Spectrally-consistent regularization of turbulent Rayleigh-Bénard convection

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Outline

1 Introduction

2 Direct numerical simulation
   - Problem parameters and resolution requirements
   - Fine-scale features of thermal and kinetic energy dissipation rates

3 Turbulence modeling
   - $C_4$-regularization
   - Averaged results
   - Structure and dynamics of fine-scale motions

4 Conclusions
Rayleigh-Bénard convection (RBC)

- **Definition**: a convective cell heated from below and cooled from above.
- Flow dynamics are characterized by:
  - Rayleigh number: \( \text{Ra} = \frac{g \alpha H^3 \Delta T}{\nu \kappa} \)
  - Prandtl number: \( \text{Pr} = \frac{\nu}{\kappa} \)
  - System respond
    \( \text{Nu} = \frac{w T - \kappa \partial T/\partial z}{\kappa \Delta T/H} \)
- Different flow behaviour between the near-isothermal walls and the bulk that is connected by the thermal plumes [Chillá & Schumacher, Eur. J. Phys. 2012]
Non-dimensional governing equations:
\[
\begin{align*}
\nabla \cdot u &= 0 , \\
\partial_t u + C(u,u) &= -\nabla p + \mathcal{D}u + f , \\
\partial_t T + C(u,T) &= Pr^{-1}\mathcal{D}T \\
\end{align*}
\]
- \( C(u,u) = (u \cdot \nabla)u \) is the convective term.
- \( \mathcal{D}u = (Pr/Ra)^{1/2}\nabla^2 u \) is the diffusive term.
- \( f = (0, 0, T) \) is the body forces term.
- 4\textsuperscript{th}-order symmetry-preserving spatial discretizations [Verstappen & Veldman, J. Comp. Phys. 2003].

Parameters
- \( Ra = 10^8, \ Pr = 0.7 \), rectangular cell of \( \Gamma = 1 \) and \( L = \pi H \).
- Periodic boundaries at \( L \) direction.
- Adiabatic side walls.
Non-dimensional governing equations:
\[
\nabla \cdot \mathbf{u} = 0,
\]
\[
\partial_t \mathbf{u} + \mathbf{C}(\mathbf{u}, \mathbf{u}) = -\nabla p + \mathbf{D} \mathbf{u} + \mathbf{f},
\]
\[
\partial_t T + \mathbf{C}(\mathbf{u}, T) = Pr^{-1} \mathbf{D} T
\]

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4th-order symmetry-preserving spatial discretizations \cite{Verstappen & Veldman, J. Comp. Phys. 2003}.

Global heat flux \( NuV = 1 + (RaPr)^{1/2} \left\langle wT \right\rangle V,t \)

Parameters

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Grötzbach criterion \cite{Grötzbach, J. Comp. Phys. 1983}

maximum cell size
\[
\eta_{Grö} \leq \pi \left( \frac{Pr^2}{(Nu - 1)Ra} \right)^{1/4}, Pr \leq 1
\]
\[
\eta_{Grö} \leq \pi \left( \frac{1}{(Nu - 1)Ra} \right)^{1/4}, Pr > 1
\]

<table>
<thead>
<tr>
<th>ref. ratio</th>
<th>( N_x \times N_y \times N_z )</th>
<th>( Nu )</th>
<th>( NuV/Nu )</th>
<th>( (\left\langle \epsilon \right\rangle_{V,t}(RaPr)^{1/2} + 1)/Nu )</th>
<th>( (\left\langle \epsilon_T \right\rangle_{V,t}(RaPr)^{1/2})/Nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>256 \times 150 \times 150</td>
<td>31.44</td>
<td>0.997</td>
<td>0.966</td>
<td>0.983</td>
</tr>
<tr>
<td>1.0</td>
<td>288 \times 158 \times 158</td>
<td>31.10</td>
<td>0.997</td>
<td>0.971</td>
<td>0.985</td>
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<tr>
<td>0.9</td>
<td>320 \times 174 \times 174</td>
<td>31.00</td>
<td>0.999</td>
<td>0.976</td>
<td>0.988</td>
</tr>
<tr>
<td>0.8</td>
<td>342 \times 192 \times 192</td>
<td>30.93</td>
<td>0.999</td>
<td>0.982</td>
<td>0.991</td>
</tr>
<tr>
<td>0.7</td>
<td>400 \times 208 \times 208</td>
<td>30.86</td>
<td>1.001</td>
<td>0.984</td>
<td>0.993</td>
</tr>
</tbody>
</table>
\[ \epsilon(x, t) = \left( Pr/Ra \right)^{1/2} (\nabla u + \nabla u^t)^2 \quad \text{and} \quad \epsilon_T(x, t) = (RaPr)^{-1/2}(\nabla T)^2 \]

- \( \epsilon \) and \( \epsilon_T \) are highly correlated through BLs and tend to decorrelate in the bulk when the plumes are shedded and well mixed.

**Figure:** Snapshot of kinetic \( \epsilon \) and thermal \( \epsilon_T \) dissipation rates.
Spatial PDF of normalized dissipation rates through the whole cavity (a) and the bulk (b).

