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Regularization models for the simulation of turbulence in a differentially heated cavity

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Presentation outline

1. Introduction
   - Problem definition: Differentially Heated Cavity
   - DNS results for $Ra = 10^{10}$, $Pr = 0.71$
   - Governing equations
2. Regularization models for the simulation of turbulence
   - Existing regularization: Leray and Navier-Stokes-α models
   - Symmetry-preserving regularization models
   - Discretization of the convective operator: a symmetry-preserving discretization
   - Choice of the filter
3. Results for a Differentially Heated Cavity
   - Description of cases
   - Mean fields
   - Convergence studies
   - Turbulent statistics
4. Conclusions and Future Research

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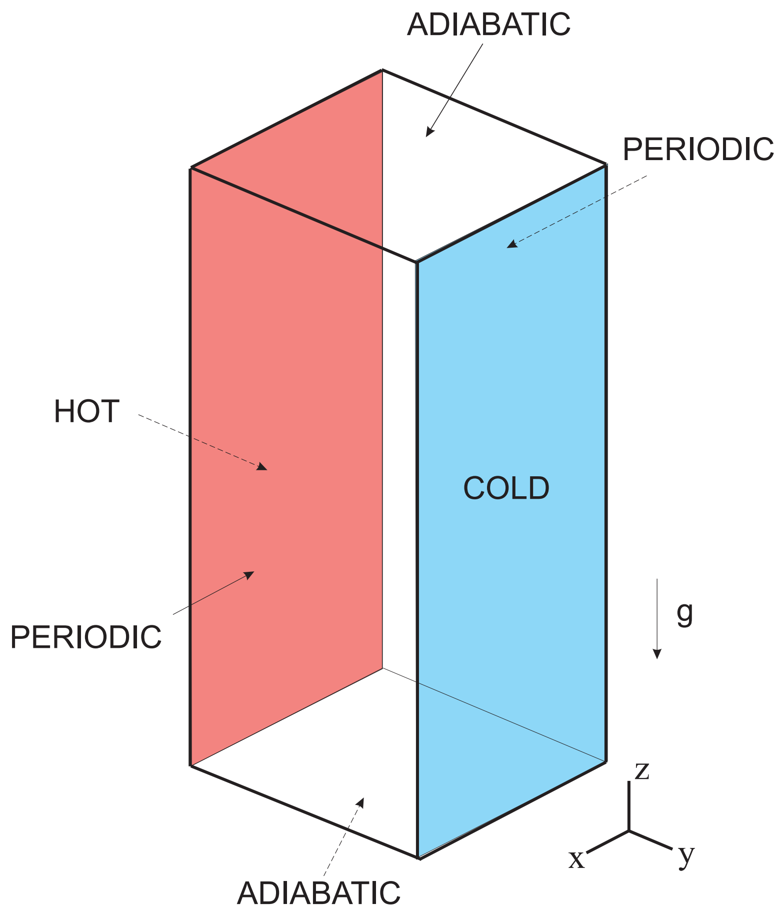
Problem definition: Differentially Heated Cavity



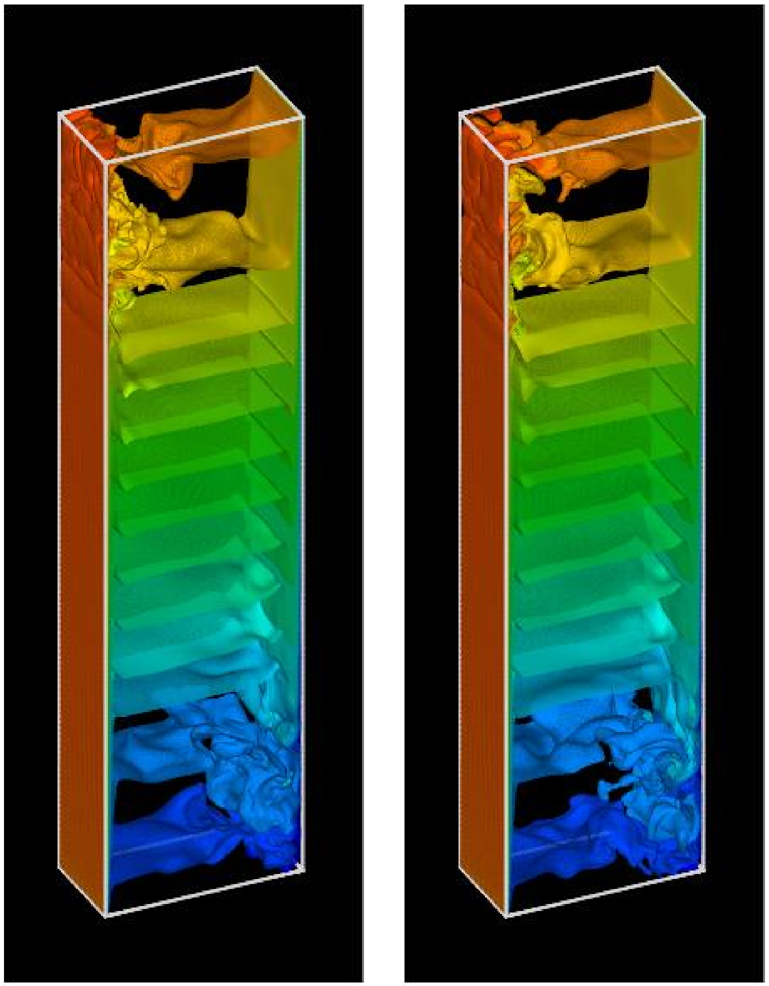
Boundary conditions:

- **Isothermal vertical walls**
- **Adiabatic horizontal walls**
- **Periodic** boundary conditions in the x-direction, orthogonal to the main flow

Dimensionless governing numbers:

- $Ra_z = \frac{\beta \Delta T L_z^3 g}{\alpha \nu}$
- $Pr = \frac{\nu}{\alpha}$
- Height aspect ratio $A_z = \frac{Lz}{Ly}$
- Depth aspect ratio $A_x = \frac{Lx}{Ly}$

DNS results for $Ra = 10^{10}$, $Pr = 0.71$



Some details about **DNS simulations**:

- Mesh size: $64 \times 136 \times 324$
- Computing Time: ≈ 1 month - 24 CPUs
- 4^{th}-order symmetry-preserving discretization

Complexity of the flow:

- Boundary layers
- Stratified cavity core
- Internal waves
- Recirculation areas

Governing equations

Incompressible Navier-Stokes coupled with energy transport equation:

$$\begin{aligned}
\nabla \cdot u &= 0 \\
\partial_t u + \mathcal{C}(u, u) &= Pr\mathcal{D}(u) - \nabla p + f \\
\partial_t T + \mathcal{C}(u, T) &= \mathcal{D}(T)
\end{aligned}$$

where $f = (0, 0, RaPrT)$ (Boussinesq approximation) and the **nonlinear convective term** is given by

$$\mathcal{C}(u, v) = (u \cdot \nabla)v$$

and the linear dissipative term is given by

$$\mathcal{D}(u) = \frac{1}{Ra^{0.5}} \nabla^2 u$$

Regularization modelling

A **dynamically less complex mathematical formulation** is sought. We consider smooth approximations (regularizations) of the nonlinearity,

$$
\begin{aligned}
\partial_t u_\epsilon + \widetilde{\mathcal{C}}(u_\epsilon, u_\epsilon) &= Pr\mathcal{D}(u_\epsilon) - \nabla p_\epsilon + f \\
\partial_t T_\epsilon + \widetilde{\mathcal{C}}(u_\epsilon, T_\epsilon) &= \mathcal{D}(T_\epsilon)
\end{aligned}
$$

such approximations may fall in the **Large-Eddy Simulation** (LES) concept,

$$
\begin{aligned}
\partial_t \bar{u}_\epsilon + \mathcal{C}(\bar{u}_\epsilon, \bar{u}_\epsilon) &= Pr\mathcal{D}(\bar{u}_\epsilon) - \nabla \bar{p}_\epsilon + f + \mathcal{M}_1(\bar{u}_\epsilon, \bar{u}_\epsilon) \\
\partial_t \overline{T}_\epsilon + \mathcal{C}(\bar{u}_\epsilon, \overline{T}_\epsilon) &= \mathcal{D}(\overline{T}_\epsilon) + \mathcal{M}_2(\bar{u}_\epsilon, \overline{T}_\epsilon)
\end{aligned}
$$

if the model terms were given by

$$
\begin{aligned}
\mathcal{M}_1(\bar{u}_\epsilon, \bar{u}_\epsilon) &= \mathcal{C}(\bar{u}_\epsilon, \bar{u}_\epsilon) - \overline{\widetilde{C}(\bar{u}_\epsilon, \bar{u}_\epsilon)} \\
\mathcal{M}_2(\bar{u}_\epsilon, \overline{T}_\epsilon) &= \mathcal{C}(\bar{u}_\epsilon, \overline{T}_\epsilon) - \overline{\widetilde{C}(\bar{u}_\epsilon, \overline{T}_\epsilon)}
\end{aligned}
$$

Existing regularization modellings

Leray and Navier-Stokes-α models

The regularization methods basically **alters the convective term** to **restrain the production of small scales** of motion.

