# ASSESSMENT OF LARGE-EDDY SIMULATION MODELS IN COMPLEX FLOWS USING UNSTRUCTURED MESHES

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The main ojectives of this work are:

1. Present a non disipative formulation for the discretitzation of the Navier-Stokes equations suitable for DNS or LES modelling under general unstructured meshes.

2. Present a suitable SGS model for the simulation of complex flows and engineering applications.

3. Perform some tests in order to validate the presented formulation and LES modelling. The selected cases are:

   - Flow over a circular cylinder of $Re = 3900$
   - Turbulent plane impingnig jet of Reynolds number $20000$ and $H/B = 4$
   - Flow over an Ahmed Car of Reynolds number of $4 \cdot 10^6$ and a slot angle of $25^o$

**Solving the pressure-velocity coupling**:

The finite volume discretization of the Navier-Stokes and continuity equations on an arbitrary mesh can be written by

$$\rho\Omega_s\frac{d\boldsymbol{u}_s}{dt} + \mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_s + \mathsf{D}\boldsymbol{u}_s + \Omega_s\mathsf{G}\boldsymbol{p}_c = \boldsymbol{0}_s \quad (1)$$

$$\mathsf{M}\boldsymbol{u}_s = \boldsymbol{0}_c \quad (2)$$

Or with a simplified notation,

$$\frac{d\boldsymbol{u}_c}{dt} = \mathsf{R}\left(\boldsymbol{u}_c\right) - \mathsf{G}_c\boldsymbol{p}_c \quad (3)$$

where $\mathsf{R}\left(\boldsymbol{u}_c\right)$ represents the right-hand-side terms of the momentum equation except for the pressure gradient,

$$\mathsf{R}\left(\boldsymbol{u}_c\right) \equiv -\mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_c - \mathsf{D}\boldsymbol{u}_c \quad (4)$$

For the temporal discretization, a central difference scheme is used for the time derivative term, a fully explicit second-order one-leg scheme for $\mathsf{R}(\boldsymbol{u}_c)$ and a first-order backward Euler scheme for the pressure-gradient term. Incompressibility constraint is treated implicit. Thus, we obtain the fully-discretized Navier-Stokes equations

$$\mathsf{M}\boldsymbol{u}_s^{n+1} = \mathbf{0} \tag{5}$$

$$\frac{(\beta+1/2)\,\boldsymbol{u}_c^{n+1} - 2\beta\boldsymbol{u}_c^{n} + (\beta-1/2)\,\boldsymbol{u}_c^{n-1}}{\Delta t} = \mathsf{R}\left((1+\beta)\,\boldsymbol{u}_c^{n} - \beta\boldsymbol{u}_c^{n-1}\right) - \mathsf{G}_c\boldsymbol{p}_c^{n+1} \tag{6}$$

where the parameter $\beta$ is computed each time-step to adapt the linear stability domain of the time-integration scheme to the instantaneous flow conditions in order to use the maximum $\Delta t$ possible.

To solve the velocity-pressure coupling a classical fractional step projection method is used.

$$\boldsymbol{u}_c^p = \boldsymbol{u}_c^{n+1} + \mathsf{G}\tilde{\boldsymbol{p}}_c \tag{7}$$

$$\boldsymbol{u}_c^p = \frac{2\beta \boldsymbol{u}_c^n - (\beta - 1/2)\, \boldsymbol{u}_c^{n-1}}{\beta + 1/2} + \frac{\Delta t}{\beta + 1/2}\mathsf{R}\left((1+\beta)\, \boldsymbol{u}_c^n - \beta \boldsymbol{u}_c^{n-1}\right) \tag{8}$$

and the pseudo-pressure is $\tilde{\boldsymbol{p}}_c = \Delta t / (\beta + 1/2)\, \boldsymbol{p}_c^{n+1}$. Taking the divergence of the previous equation yields a discrete Poisson equation for $\tilde{\boldsymbol{p}}_c$

$$\mathsf{L}\boldsymbol{p}_c = \mathsf{M}\boldsymbol{u}_c^p \tag{9}$$

where discrete laplacion operator $\mathsf{L} \in \mathbb{R}^{n \times n}$ is, by construction, a symmetric positive definite matrix.

Once the solution is obtained, $\boldsymbol{u}_c^{n+1}$ results from the correction

$$\boldsymbol{u}_c^{n+1} = \boldsymbol{u}_c^p - \mathsf{G}\tilde{\boldsymbol{p}}_c \tag{10}$$

In summary, the algorithm for the integration of each time-step is

1. Evaluate R $\left((1+\beta)\,\boldsymbol{u}_c^n - \beta \boldsymbol{u}_c^{n-1}\right)$.
2. Evaluate the predictor velocity $\boldsymbol{u}_s^p$..
3. Evaluate M$\boldsymbol{u}_s^p$ and solve the discrete Poisson equation.
4. Obtain the new velocity field with $\boldsymbol{u}_c^{n+1} = \boldsymbol{u}_c^p - \mathrm{G}\tilde{\boldsymbol{p}}_c$.

On the context of the presented algorithm, the Poisson equation which arises from the incompressibility constraint, which has to be solved at least once at each time step, is usually the main bottleneck from a parallel point of view. On these circumstances, it is important to use efficient and scalable algorithms for the solution of the Poisson equation on a wide range of parallel systems.

## Kinetic energy conservation

The global discrete energy is defined as

$$||\boldsymbol{u}_h||^2 \equiv \boldsymbol{u}_h^* \Omega_h \boldsymbol{u}_h \tag{11}$$

The evolution equation of $||\boldsymbol{u}_h||^2$ can be obtained by left-multiplying Eq.(1) by $\boldsymbol{u}_h^*$ and summing the resulting expression with its conjugate transpose

$$\rho \Omega_s \frac{d}{dt}||\boldsymbol{u}_h||^2 = -\boldsymbol{u}_h^* \left(\mathsf{C}\left(\boldsymbol{u}_h\right) + \mathsf{C}^*\left(\boldsymbol{u}_h\right)\right) \boldsymbol{u}_h - \boldsymbol{u}_h^* \left(\mathsf{D} + \mathsf{D}^*\right) \boldsymbol{u}_h - \boldsymbol{u}_h^* \Omega_h \mathsf{G}_h \boldsymbol{p}_h - \boldsymbol{p}_h^* \mathsf{G}_h^* \Omega_h \boldsymbol{u}_h \tag{12}$$

Kinetic energy conservation is held if and only if the discrete convective operator is skew-symmetric and if the negative conjugate transpose of the discrete gradient operator is exactly equal to the divergence operator.

$$\mathsf{C}\left(\boldsymbol{u}_h\right) = -\mathsf{C}^*\left(\boldsymbol{u}_h\right) \tag{13}$$

$$-\left(\Omega_h \mathsf{G}_h\right)^* = \mathsf{M}_h \tag{14}$$

Therefore, if the convective and gradient operators are properly chosen, the global kinetic energy equation (12) reduces to:

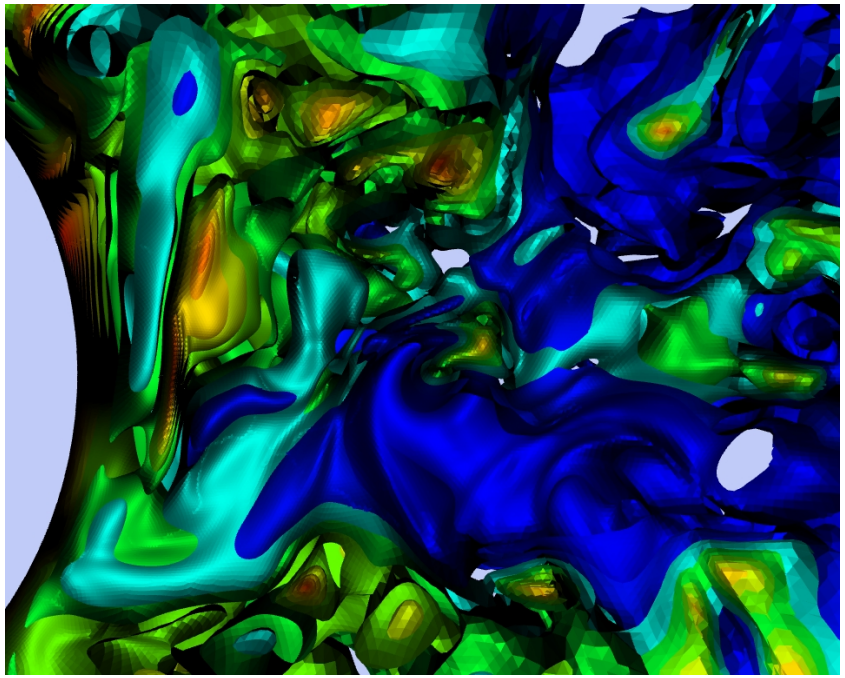
$$\frac{d}{dt}||\boldsymbol{u}_h||^2 = -\boldsymbol{u}_h^*\left(\mathsf{D}+\mathsf{D}^*\right)\boldsymbol{u}_h \tag{15}$$

Since the diffusive terms must be strictly dissipative,

$$\boldsymbol{u}_h^*\left(\mathsf{D}+\mathsf{D}^*\right)\boldsymbol{u}_h \geq 0, \quad \forall \boldsymbol{u}_h \tag{16}$$

and, therefore, the diffusive operator, $\mathsf{D}$, must be symmetric and positive-definite.

## Turbulence modelling: Large Eddy Simulations



In Large Eddy Simulation the Navier Stokes equations are filtered spatially. Doing so, the filtered non-linear convective term must be modelled,

$$\rho\Omega_s \frac{d\overline{\boldsymbol{u}_s}}{dt} + \mathsf{C}\left(\overline{\boldsymbol{u}_s}\right)\overline{\boldsymbol{u}_s} + \mathsf{D}\overline{\boldsymbol{u}_s} + \mathsf{G}\overline{\boldsymbol{p}_c} \quad =$$

$$\mathsf{C}\left(\overline{\boldsymbol{u}_s}\right)\overline{\boldsymbol{u}_s} - \overline{\mathsf{C}\left(\boldsymbol{u}_s\right)\boldsymbol{u}_s} \approx model(\overline{\boldsymbol{u}_s})$$

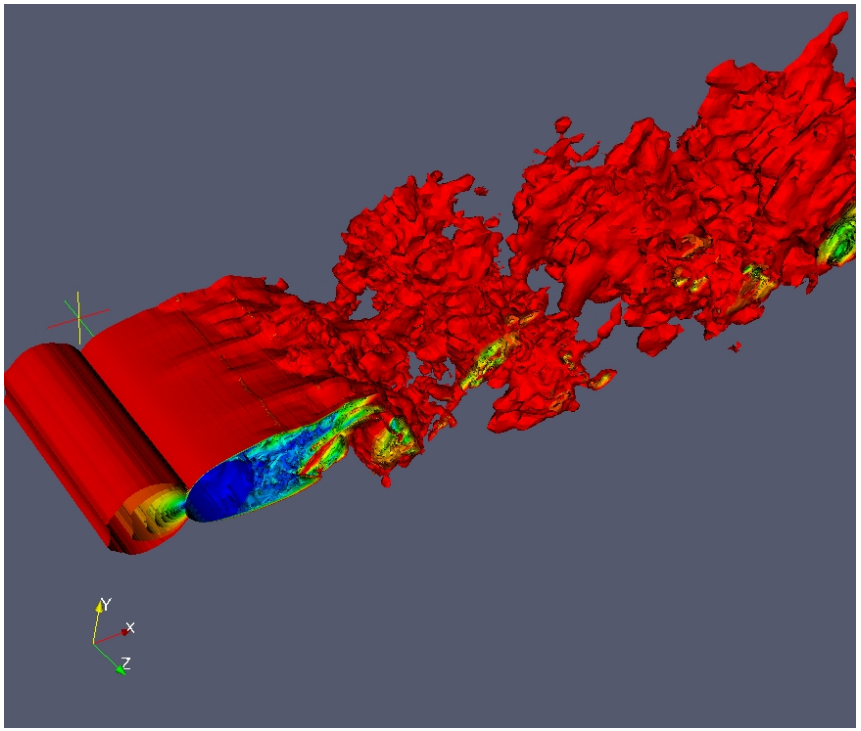
## Yoshizawa nonequilibrium fixed-parameter subgrid-scale model