Normalized joint statistics $\Pi(\zeta,\zeta_T)$ in the bulk region that shows the correlation of $(\epsilon,\epsilon_T)$ at the rare high-magnitude events exceeding their averaged values.

- Spatial PDF of normalized dissipation rates through the whole cavity (a) and the bulk (b).
- Normalized joint statistics $\Pi(\zeta,\zeta_T)$ in the bulk region that shows the correlation of $(\epsilon,\epsilon_T)$ at the rare high-magnitude events exceeding their averaged values.
High correlation of $(\epsilon, \epsilon_T)$ at the strong kinetic-thermal interactions that associate with the role of thermal plumes in feeding the momentum through the bulk.

- 1st-order gradient similarity of $(\epsilon, \epsilon_T)$
  \[ s(G, G_T) = s_m \cdot s_d, \]
  \[ G = \nabla \epsilon, \quad G_T = \nabla \epsilon_T. \]

- Magnitude similarity
  \[ s_m = 4 \frac{\|G\| \cdot \|G_T\|}{(\|G\| + \|G_T\|)^2} \]

- Direction similarity
  \[ s_d = \left( \frac{G \cdot G_T}{\|G\| \cdot \|G_T\|} \right)^2 \]
DNS is not feasible at high $Ra$ number.

- **Symmetry-preserving regularizations** on the convective non-linearity that restrain the production of the smallest scales of motion, proposed by Verstappen [Verstappen, Comp. & Fluids, 2008]

- **DNS resolved equation:**

  $$\partial_t u + C(u, u) = -\nabla p + \mathcal{D}u + f$$

- **Alerted non-linearity equation:**

  $$\partial_t u_\epsilon + C_4(u_\epsilon, u_\epsilon) = -\nabla p_\epsilon + \mathcal{D}u_\epsilon + f_\epsilon$$

- **$O(\epsilon^4)$-accurate smooth approximation is given:**

  $$C_4(u, v) = C(\overline{u}, \overline{v}) + \overline{C}(\overline{u}, v') + \overline{C}(u', \overline{v})$$

  $$u' = u - \overline{u}$$ the residual of the filtered field $\overline{u}$

**Figure:** One-dimensional spanwise energy spectra at $y = 0.5$. 
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The low modes of $u_\varepsilon$ approximate the corresponding low modes $u$, whereas the high modes vanish faster than $u$.

Kinetic energy, enstrophy in 2D and helicity in 3D are exactly preserved leading to an inevitable pile-up energy at the intermediate scales.

**Figure:** One-dimensional spanwise energy spectra at $y = 0.5$. 
Very good assessment of the regularization modeling results with DNS is found in channel flow [Verstappen, Comp. & Fluids, 2008] and differentially heated cavity [F.X. Trias, Int. J. Heat Mass Transfer, 2013]
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Linear filters based on polynomial functions of the discrete diffusive operator

\[ F = I + \sum_{m=1}^{M} d_m \tilde{D}^m \quad \text{with} \quad \tilde{D} = -(Pr/Ra)^{-1/2}\Omega^{-1}D \]

Filter length is decided from the requirements of stopping the vortex-stretching mechanism at the smallest scales.

\[ f_4 = \min \left\{ \lambda_\Delta \sqrt{\frac{Pr}{Ra} \frac{Q_s}{|R_s|}}, 1 \right\} \]

\( \lambda_\Delta \) largest negative non-zero eigenvalue of the Laplacian operator \( \Delta \)

\[
\int_{\Omega} \omega \cdot S\omega = 4 \int_{\Omega} R_s d\Omega \\
- \int_{\Omega} \omega \cdot \Delta \omega \leq 4 \int_{\Omega} \lambda_\Delta Q_s d\Omega \\
S = 1/2(\nabla u + \nabla u^t) \\
Q_s = -1/2tr(S^2) \\
R_s = -1/3tr(S^3) = -\det(S) \]
Figure: Temperature variance $\langle T' T' \rangle$ (a), and averaged turbulent kinetic energy $\langle k' \rangle$ (b)

- Weaker modeling resolution at very coarse grids but better performance at finer ones returned to the plumes dynamics at this moderate $Ra$ number.
The universal inclined teardrop shape of the joint PDF map of the invariants $(Q, R)$ of velocity gradient tensor, is found in RBC in the bulk region.
Spectrally-consistent regularization of turbulent Rayleigh-Bénard convection

- **(g)** DNS
- **(h)** model(mesh B)
- **(i)** no model(mesh B)
A complete DNS study of RBC have done at $Ra = 10^8$ in rectangular cell using energy-conserving discretizations to show a good agreement with [M. Kaczorowiski & Wagner, J. Fluid Mech, 2009]

Fine-scales structure of thermal and kinetic energy dissipation rates are correlated at the strong thermal-kinetic interactions related with the evolution of thermal plumes.

Symmetry-preserving regularization models reduce effectively the convective production of the smallest scales at high turbulent RBC when the turbulent background dominates the thermal plumes.

Ongoing DNS at $Ra = 10^{10}$ to assess the good regularization modeling at coarser grids.
Thanks for your attention!