- Leray model:

$$\partial_t u_\epsilon + \mathcal{C}(\bar{u}_\epsilon, u_\epsilon) = \mathcal{D}(u_\epsilon) - \nabla p_\epsilon$$

- Navier-Stokes-α model:

$$\partial_t u_\epsilon + \mathcal{C}_r(u_\epsilon, \bar{u}_\epsilon) = \mathcal{D}(u_\epsilon) - \nabla \pi_\epsilon$$

where the $\pi = p + u^2/2$ and the convetive operator in rotational form is defined as

$$\mathcal{C}_r(u, v) = (\nabla \times u) \times v$$

However, in doing so some of the **inviscid invariants** (kinetic energy, enstrophy in 2D and helicity in 3D) are **not conserved**.

Symmetry-preserving regularization models (1/2)

In order to conserve the following inviscid invariants

- Kinetic energy

$$\int_\Omega u \cdot u d\Omega$$

- Enstrophy (in 2D)

$$\int_\Omega (\nabla \times u) \cdot (\nabla \times u) d\Omega$$

- Helicity (in 3D)

$$\int_\Omega (\nabla \times u) \cdot u d\Omega$$

the **approximate convective operator** has to be **skew-symmetric**:

$$\left(\widetilde{\mathcal{C}}(u,v),w\right) = -\left(\widetilde{\mathcal{C}}(u,w),v\right)$$

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Symmetry-preserving regularization models (2/2)

This criterion yields the following class of approximations...

$$\partial_t u_\epsilon + \mathcal{C}_n(u_\epsilon, u_\epsilon) = \mathcal{D}(u_\epsilon) - \nabla p_\epsilon$$

in which the convective term in smoothened according to:

$$\begin{aligned}
\mathcal{C}_2(u,v) &= \overline{\mathcal{C}(\bar{u},\bar{v})} \\
\mathcal{C}_4(u,v) &= \mathcal{C}(\bar{u},\bar{v}) + \overline{\mathcal{C}(\bar{u},v')} + \overline{\mathcal{C}(u',\bar{v})} \\
\mathcal{C}_6(u,v) &= \mathcal{C}(\bar{u},\bar{v}) + \mathcal{C}(\bar{u},v') + \mathcal{C}(u',\bar{v}) + \overline{\mathcal{C}(u',v')}
\end{aligned}$$

where $u' = u - \bar{u}$ and $\mathcal{C}_n(u,v) = \mathcal{C}(u,v) + \mathcal{O}(\epsilon^n)$ for **any symmetric filter**.

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Discretization of the convective operator: a symmetry-preserving discretization

The spatially discrete incompressible Navier-Stokes equations can be expressed as

$$\begin{aligned} \mathsf{H}\frac{du_h}{dt} + \mathsf{C}(u_h)u_h + \mathsf{D}u_h - \mathsf{M}^T p_h &= 0 \\ \mathsf{M}u_h &= 0 \end{aligned}$$

It can be shown that the **convective matrix** $\mathsf{C}(u_h)$ has to be **skew-symmetric**,

$$\mathsf{C}(u_h) + \mathsf{C}^T(u_h) = 0$$

to **preserve** the continuous **invariants** (kinetic energy, enstrophy in 2D and helicity in 3D) in a **discrete sense**.

Choice of the filter

Let us consider a generic linear filter

$$\bar{u}_\epsilon = F u_\epsilon$$

Then, three basic properties are required for the filter:

$$\begin{aligned} \bar{u}_\epsilon &= u_\epsilon + \mathcal{O}(\epsilon^2) \\ (HF) &= (HF)^T \\ F1 &= 1 \end{aligned}$$

$\Longrightarrow$ Our filter is based on the elliptic differential operator

$$(1 - \alpha_1^2 \partial_{xx}^2 - \alpha_2^2 \partial_{yy}^2 - \alpha_3^2 \partial_{zz}^2)\bar{u}_\epsilon = u_\epsilon$$

where filter length is defined by

$$\epsilon_i = \alpha_i \sqrt{24}$$

Results for differentially heated cavity at $Ra = 10^{10}$

- Regularization model $\mathcal{C}_4$ is tested.
- Two very coarse meshes are considered

	DNS	RM1	RM2
Nx	64	8	8
Ny	136	17	13
Nz	324	40	30
Δx_{min}	7.81×10^{-3}	6.25×10^{-2}	6.25×10^{-2}
Δy_{min}	1.11×10^{-3}	8.88×10^{-3}	1.16×10^{-2}
Δz_{min}	1.23×10^{-2}	9.96×10^{-2}	1.33×10^{-1}

- Ratio ϵ/h (filter length to the average grid width) is kept constant in all three spatial directions.