Main features of the model:

- The equilibrium of SGS fluctuations is not assumed
- No use is made of a wall-unit distance based on the friction velocity
- The near-wall asymptotic behavior of the SGS viscosity is fulfilled
- The effect of each filter function occurs explicitly friction velocity through the doubly-filtered velocity

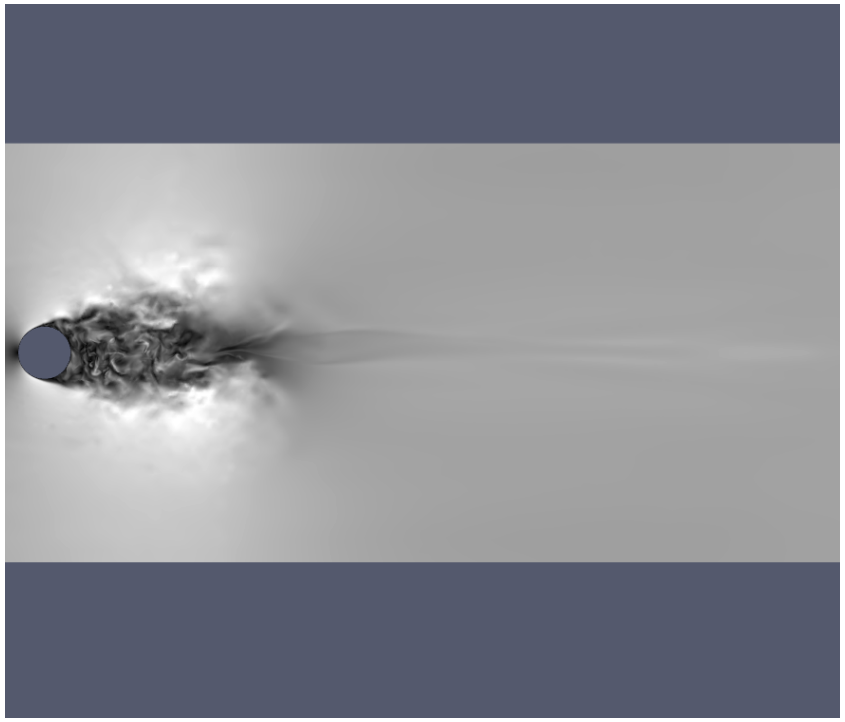
$$\nu_s = C_{\nu S}\Delta(\sqrt{(\overline{u} - \overline{\overline{u}})^2})(1 - exp(-(C_W \frac{\sqrt{(\overline{u} - \overline{\overline{u}})^2}}{\overline{s}\Delta})^2)) \quad (17)$$

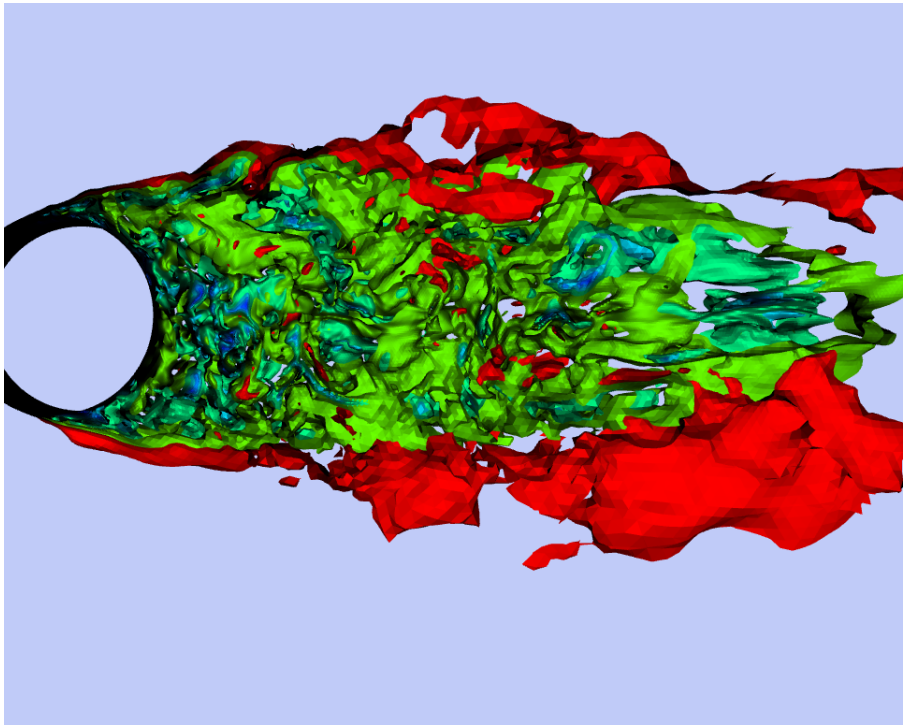
## Flow around a circular cylinder $Re = 3900$



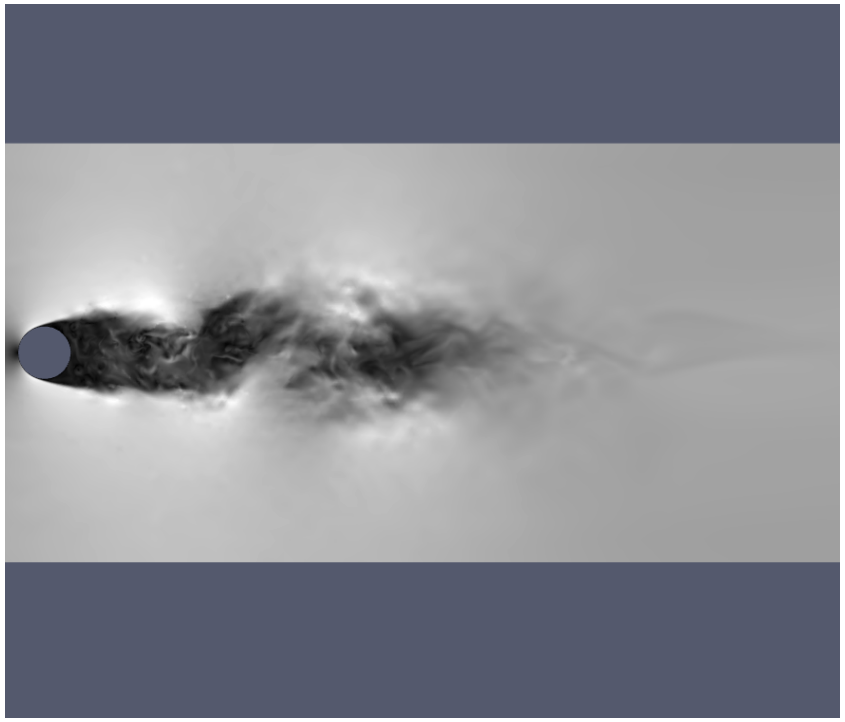
- A detailed validation using experimental data from Lourenco and Shih and DNS data from Dröge is presented.
- The computational domain to be simulated is a rectangular box of dimensions [-4D, 20D],[-4D, 4D],[0,$\pi$D] with a circular cylinder of a diameter D at (0,0,0). At the inflow (u,v,w) = (1,0,0) is prescribed. Periodic boundary conditions are used for the spanwise direction, being this direction Fourier transformed. At all other boundaries, pressure based conditions are used (outflow boundaries).
- Two different grids are used two evaluate the accuracy of the symmetry preserving formulation. The first one is a coarse grid of 160.000 Control Volumes (CV) and the last one a medium sized grid of 1.5 M CV.

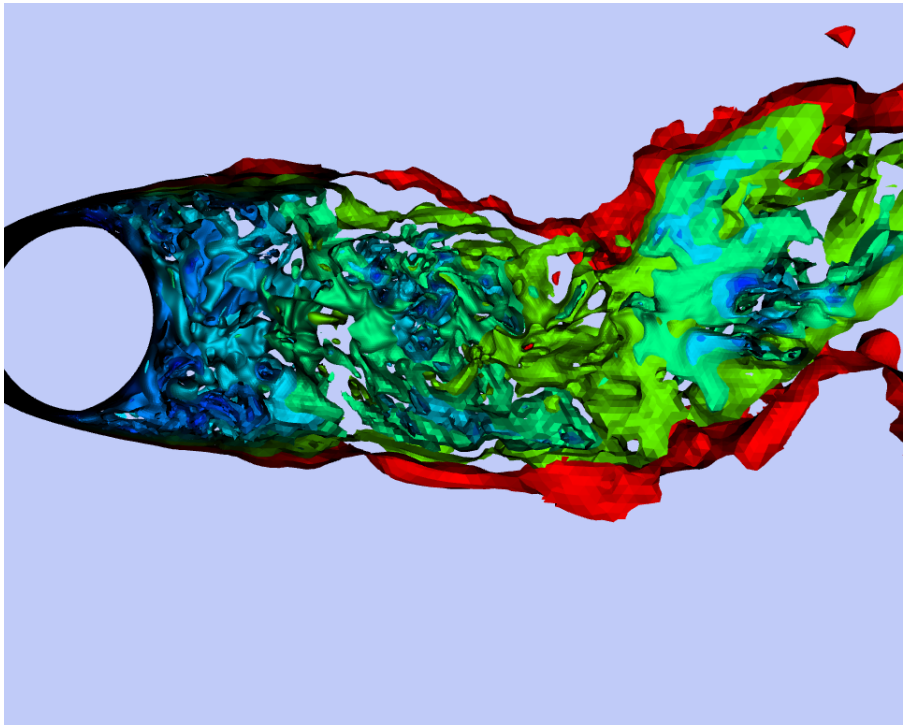
Instantaneous wake flow structures evolution in the symmetry plane



