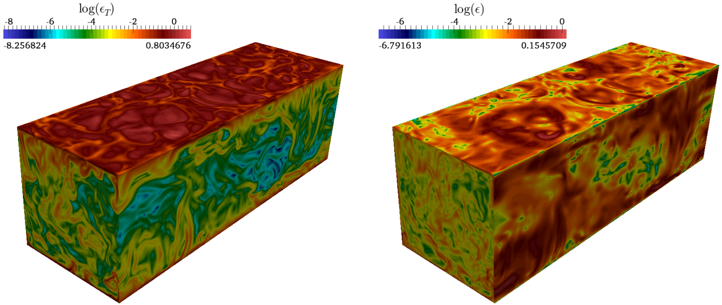
$$\eta_{Gr\ddot{o}} \le \pi\left(\frac{Pr^2}{(Nu-1)Ra}\right)^{1/4}, Pr \le 1$$

$$\eta_{Gr\ddot{o}} \le \pi\left(\frac{1}{(Nu-1)Ra}\right)^{1/4}, Pr > 1$$

ref. ratio	$N_x \times N_y \times N_z$	Nu	Nu_V/Nu	$(\langle\epsilon\rangle_{V,t}(RaPr)^{1/2}+1)/Nu$	$(\langle\epsilon_T\rangle_{V,t}(RaPr)^{1/2})/Nu$
1.1	$256\times150\times150$	31.44	0.997	0.966	0.983
1.0	$288\times158\times158$	31.10	0.997	0.971	0.985
0.9	$320\times174\times174$	31.00	0.999	0.976	0.988
0.8	$342\times192\times192$	30.93	0.999	0.982	0.991
0.7	$400\times208\times208$	30.86	1.001	0.984	0.993

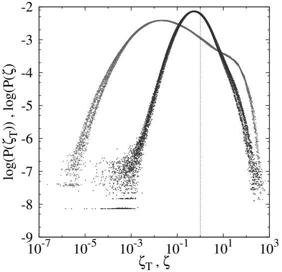
$$\epsilon(\mathbf{x},t) = (Pr/Ra)^{1/2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^t)^2 \quad \text{and} \quad \epsilon_T(\mathbf{x},t) = (RaPr)^{-1/2}(\nabla T)^2$$

- ϵ and ϵ_T are highly correlated through BLs and tend to decorrelate in the bulk when the plumes are shedded and well mixed.

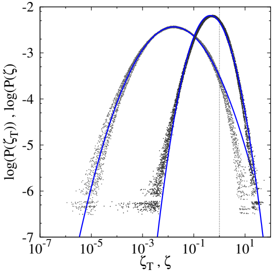


(a) (b)

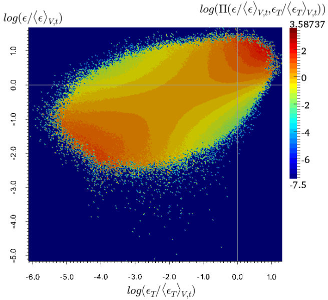
Figure: Snapshot of kinetic ϵ and thermal ϵ_T dissipation rates.



(a)

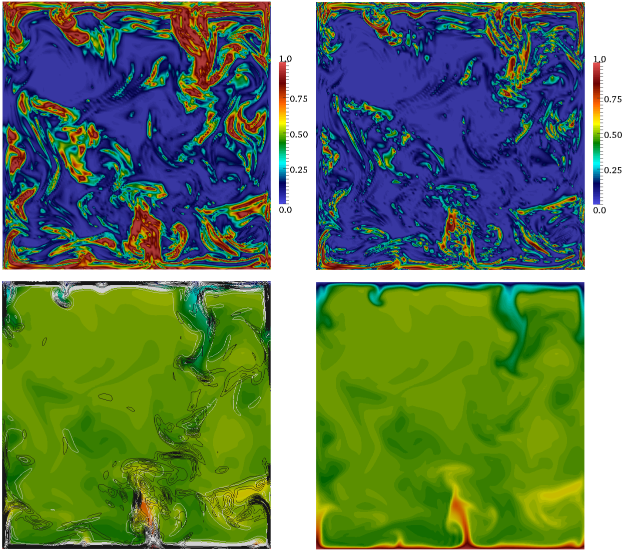


(b)



- Spatial PDF of normalized dissipation rates through the whole cavity (a) and the bulk (b).
- Normalized joint statistics $\Pi(\zeta, \zeta_T)$ in the bulk region that shows the correlation of (ϵ, ϵ_T) at the rare high-magnitude events exceeding their averaged values.

High correlation of (ϵ, ϵ_T) at the strong kinetic-thermal interactions that associate with the role of thermal plumes in feeding the momentum through the bulk.