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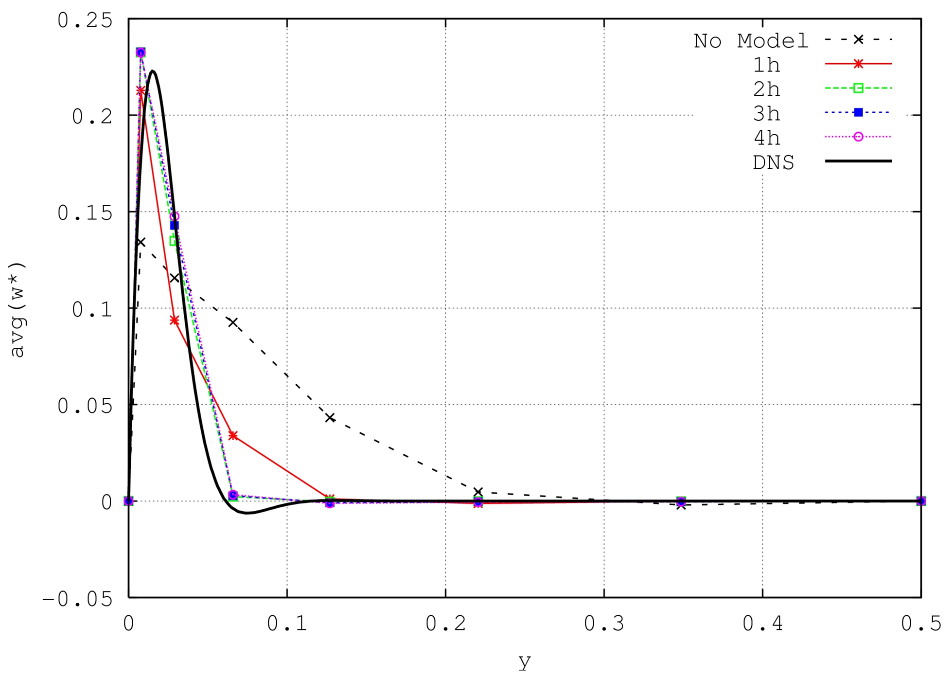
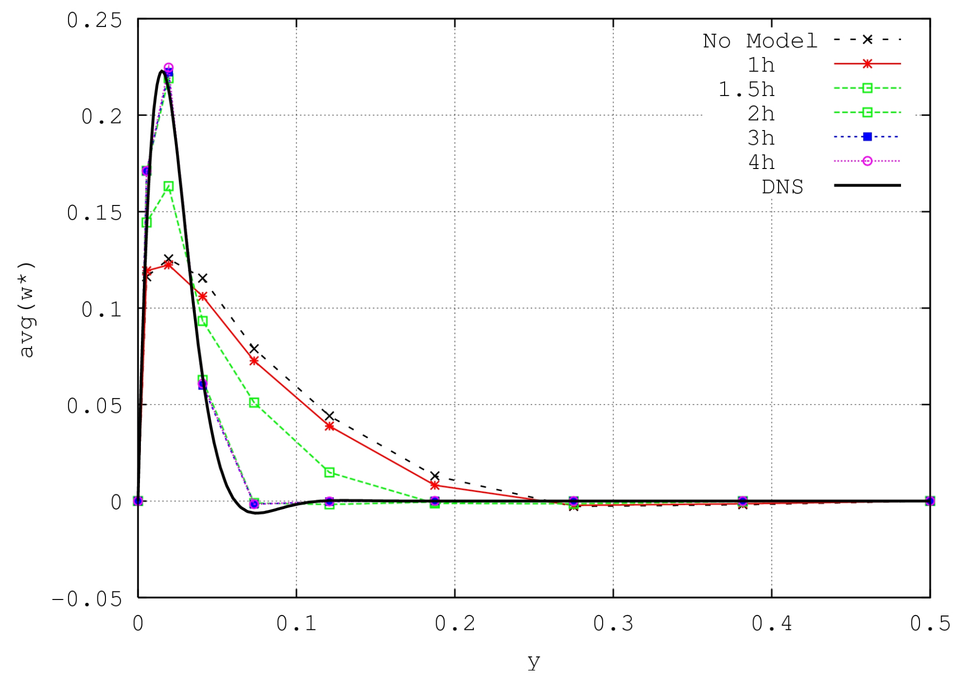
Results for differentially heated cavity at $Ra = 10^{10}$