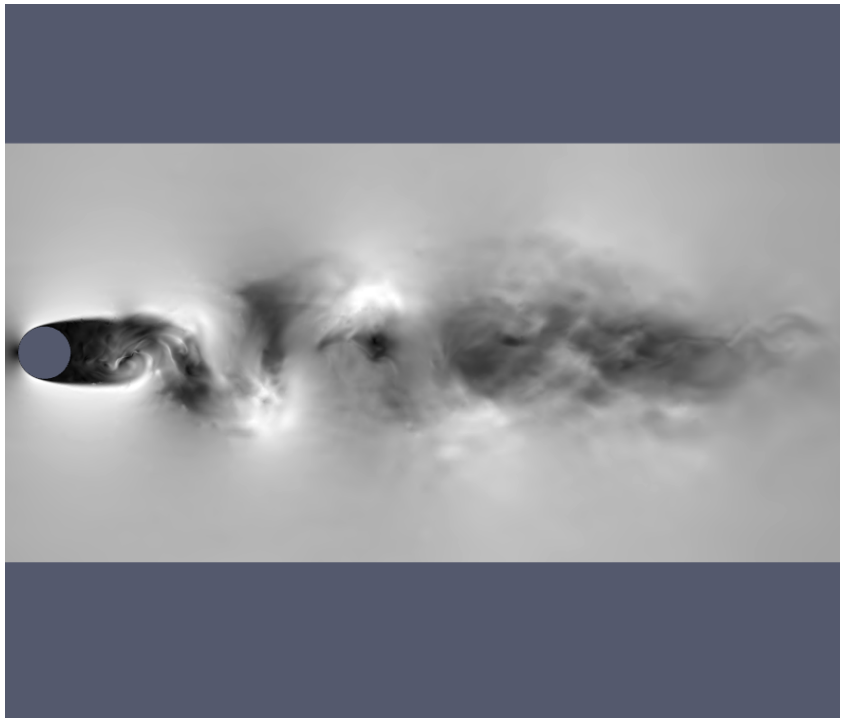


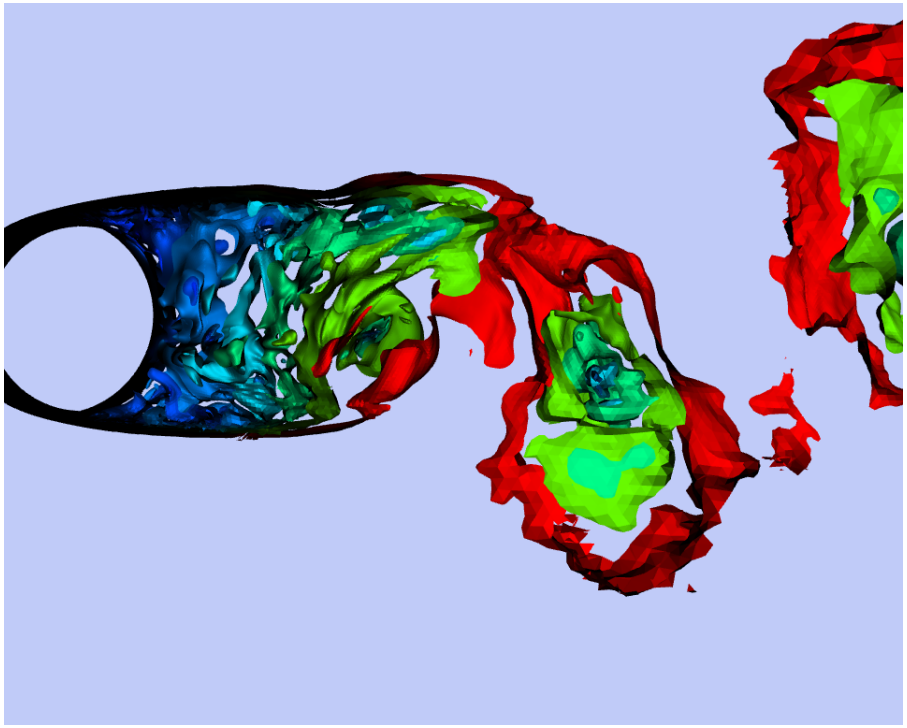
Instantaneous wake flow structures evolution in the symmetry plane

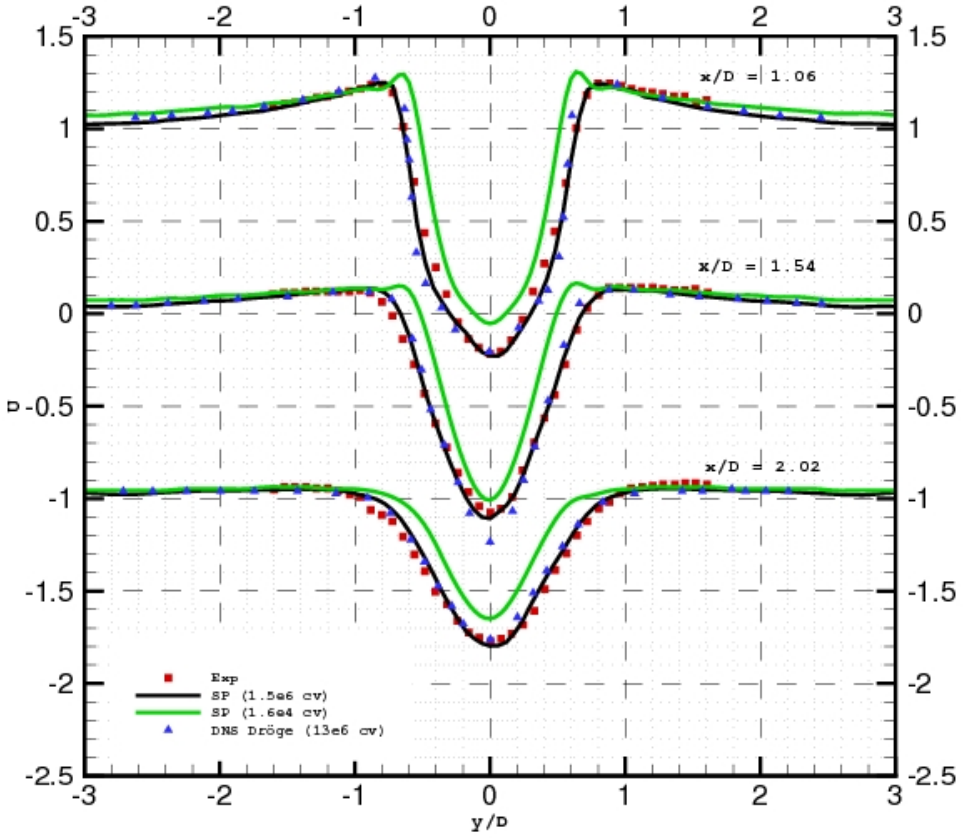


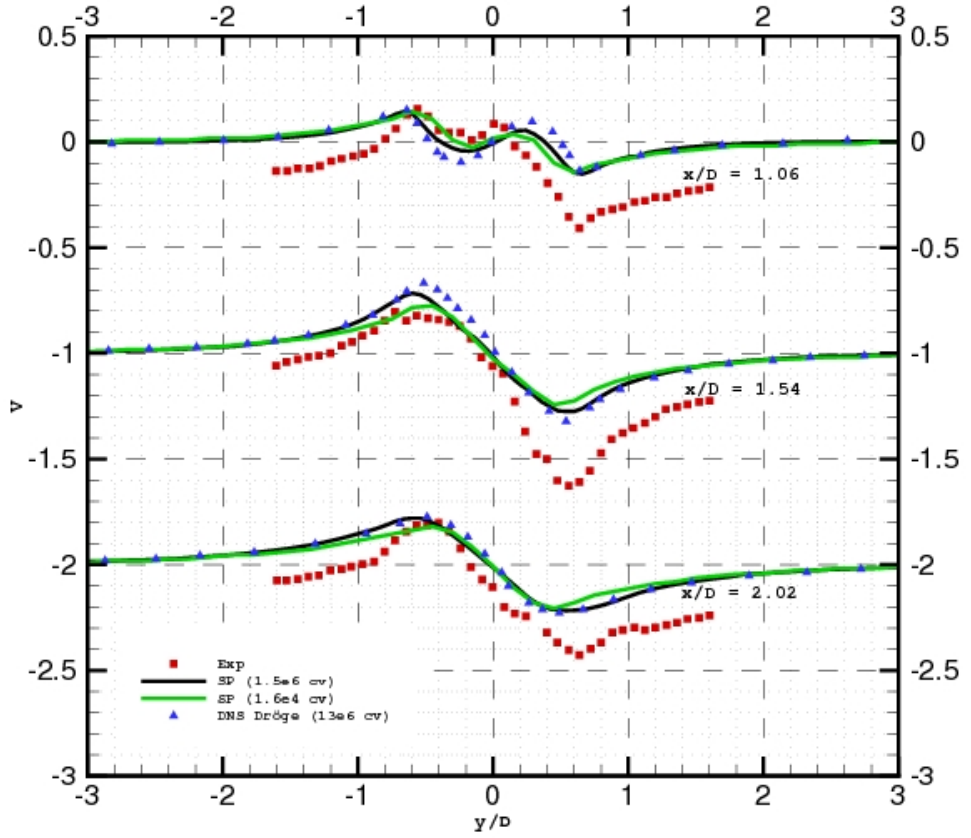


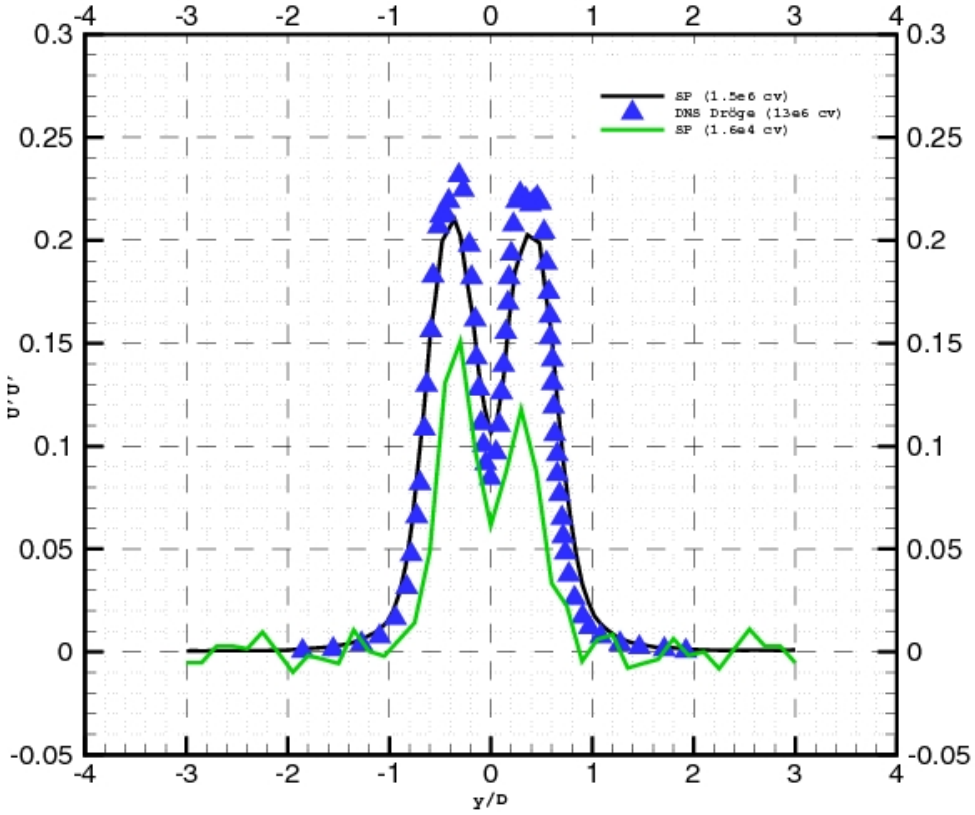
Instantaneous wake flow structures evolution in the symmetry plane

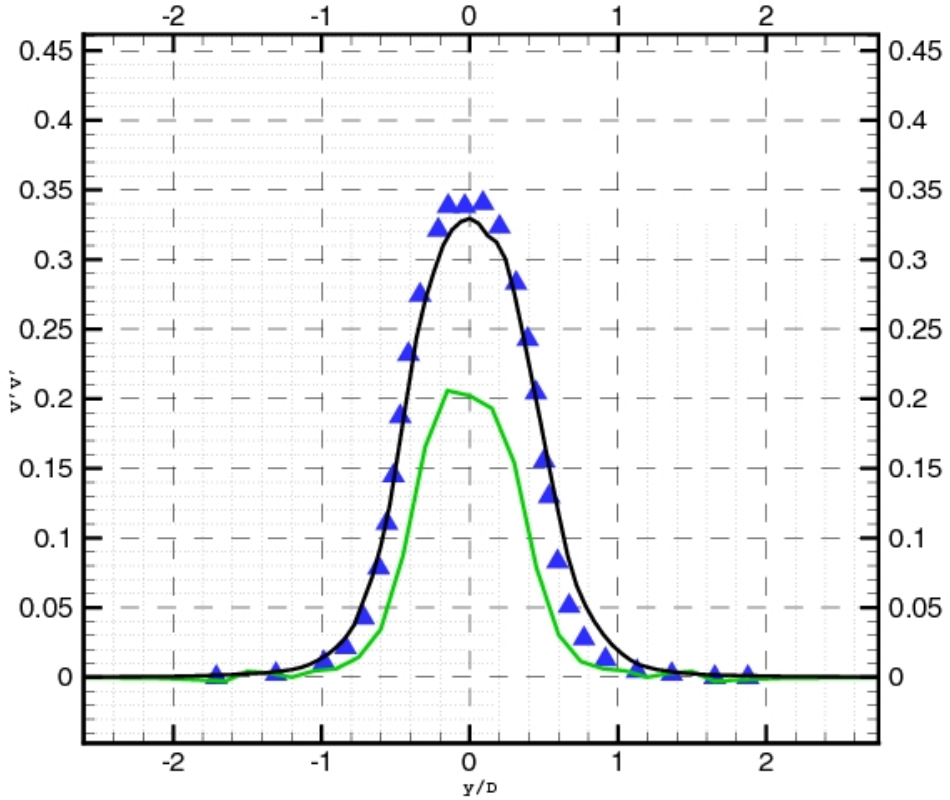





x/D = 1.06
x/D = 1.54
x/D = 2.02
Exp
SP (1.5e6 cv)
SP (1.6e4 cv)
DNS Dröge (13e6 cv)
y/D