- 1st-order gradient similarity of (ϵ, ϵ_T) $s(G, G_T) = s_m \cdot s_d$, $G = \nabla\epsilon,\ G_T = \nabla\epsilon_T$.
- Magnitude similarity

$$s_m = 4\frac{\|G\| \cdot \|G_T\|}{(\|G\| + \|G_T\|)^2}$$

- Direction similarity

$$s_d = \left(\frac{G \cdot G_T}{\|G\| \cdot \|G_T\|}\right)^2$$

- DNS is not feasible at high *Ra* number.
- Symmetry-preserving regularizations on the convective non-linearity that restrain the production of the smallest scales of motion, proposed by Verstappen [Verstappen, Comp. & Fluids, 2008]

- DNS resolved equation:

$$\partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u}, \boldsymbol{u}) = -\nabla p + \mathcal{D}\boldsymbol{u} + \boldsymbol{f}$$

- Alerted non-linearity equation:

$$\partial_t \boldsymbol{u}_\varepsilon + \mathcal{C}_4(\boldsymbol{u}_\varepsilon, \boldsymbol{u}_\varepsilon) = -\nabla p_\varepsilon + \mathcal{D}\boldsymbol{u}_\varepsilon + \boldsymbol{f}_\varepsilon$$

- $\mathcal{O}(\varepsilon^4)$-accurate smooth approximation is given:

$$\mathcal{C}_4(\boldsymbol{u}, \boldsymbol{v}) = \mathcal{C}(\overline{\boldsymbol{u}}, \overline{\boldsymbol{v}}) + \overline{\mathcal{C}(\overline{\boldsymbol{u}}, \boldsymbol{v}')} + \overline{\mathcal{C}(\boldsymbol{u}', \overline{\boldsymbol{v}})}$$

$\boldsymbol{u}' = \boldsymbol{u} - \overline{\boldsymbol{u}}$ the residual of the filtered field $\overline{\boldsymbol{u}}$

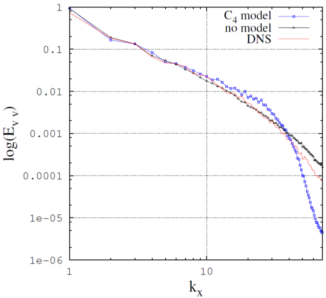


Figure: One-dimensional spanwise energy spectra at $y = 0.5$.

- DNS is not feasible at high *Ra* number.
- Symmetry-preserving regularizations on the convective non-linearity that restrain the production of the smallest scales of motion, proposed by Verstappen [Verstappen, Comp. & Fluids, 2008]

- DNS resolved equation:

$$\partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u},\boldsymbol{u}) = -\nabla p + \mathcal{D}\boldsymbol{u} + \boldsymbol{f}$$

- Alerted non-linearity equation:

$$\partial_t \boldsymbol{u}_\varepsilon + \mathcal{C}_4(\boldsymbol{u}_\varepsilon,\boldsymbol{u}_\varepsilon) = -\nabla p_\varepsilon + \mathcal{D}\boldsymbol{u}_\varepsilon + \boldsymbol{f}_\varepsilon$$

- $\mathcal{O}(\varepsilon^4)$-accurate smooth approximation is given:

$$\mathcal{C}_4(\boldsymbol{u},\boldsymbol{v}) = \mathcal{C}(\overline{\boldsymbol{u}},\overline{\boldsymbol{v}}) + \overline{\mathcal{C}(\overline{\boldsymbol{u}},\boldsymbol{v}')} + \overline{\mathcal{C}(\boldsymbol{u}',\overline{\boldsymbol{v}})}$$

$\boldsymbol{u}' = \boldsymbol{u} - \overline{\boldsymbol{u}}$ the residual of the filtered field $\overline{\boldsymbol{u}}$

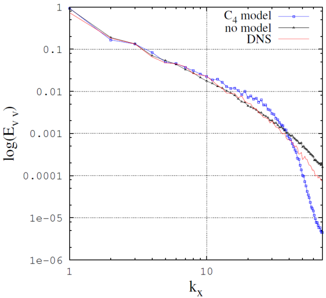


Figure: One-dimensional spanwise energy spectra at $y = 0.5$.

- The low modes of $\boldsymbol{u}_\varepsilon$ approximate the corresponding low modes $\boldsymbol{u}$, whereas the high modes vanish faster than $\boldsymbol{u}$.
- Kinetic energy, enstrophy in 2D and helicity in 3D are exactly preserved leading to an inevitable pile-up energy at the intermediate scales.

- Very good assessment of the regularization modeling results with DNS is found in channel flow [Verstappen, Comp. & Fluids, 2008] and differentially heated cavity [F.X. Trias, Int. J. Heat Mass Transfer, 2013]

- Very good assessment of the regularization modeling results with DNS is found in channel flow [Verstappen, Comp. & Fluids, 2008] and differentially heated cavity [F.X. Trias, Int. J. Heat Mass Transfer, 2013]

- Linear filters based on polynomial functions of the discrete diffusive operator

$$\mathbf{F} = \mathbf{I} + \sum_{m=1}^{M} d_m \tilde{\mathbf{D}}^m \quad \text{with} \quad \tilde{\mathbf{D}} = -(Pr/Ra)^{-1/2}\Omega^{-1}\mathbf{D}$$

- Filter length is decided from the requirements of stopping the vortex-stretching mechanism at the smallest scales.