Mean fields

$8 \times 13 \times 30$



$8 \times 17 \times 40$

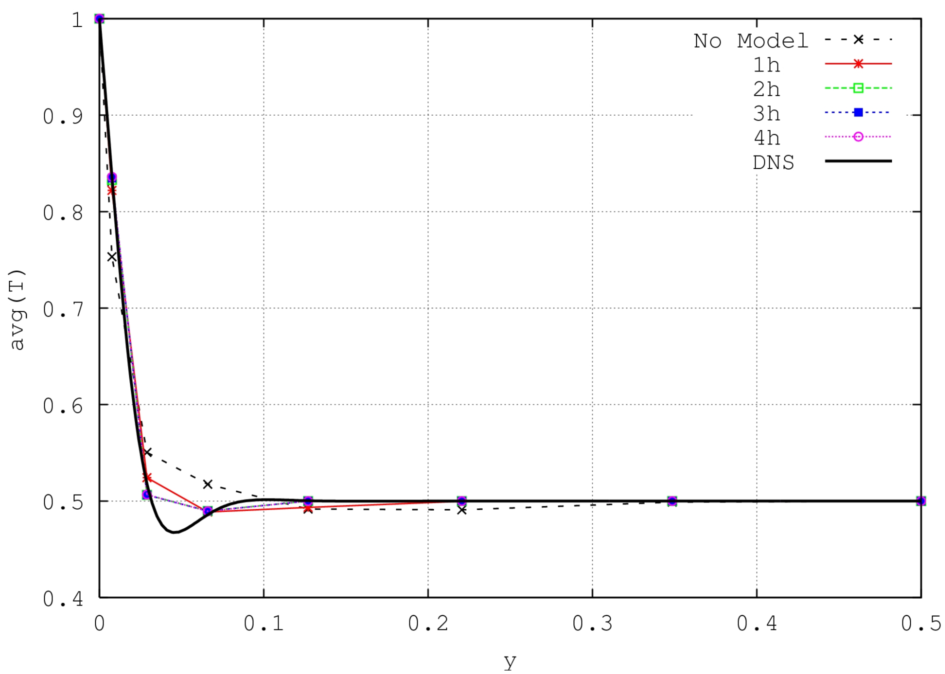


Averaged vertical velocity profile at the horizontal mid-height plane for different ϵ/h ratios.

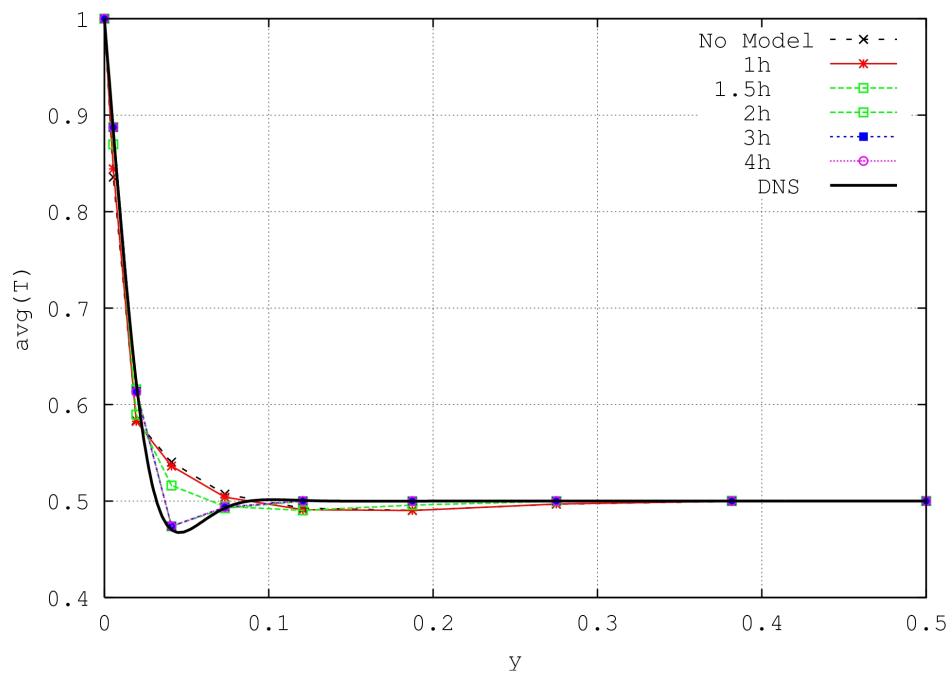
Results for differentially heated cavity at $Ra = 10^{10}$