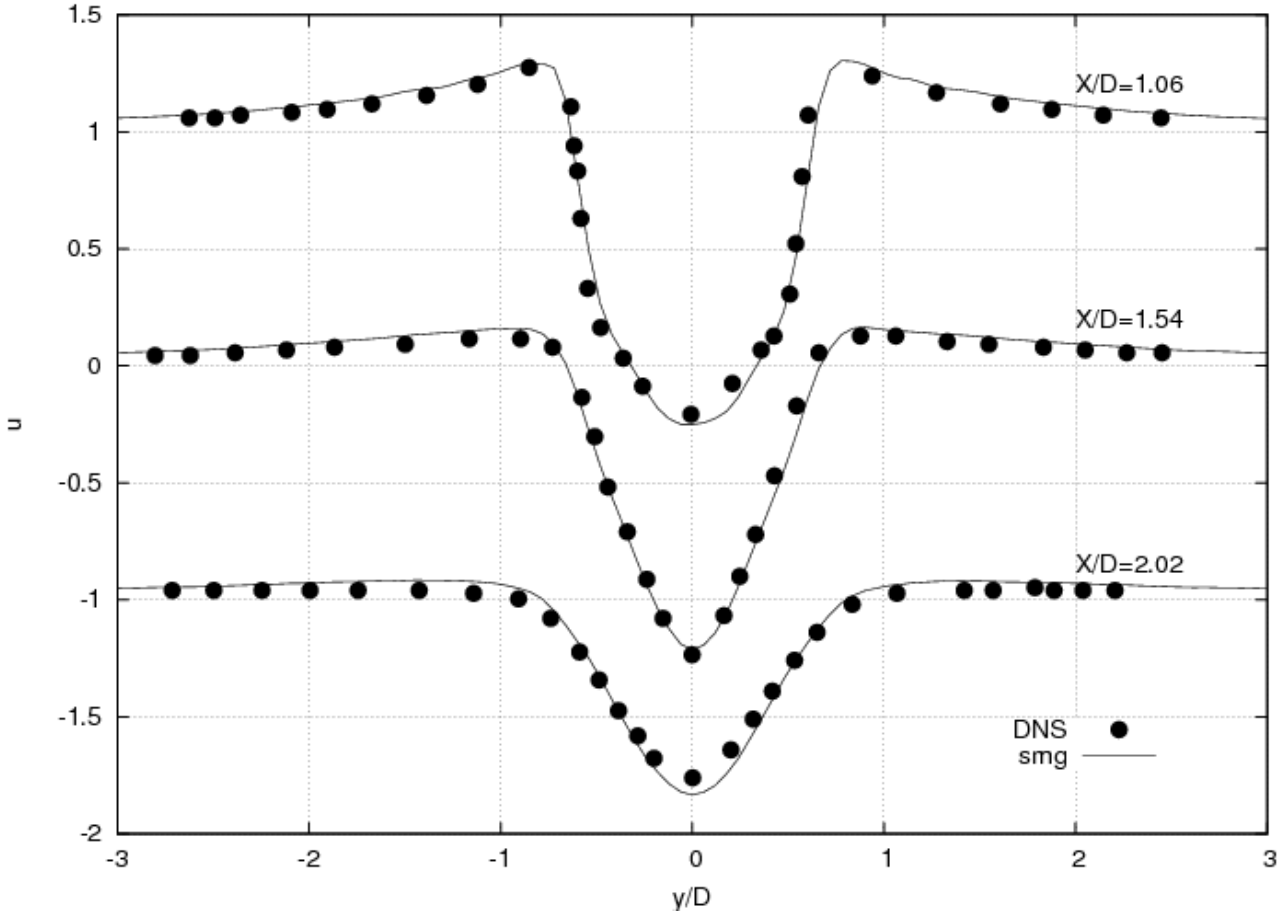

x/D = 1.06
x/D = 1.54
x/D = 2.02
Exp
SP (1.5e6 cv)
SP (1.6e4 cv)
DNS Dröge (13e6 cv)
v
y/D