$$\text{damping function} \quad f_4 = min\left\{\lambda_\Delta \sqrt{\frac{Pr}{Ra}} \frac{Q_s}{|R_s|}, 1\right\}$$

λ_Δ largest negative non-zero eigenvalue of the Laplacian operator Δ

$$\int_\Omega \omega \cdot S\omega = 4\int_\Omega R_s d\Omega$$
$$-\int_\Omega \omega \cdot \Delta\omega \leq 4\int_\Omega \lambda_\Delta Q_s d\Omega$$

$S = 1/2(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^t)$
$Q_s = -1/2tr(S^2)$
$R_s = -1/3tr(S^3) = -det(S)$

	DNS	Mesh A	Mesh B
Mesh	$400 \times 208 \times 208$	$112 \times 60 \times 60$	$168 \times 110 \times 110$

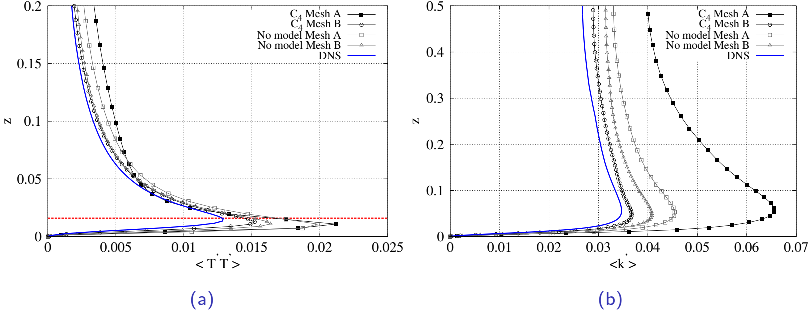
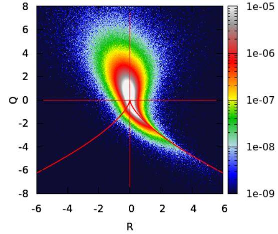


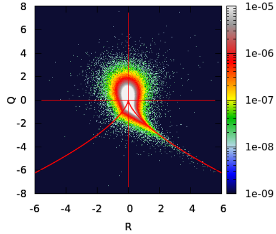
Figure: Temperature variance $\langle T'T' \rangle$ (a), and averaged turbulent kinetic energy $\langle k' \rangle$ (b)

- Weaker modeling resolution at very coarse grids but better performance at finer ones returned to the plumes dynamics at this moderate *Ra* number.

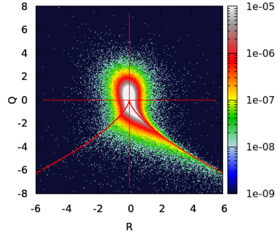
- The universal inclined teardrop shape of the joint PDF map of the invariants (Q, R) of velocity gradient tensor, is found in RBC in the bulk region.



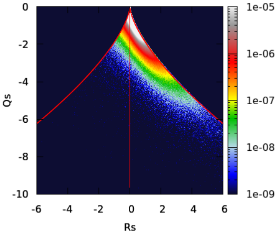
(a) DNS



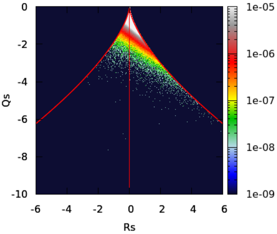
(b) model(mesh B)



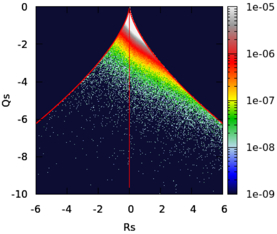
(c) no model(mesh B)



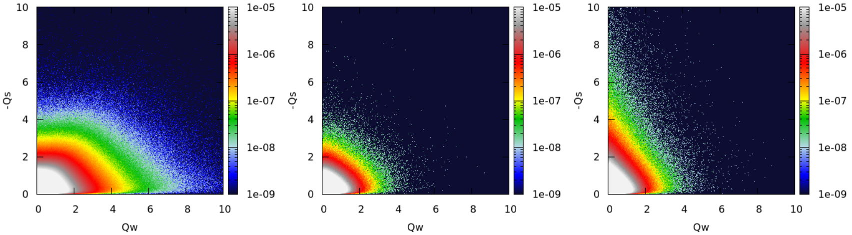
(d) DNS



(e) model(mesh B)



(f) no model(mesh B)



(g) DNS (h) model(mesh B) (i) no model(mesh B)

- A complete DNS study of RBC have done at $Ra = 10^8$ in rectangular cell using energy-conserving discretizations to show a good agreement with [M. Kaczorowiski & Wagner, J. Fluid Mech, 2009]
- Fine-scales structure of thermal and kinetic energy dissipation rates are correlated at the strong thermal-kinetic interactions related with the evolution of thermal plumes.
- Symmetry-preserving regularization models reduce effectively the convective production of the smallest scales at high turbulent RBC when the turbulent background dominates the thermal plumes.
- Ongoing DNS at $Ra = 10^{10}$ to assess the good regularization modeling at coarser grids.

Thanks for your attention!