Mean fields

$8 \times 13 \times 30$



$8 \times 17 \times 40$



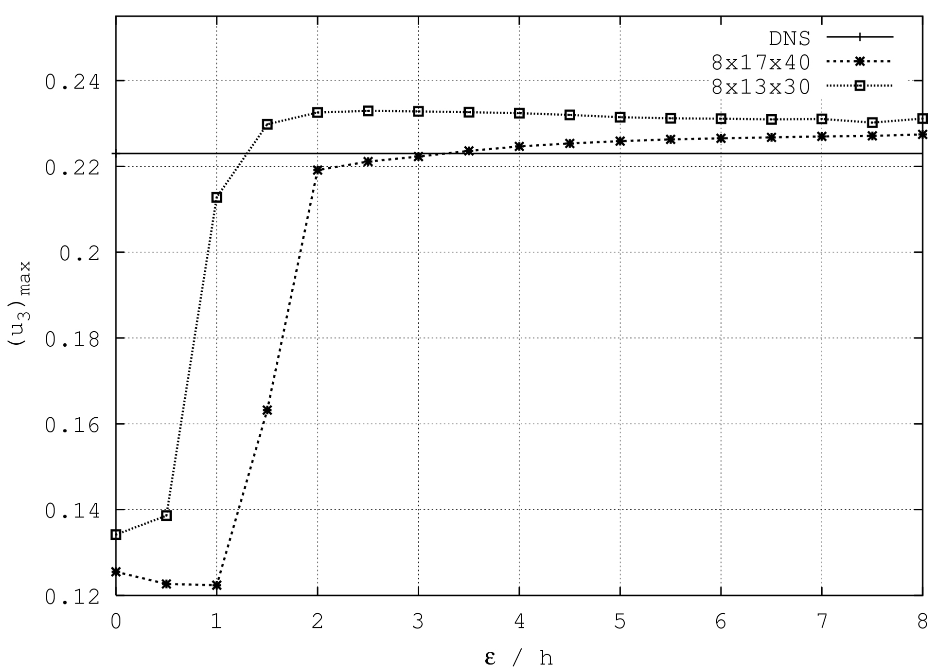
Averaged temperature profile at the horizontal mid-height plane for different ϵ/h ratios.

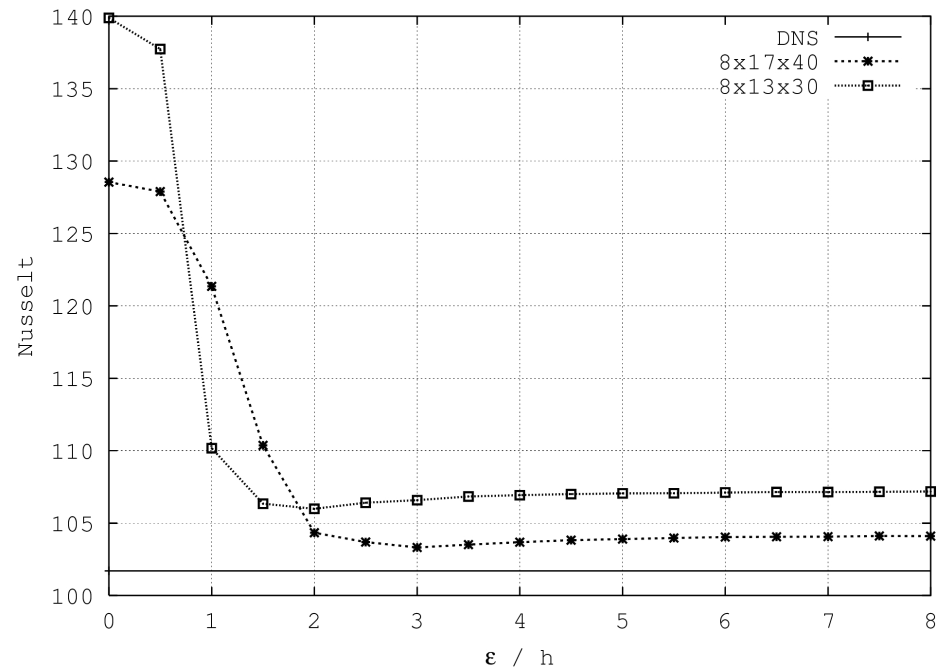
Results for differentially heated cavity at $Ra = 10^{10}$