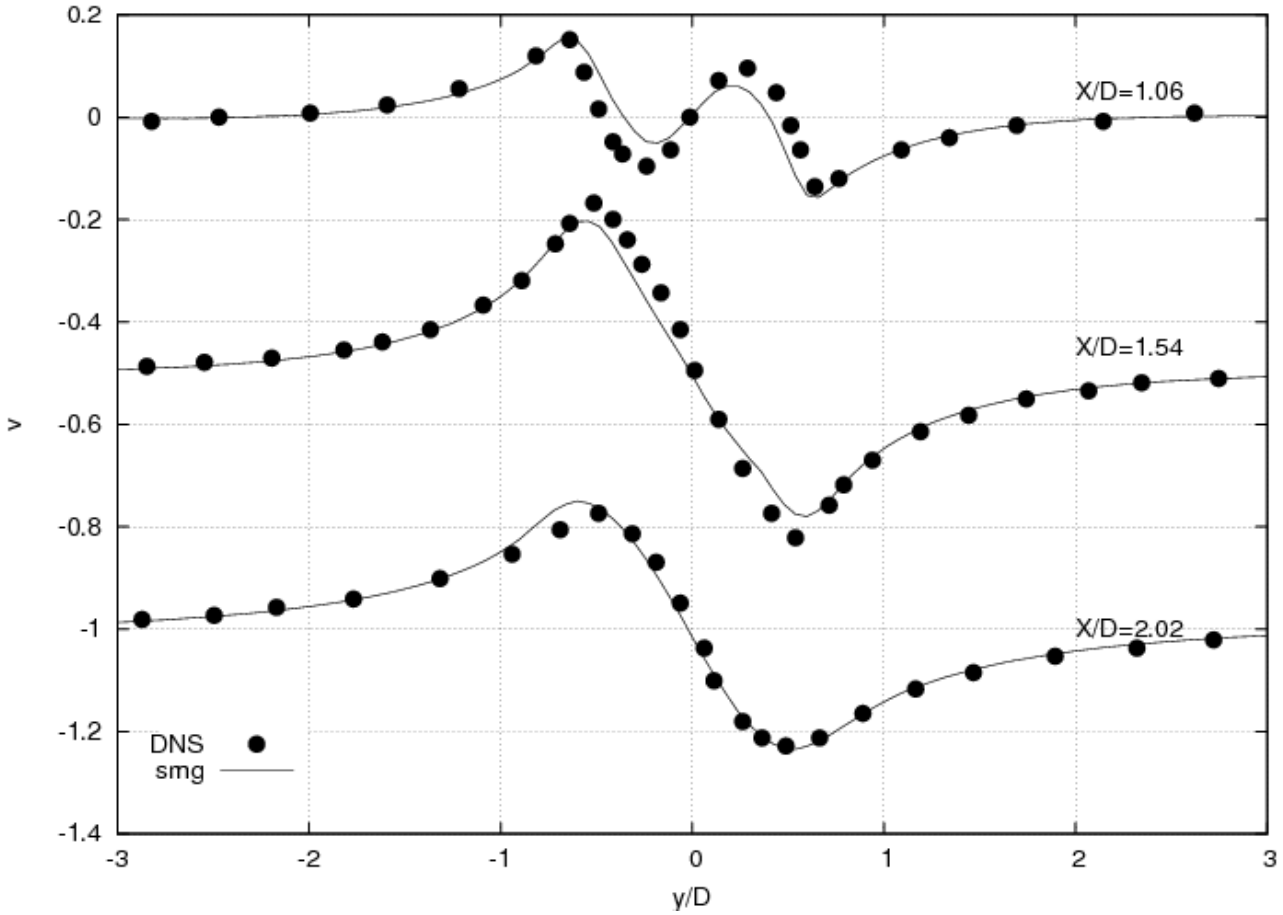

SP (1.5e6 cv)
DNS Dröge (13e6 cv)
SP (1.6e4 cv)
u'u'
y/D

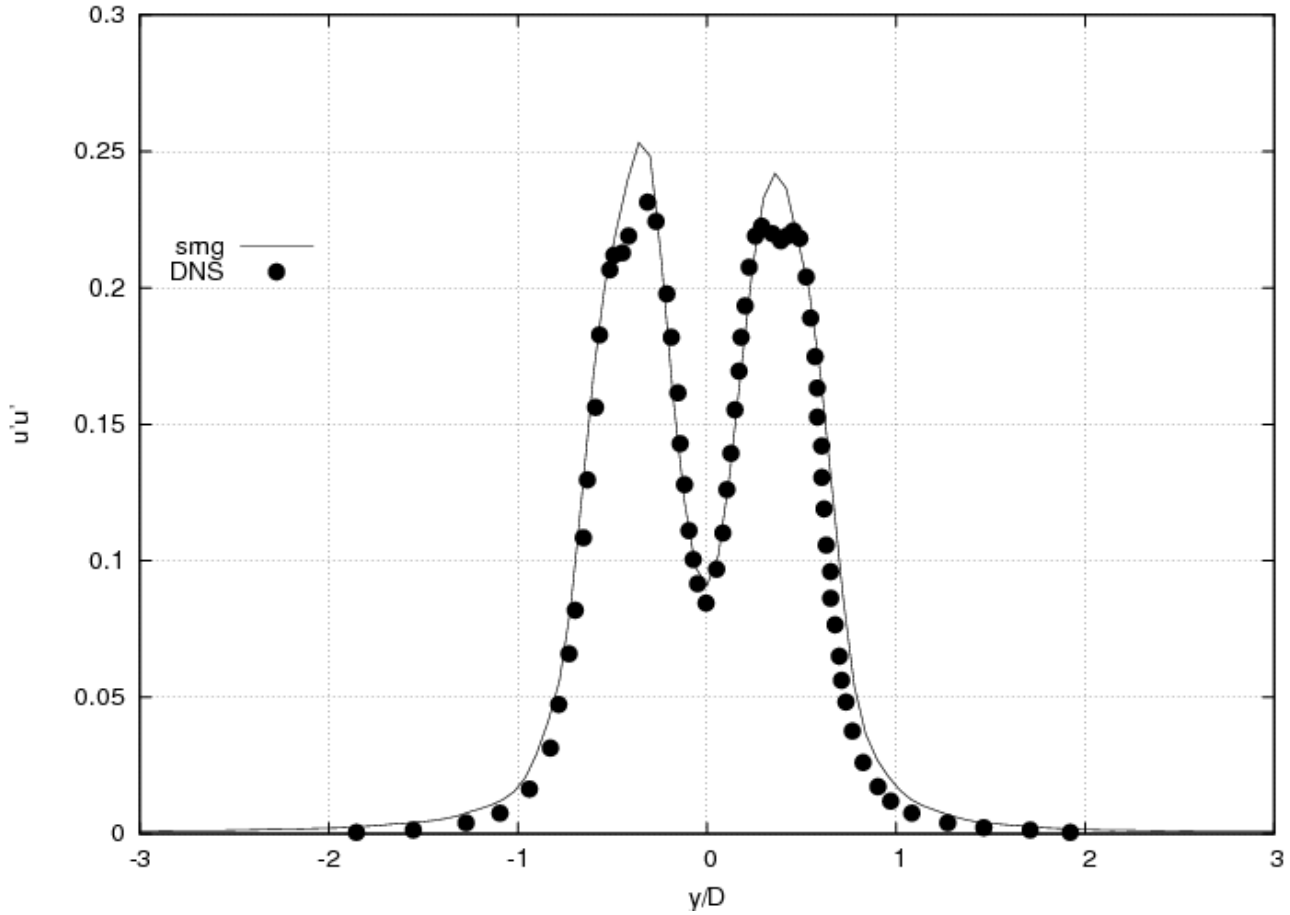

-2
-1
0
1
2
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
v'v'
y/D

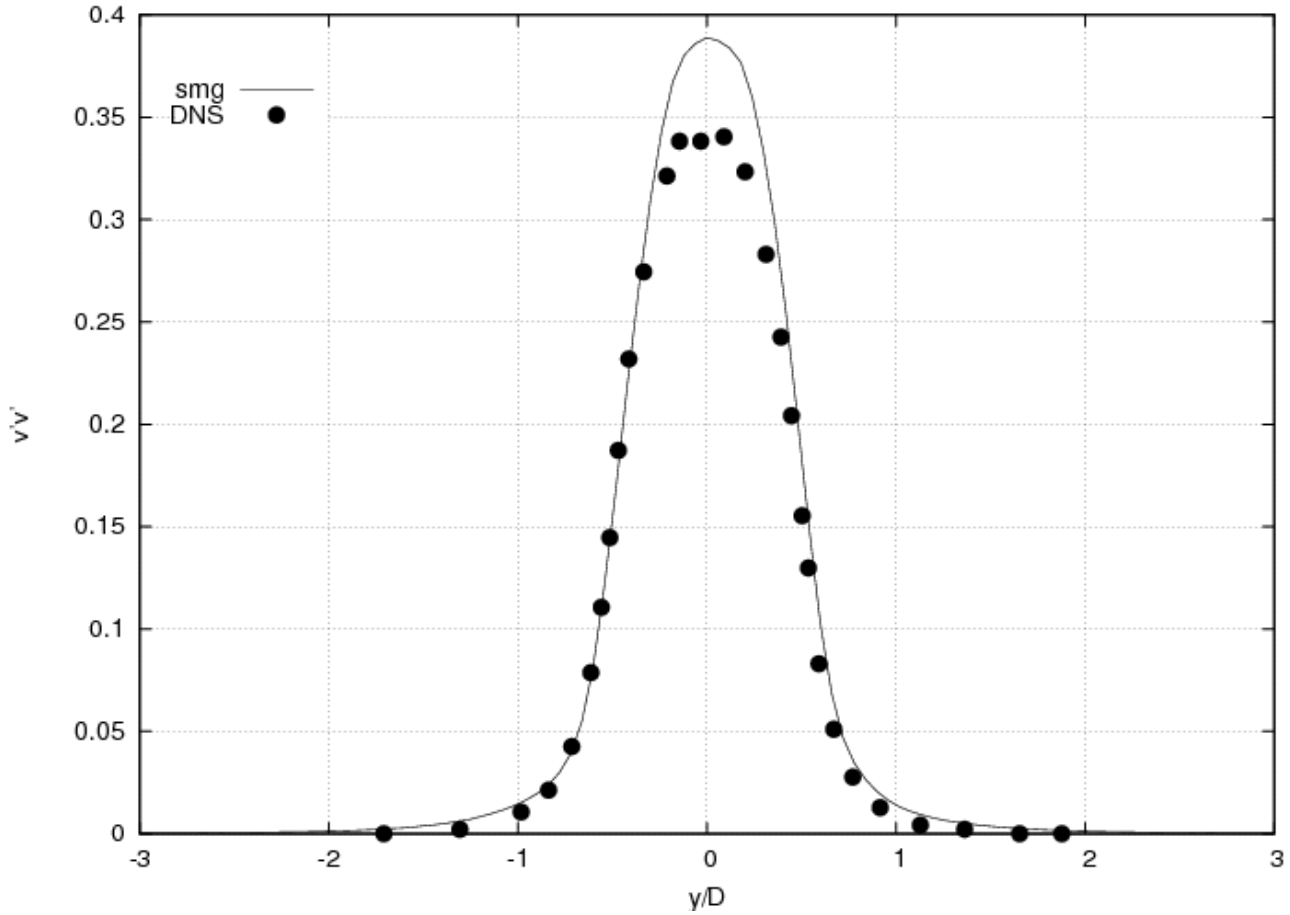
- Yoshizawa nonequilibrium fixed-parameter SGS model is tested only with the corse 160.00 cv.

- As a conclusion, overall good agreement between this modelling and reference data have been observed for both first- and second-order statistics.


1.5
1
0.5
0
-0.5
-1
-1.5
-2
u
-3
-2
-1
0
1
2
3
y/D
X/D=1.06
X/D=1.54
X/D=2.02
DNS
smg

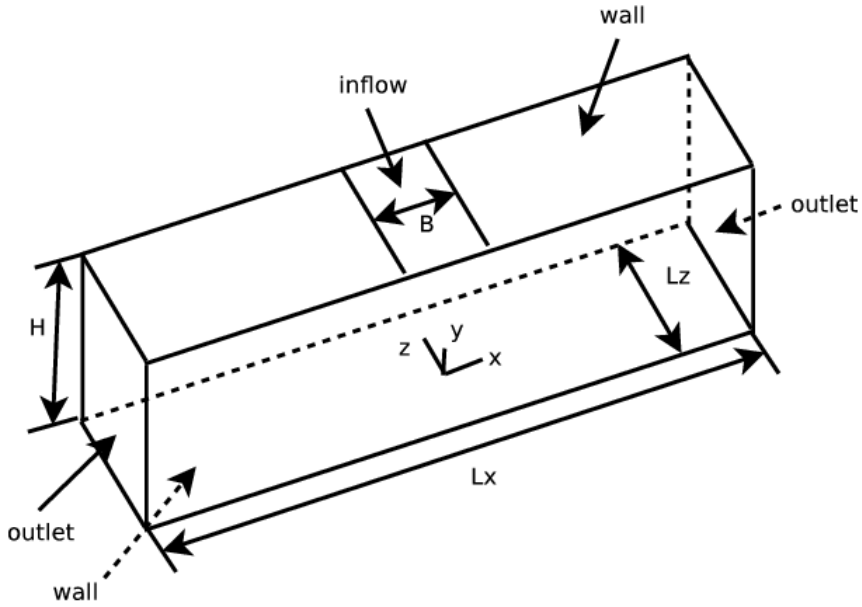

0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
v
-3
-2
-1
0
1
2
3
y/D
X/D=1.06
X/D=1.54
X/D=2.02
DNS
smg