Convergence studies

w_{max}

Nusselt



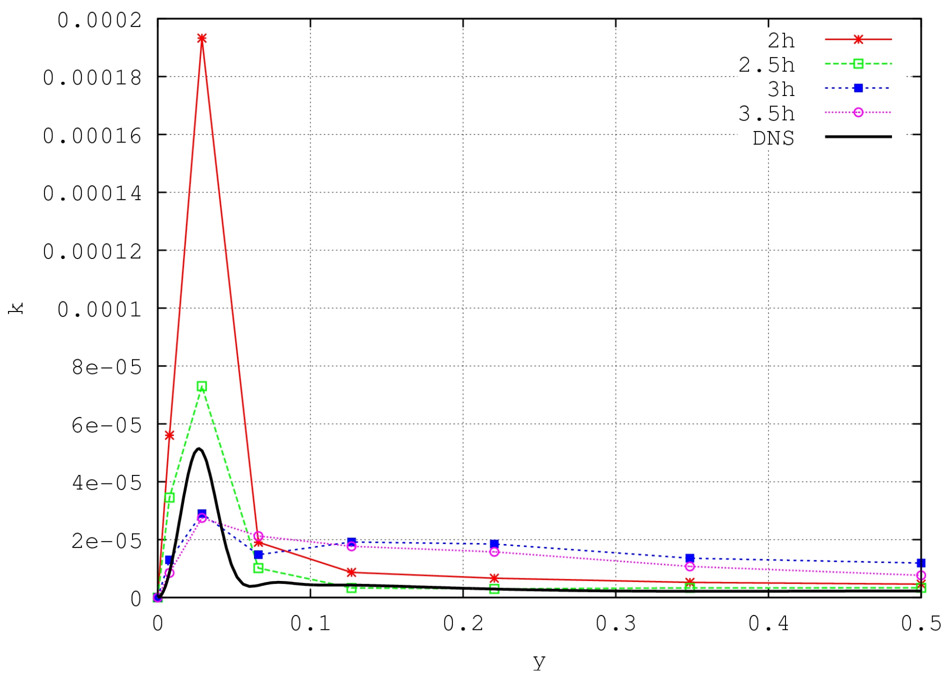


The maximum of the averaged vertical velocity at the horizontal mid-height plane and the overall averaged Nusselt number as a function of the ratio of the filter length ϵ to the average grid width h.

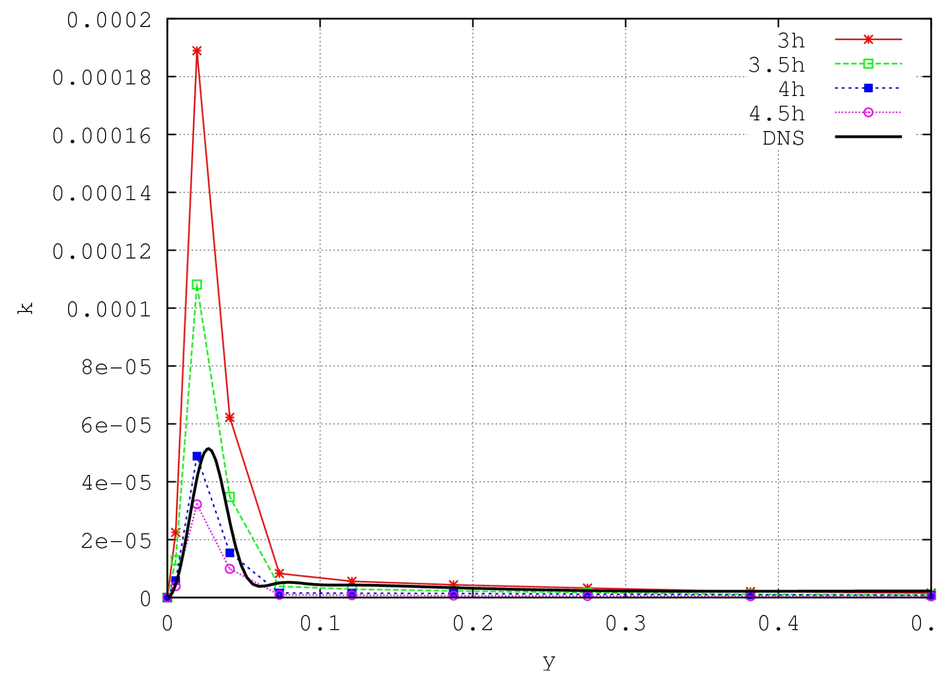
Results for differentially heated cavity at $Ra = 10^{10}$

Turbulent statistics

$8 \times 13 \times 30$



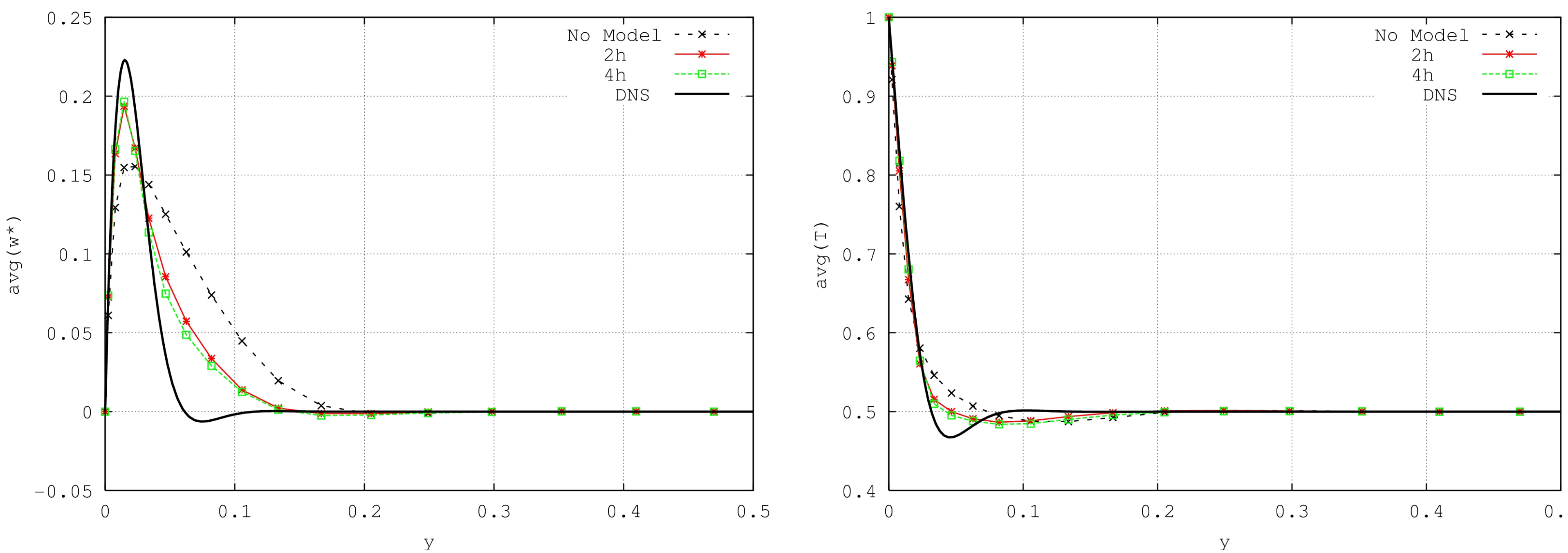
$8 \times 17 \times 40$



Turbulent kinetic energy $k = \overline{u_i' u_i'}$ profile at the horizontal mid-height plane for different filter lengths ϵ.

How the $\widetilde{\mathcal{C}}_4$ symmetry-preserving regularization modelling behave for finer grids?

First preliminary results on a finer $16 \times 34 \times 80$ grid does not seem to improve the agreement with DNS results...



Averaged vertical velocity and temperature profiles at the horizontal mid-height plane for different filter lengths ϵ.

$\Longrightarrow$ The problem seems to be related with the **linear filter**.

Conclusions and Future Research

The first results shown illustrate the potential of conservative smoothing as a new simulation shortcut.

The main advantages with respect exiting LES models can be summarized:

- **Robustnest**. As the smoothed governing equations preserve the symmetry properties of the original Navier-Stokes equations the solution can not blow up (in the energy-norm, in 2D also: enstrophy-norm). It seems that even for very coarse meshes reasonably results can be obtained.
- **Universality**. No *ad hoc* phenomenological arguments that can not be formally derived for the Navier-Stokes equations are used.

However, once the robustnest of the method has been shown for very coarse meshes, **future research** should focus on the construction of more appropriate **linear filters**.