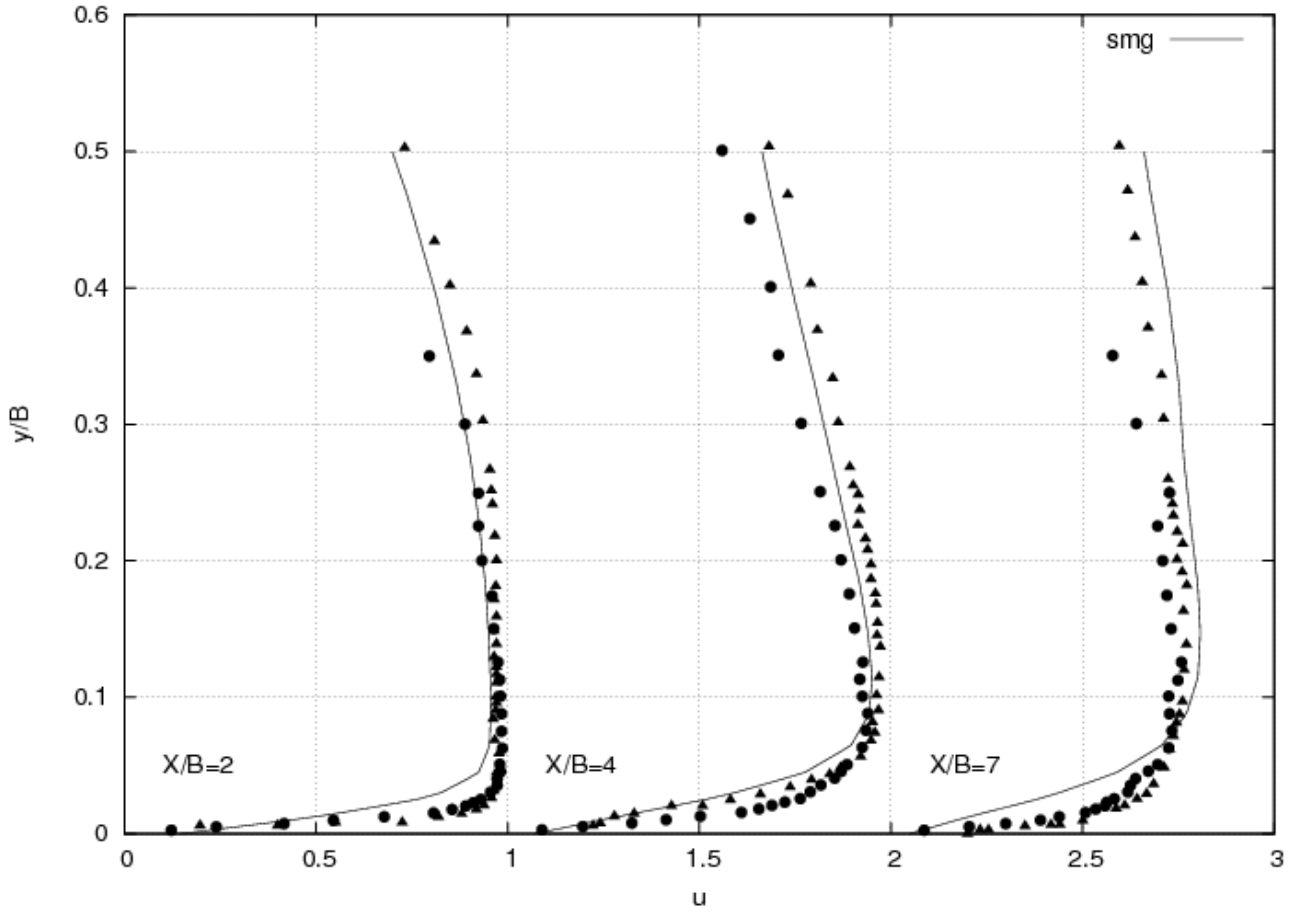

0.3
0.25
0.2
0.15
0.1
0.05
0
-3
-2
-1
0
1
2
3
u'u'
y/D
smg
DNS

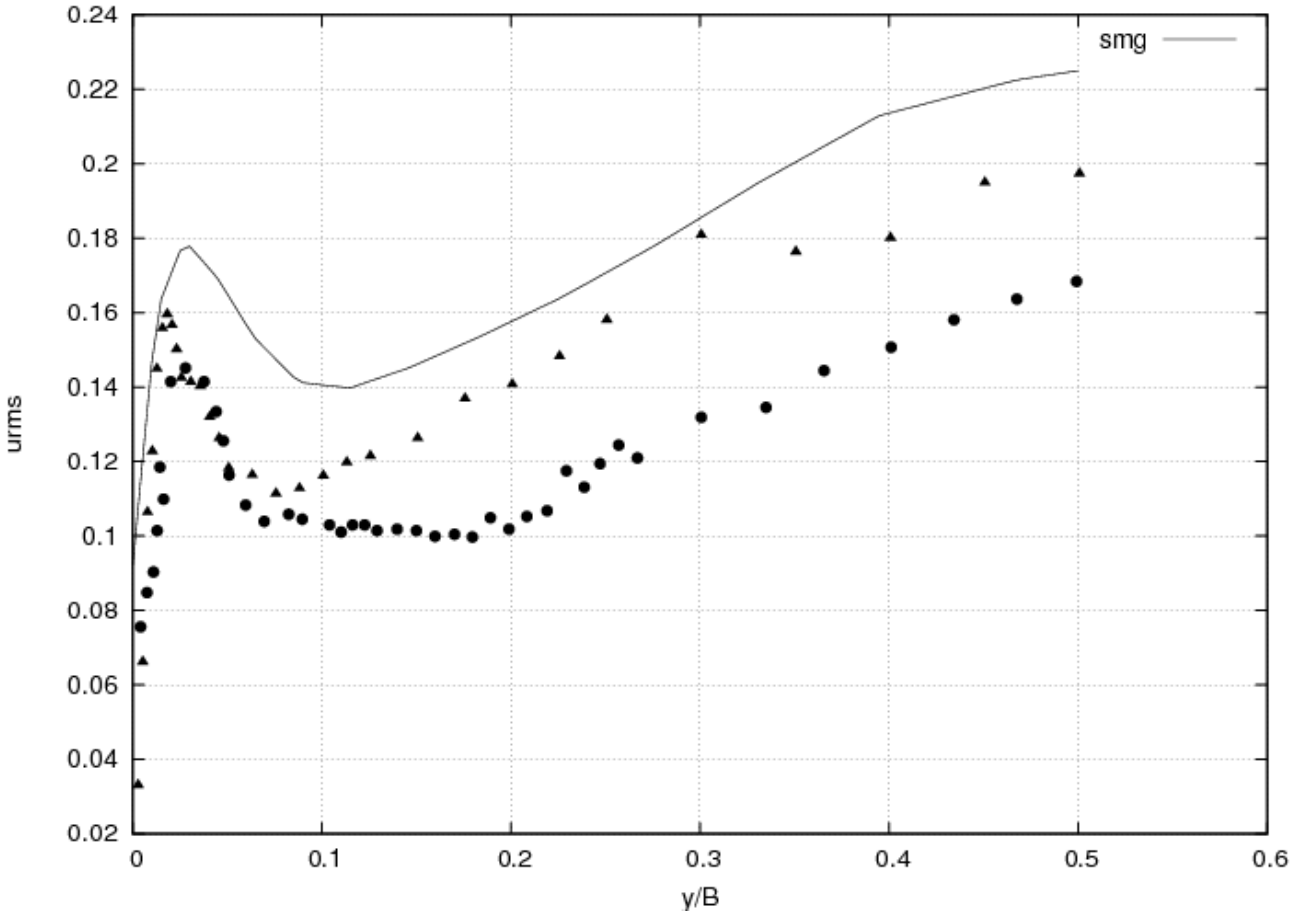

smg
DNS
v'v'
y/D
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-3
-2
-1
0
1
2
3

**LES modelling of a turbulent plane impinging jet (Reynolds number of $20 \cdot 10^3$ and $H/B = 4$):**

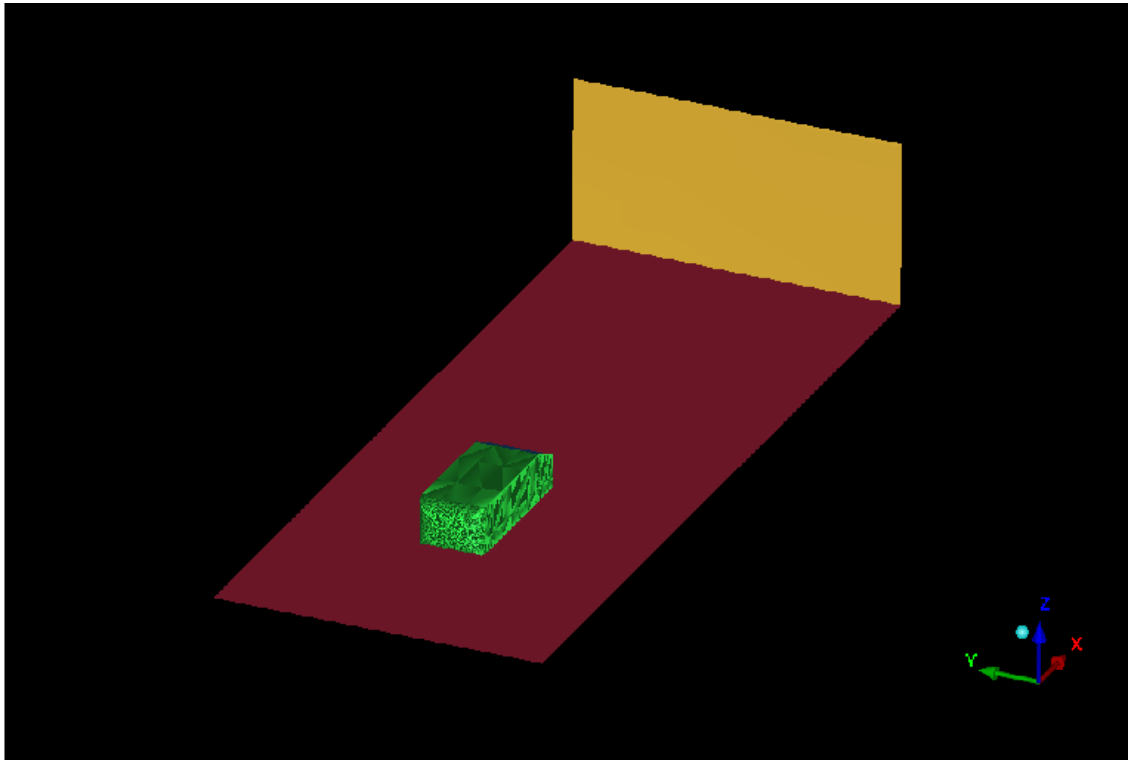
- A 96000 cv is used to test the validity of the Yoshizawa nonequilibrium fixed-parameter SGS model.
- Some minor discrepances are observed between experimental data and the model for this flow for both first and second order statistics




smg
X/B=2
X/B=4
X/B=7
y/B
u
0
0.1
0.2
0.3
0.4
0.5
0.6
0
0.5
1
1.5
2
2.5
3

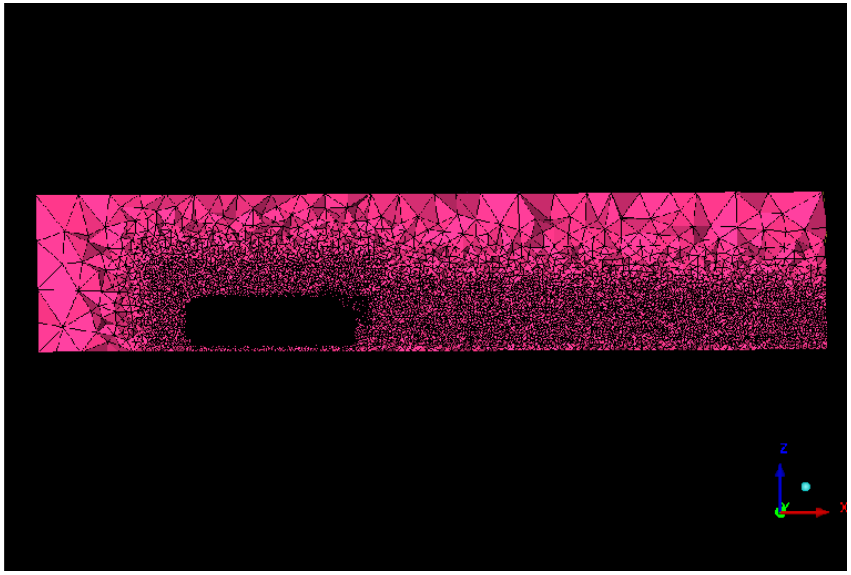

smg
urms
y/B
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.1
0.2
0.3
0.4
0.5
0.6

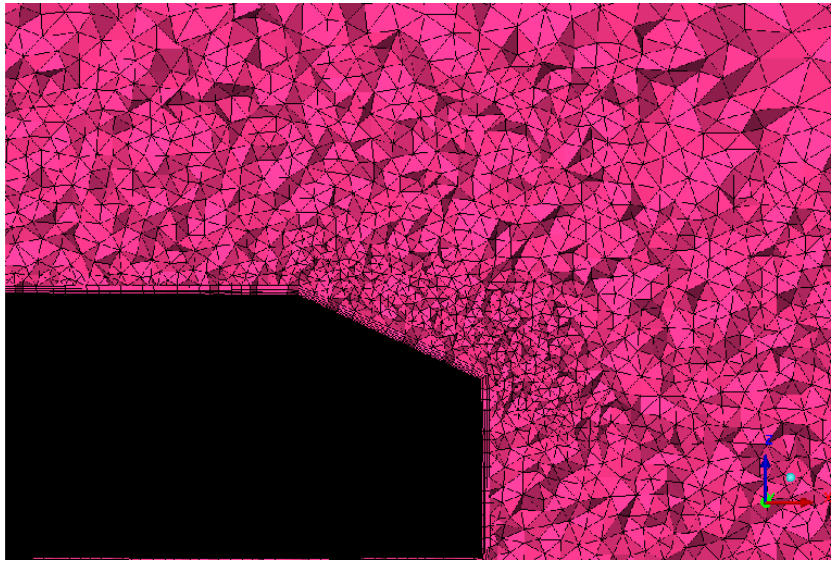
**External Ahmed car aerodynamics Re =** $4.3 \cdot 10^6$



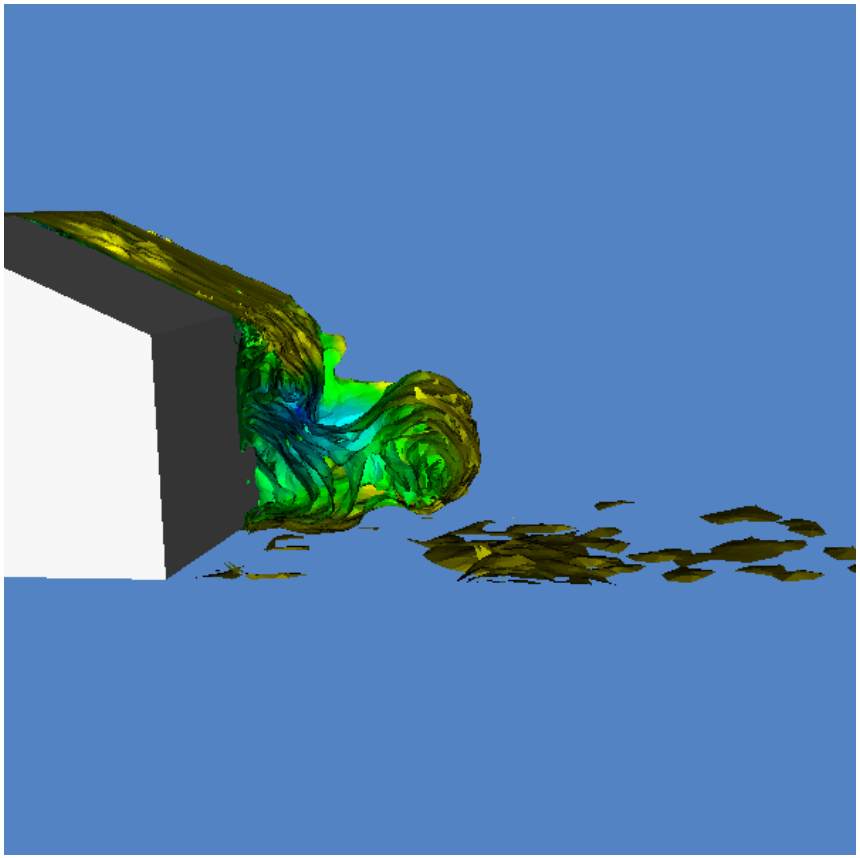

- inflow with velocity profile fixed.
- outflow with pressure profile fixed.
- solid wall boundary condition at at z=0.
- symmetry boundary condition at other planes.

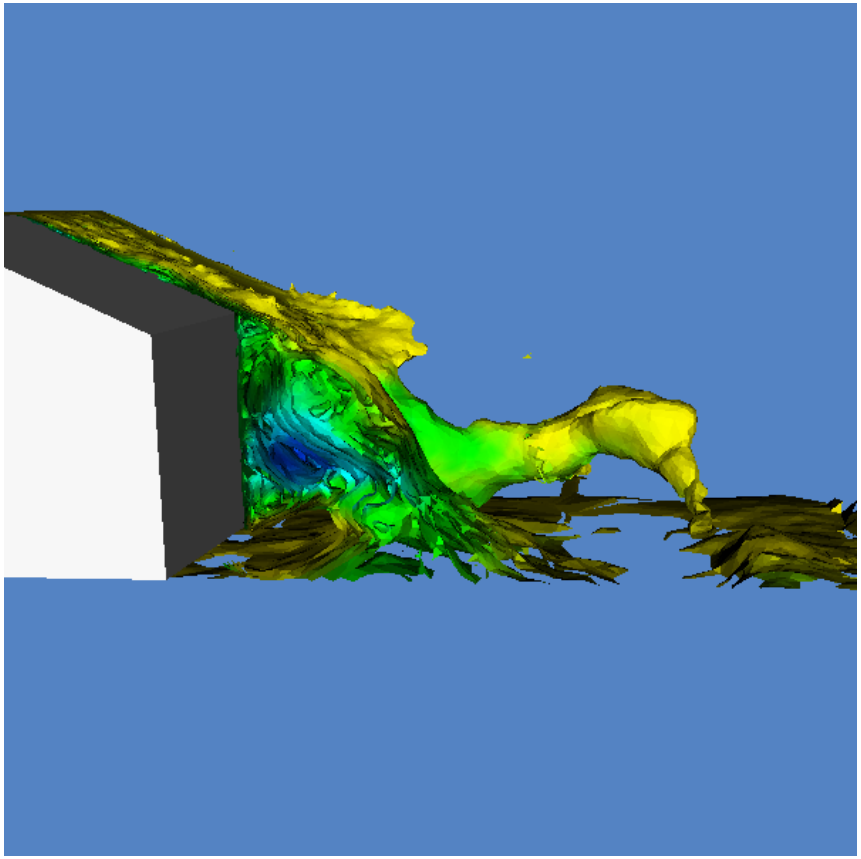
**External car aerodynamics**: mesh (over 3.000.000 CV, hybrid mesh for correct boundary layer integration).



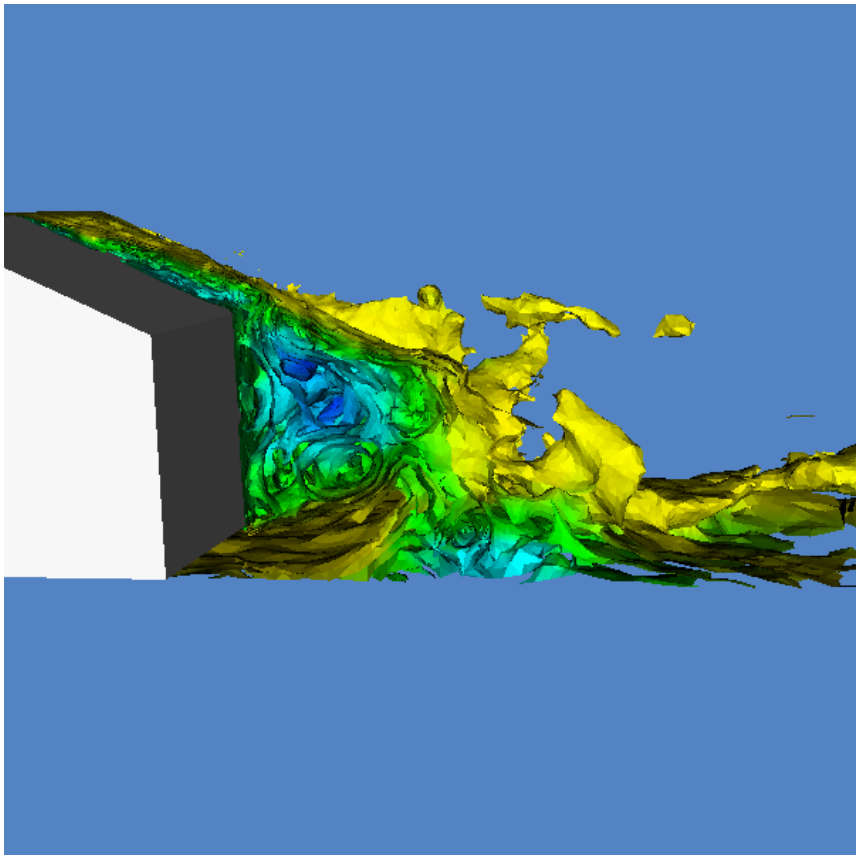


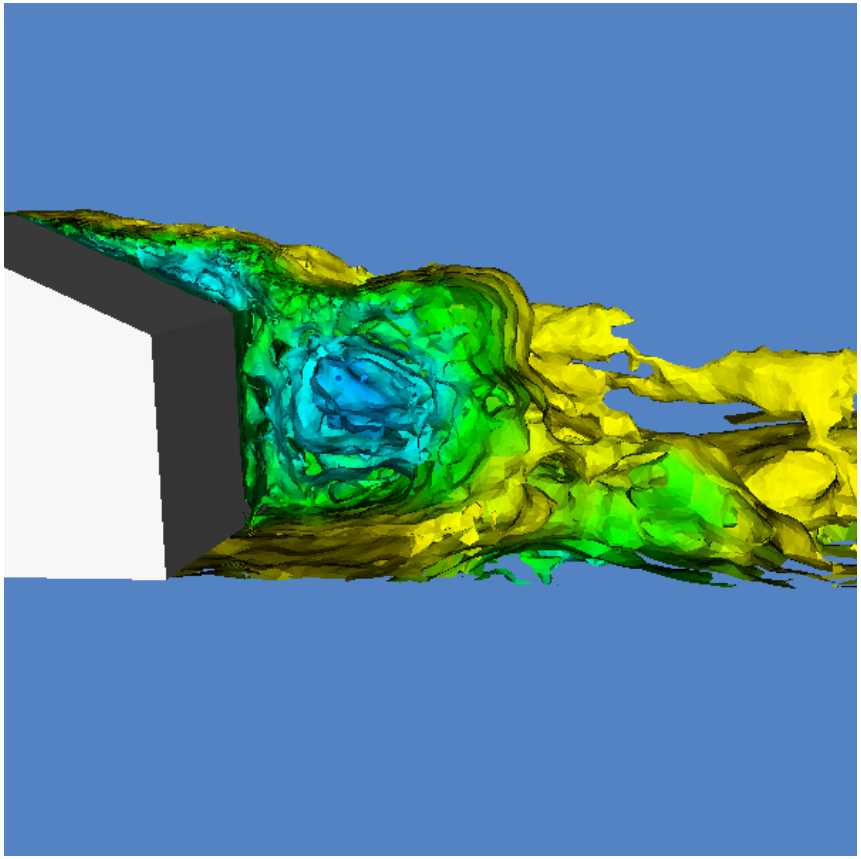
Instantaneous wake flow structures evolution in the symmetry plane ($z = 0.5m$)

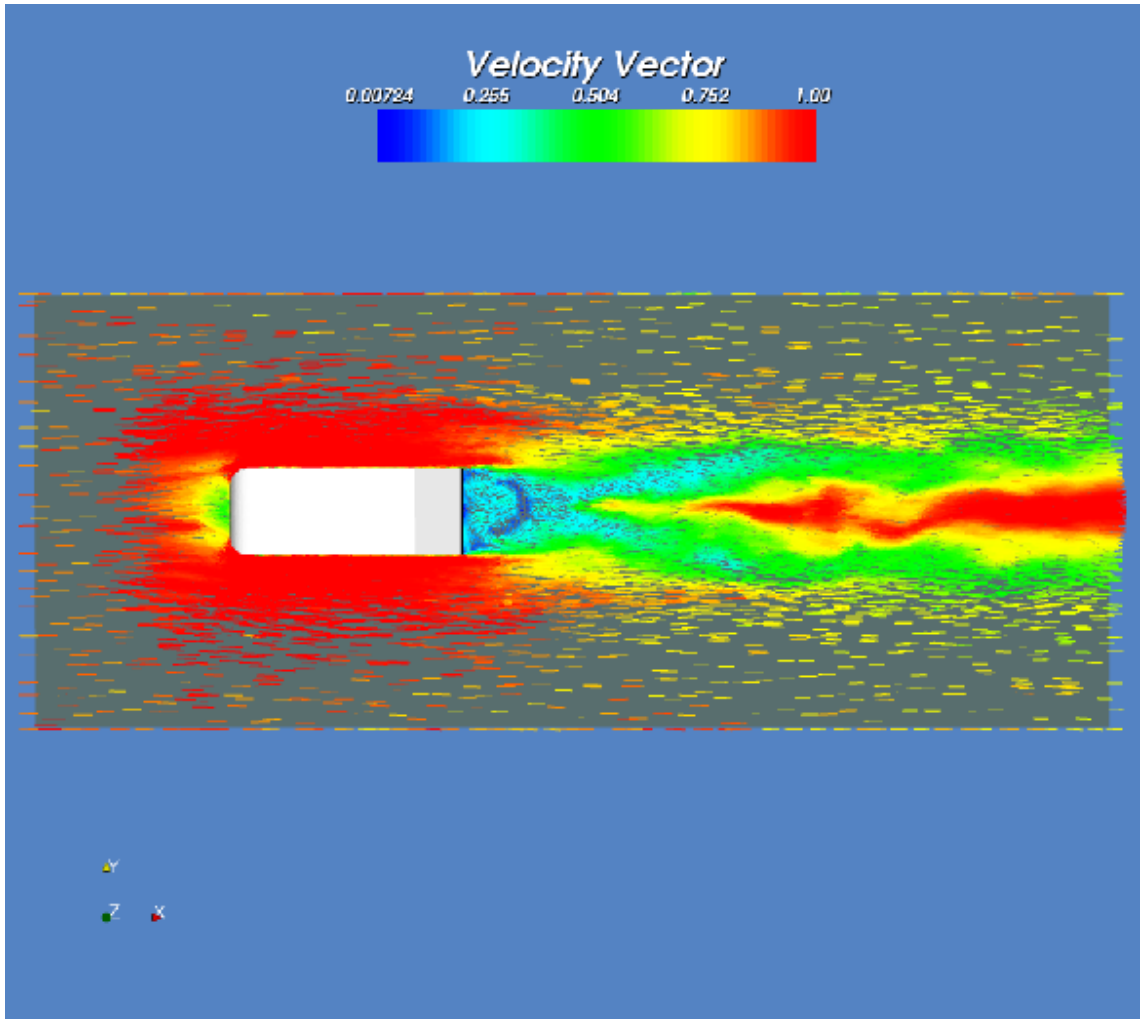


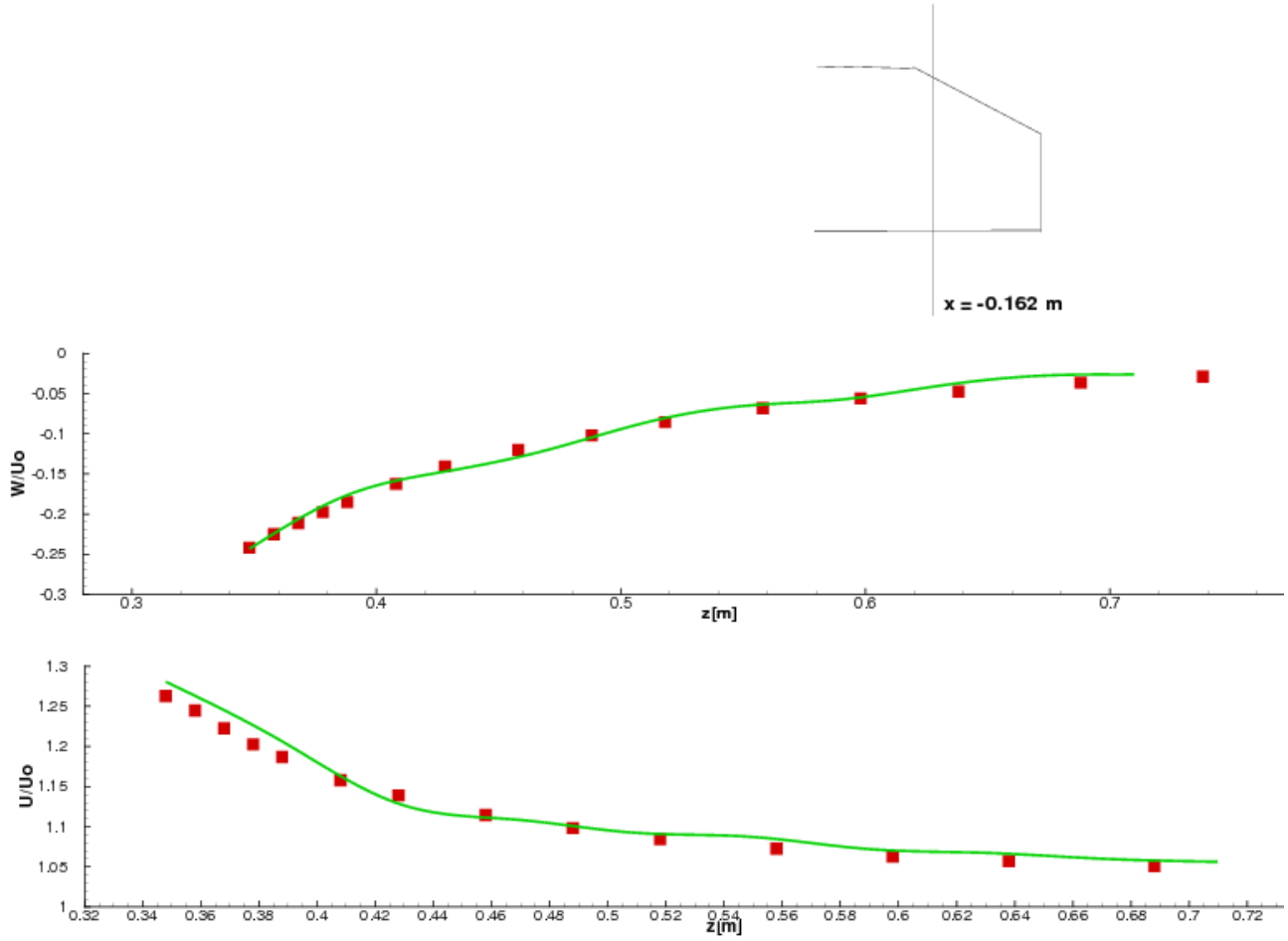


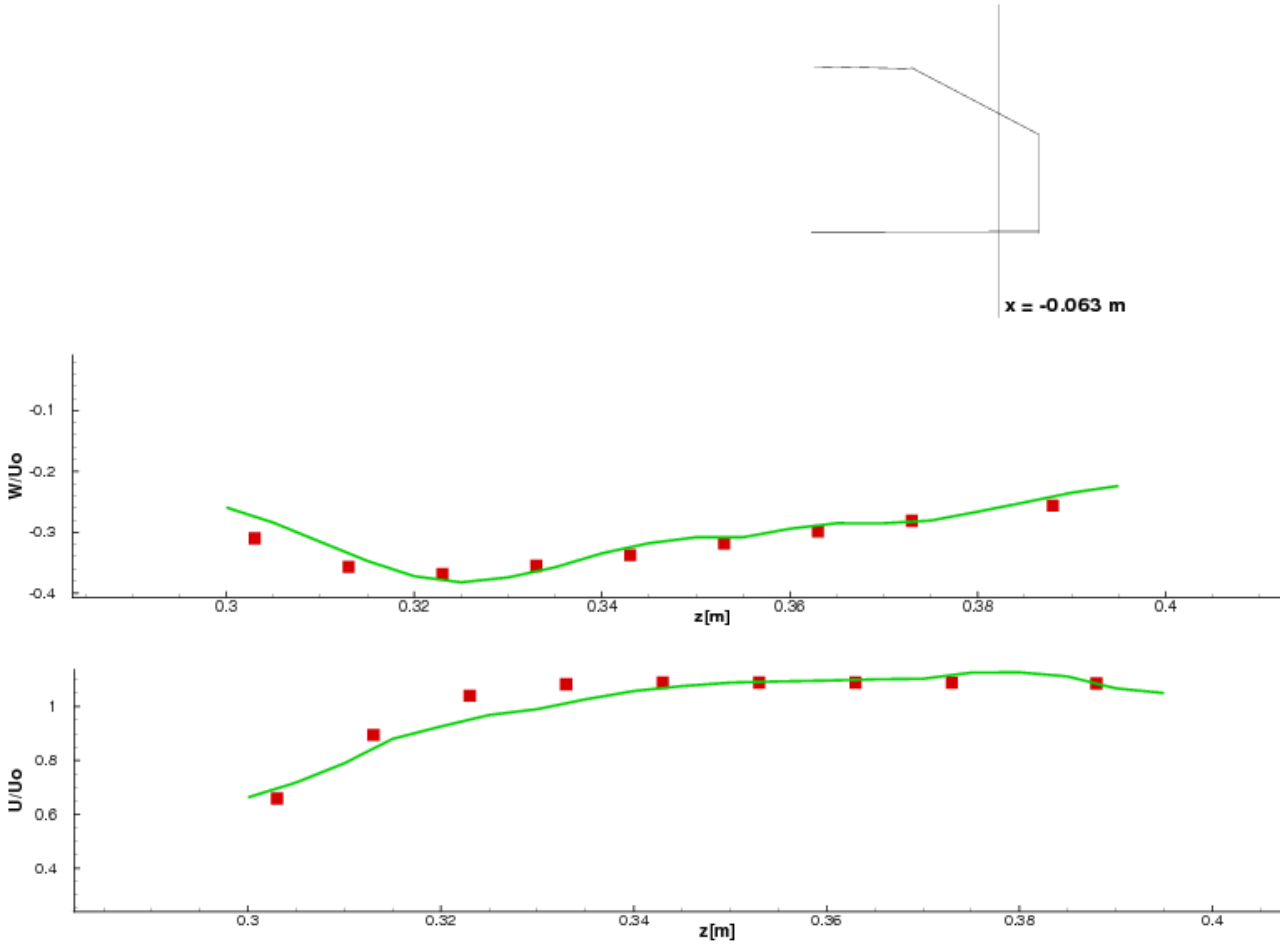
Instantaneous wake flow structures evolution in the symmetry plane ($z = 0.5m$)

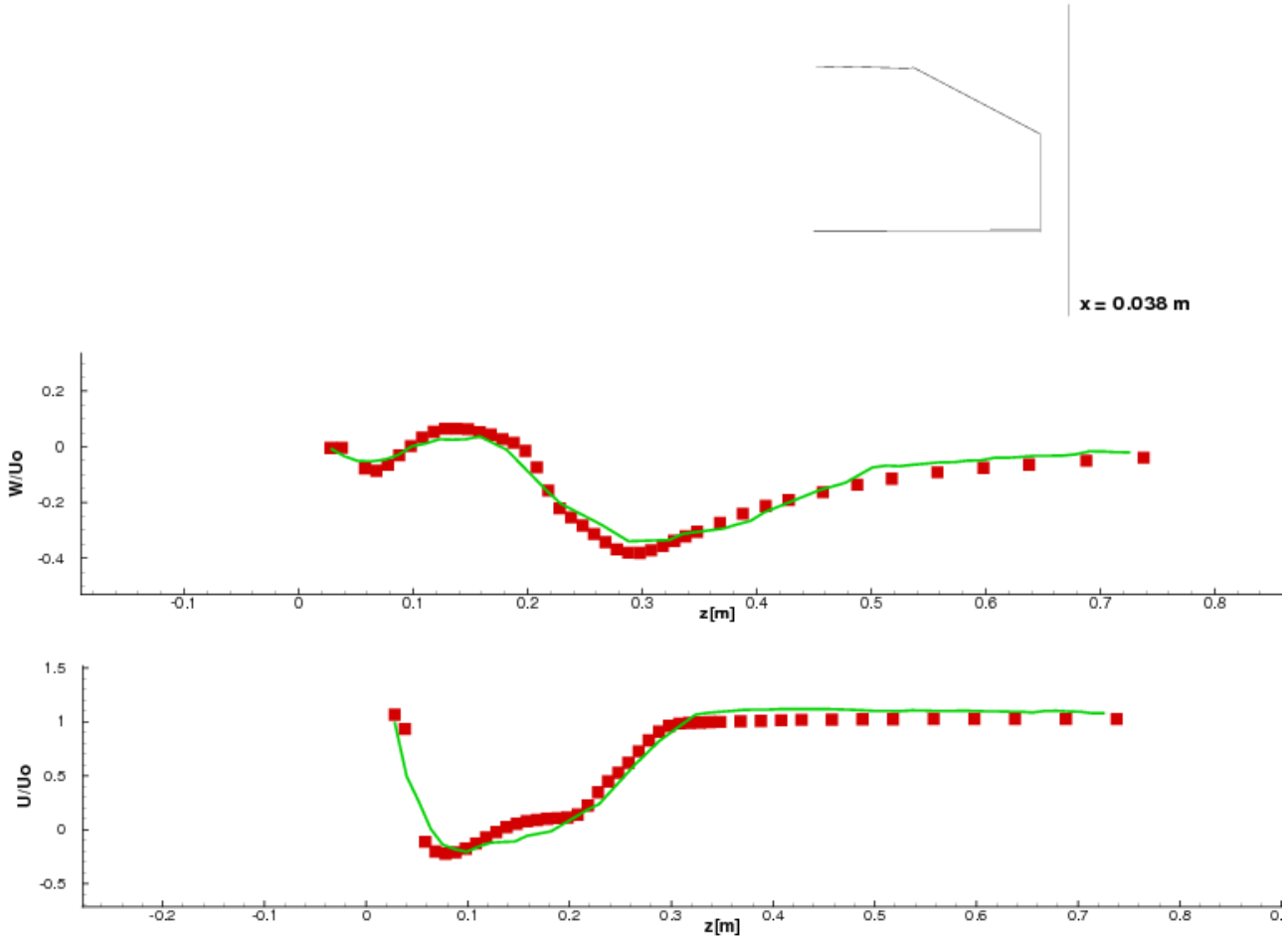





Velocity Vector
0.00724
0.255
0.504
0.752
1.00
Y
Z
X


x = -0.162 m
W/Uo
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
0.3
0.4
0.5
0.6
0.7
z[m]
U/Uo
1.3
1.25
1.2
1.15
1.1
1.05
1
0.32
0.34
0.36
0.38
0.4
0.42
0.44
0.46
0.48
0.5
0.52
0.54
0.56
0.58
0.6
0.62
0.64
0.66
0.68
0.7
0.72
z[m]


x = -0.063 m
W/Uo
-0.1
-0.2
-0.3
-0.4
0.3
0.32
0.34
0.36
0.38
0.4
z[m]
U/Uo
1
0.8
0.6
0.4
0.3
0.32
0.34
0.36
0.38
0.4
z[m]


x = 0.038 m
W/Uo
z[m]
U/Uo
z[m]

As a conclusion, in this work:

1. A non disipative formulation for the discretitzation of the Navier-Stokes equations suitable for DNS or LES modelling under general unstructured meshes have been presented.

2. A suitable SGS model for the simulation of complex flows and engineering applications have been presented and validated.

Future work:

- Performe more simulation with finer meshes of the presented test cases to assure a good asimptotic convergence of the presented results.

- We have begun to investigate Leray Regularization and Verstappen Regularization of Navier Stokes Equations. Our first results are promising, but more research in this area must to be done.