TURBULENT FLOW IN A DIFFERENTIALLY HEATED CAVITY: DIRECT NUMERICAL SIMULATION AND REGULARIZATION MODELING

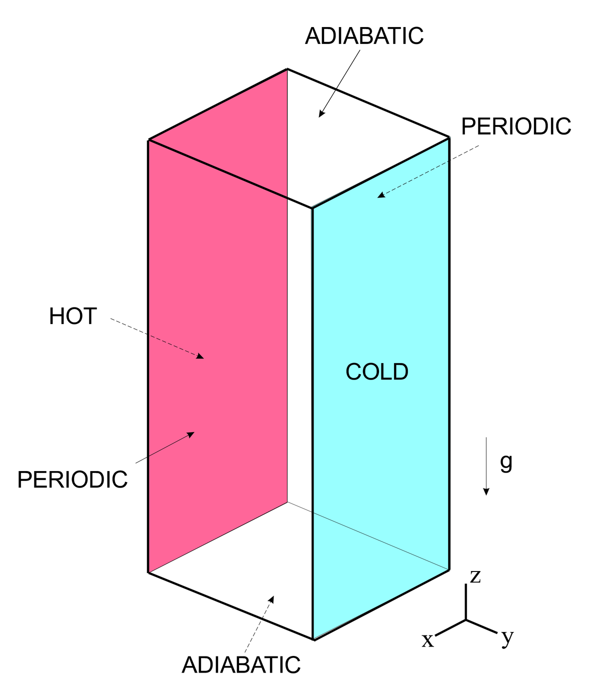
F.Xavier Trias[1,2], Andrey Gorobets[1], Assensi Oliva[1], Roel Verstappen[2]

[1]Heat and Mass Transfer Technological Center, Technical University of Catalonia, Spain
[2]Institute of Mathematics and Computing Science, University of Groningen, The Netherlands

Abstract

Since direct numerical simulations (DNS) of buoyancy-driven flows cannot be computed at high Rayleigh numbers (Ra), a dynamically less complex mathematical formulation is sought. In the quest for such a formulation, we consider regularizations (smooth approximations) of the nonlinear convective term: they basically alter the convective terms to reduce the production of small scales of motion. In this way, the new set of partial differential equations are dynamically less complex than the original Navier-Stokes (NS) equations, and therefore more amenable to be numerically solved. Here we propose to preserve the symmetry and conservation properties of the convective terms exactly. This requirement yields a family of *symmetry-preserving regularization* models. In this work, the performance of the method is tested for a turbulent differentially heated cavity (DHC).

DHC - Problem definition



Boundary conditions:

- **Isothermal vertical walls**
- **Adiabatic horizontal walls**
- **Periodic** boundary conditions in x

Dimensionless governing numbers:

- $Ra = (\beta \Delta T L_z^3 g)/(\alpha \nu)$
- $Pr = \nu/\alpha$
- Height aspect ratio $A_z = L_z/L_y$
- Depth aspect ratio $A_x = L_x/L_y$

$\mathcal{C}_4$-regularization modeling of turbulence

The incompressible NS equations form an excellent mathematical model for turbulent flows. In primitive variables they read

$$\partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u}, \boldsymbol{u}) = Pr Ra^{-1/2} \Delta \boldsymbol{u} - \nabla p + f \ ; \quad \nabla \cdot \boldsymbol{u} = 0$$
$$\partial_t T + \mathcal{C}(\boldsymbol{u}, T) = Ra^{-1/2} \Delta T$$

where Ra and Pr are the Rayleigh and Prandtl numbers and the non-linear convective term is defined by $\mathcal{C}(u, v) = (u \cdot \nabla) v$

Since the full energy spectrum, *i.e.* DNS, cannot be computed, a dynamically less complex mathematical formulation is sought. Here, we consider the $\mathcal{C}_4$ approximation: the convective term is replaced by the following $\mathcal{O}(\epsilon^4)$-accurate smooth approximation $\mathcal{C}_4(u, v)$ given by

$$\mathcal{C}_4(u, v) = \mathcal{C}(\bar{u}, \bar{v}) + \overline{\mathcal{C}(\bar{u}, v')} + \overline{\mathcal{C}(u', \bar{v})}$$

Note that here the prime indicates the residual of the filter, *e.g.* $u' = u - \bar{u}$, which can be explicitly evaluated, and $\overline{(\cdot)}$ represents a normalized self-adjoint linear filter with filter length ϵ. Therefore, the governing equations result to

$$\partial_t \boldsymbol{u} + \mathcal{C}_4(\boldsymbol{u}, \boldsymbol{u}) = Pr Ra^{-1/2} \Delta \boldsymbol{u} - \nabla p + f \ ; \quad \nabla \cdot \boldsymbol{u} = 0$$
$$\partial_t T + \mathcal{C}_4(\boldsymbol{u}, T) = Ra^{-1/2} \Delta T$$

Note that the $\mathcal{C}_4$ approximation is also a skew-symmetric operator like the original convective operator. Hence, the same inviscid invariants -kinetic energy, enstrophy in 2D and helicity- are preserved.

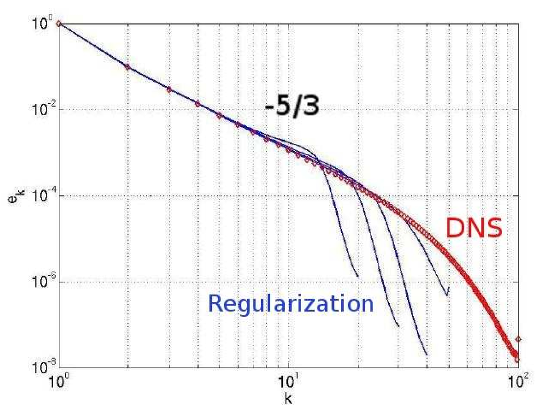
Mathematical foundation

Energy flux equation for the symmetry-preserving regularization resembles the NS

$$\frac{1}{2}\frac{d}{dt}|u_{kk'}|^2 + \nu |\nabla u_{kk'}|^2 = \widetilde{T}_k - \widetilde{T}_{k'} \quad \longrightarrow \quad \nu < |\nabla u_{kk'}|^2 > = < \widetilde{T}_k > - < \widetilde{T}_{k'} >$$

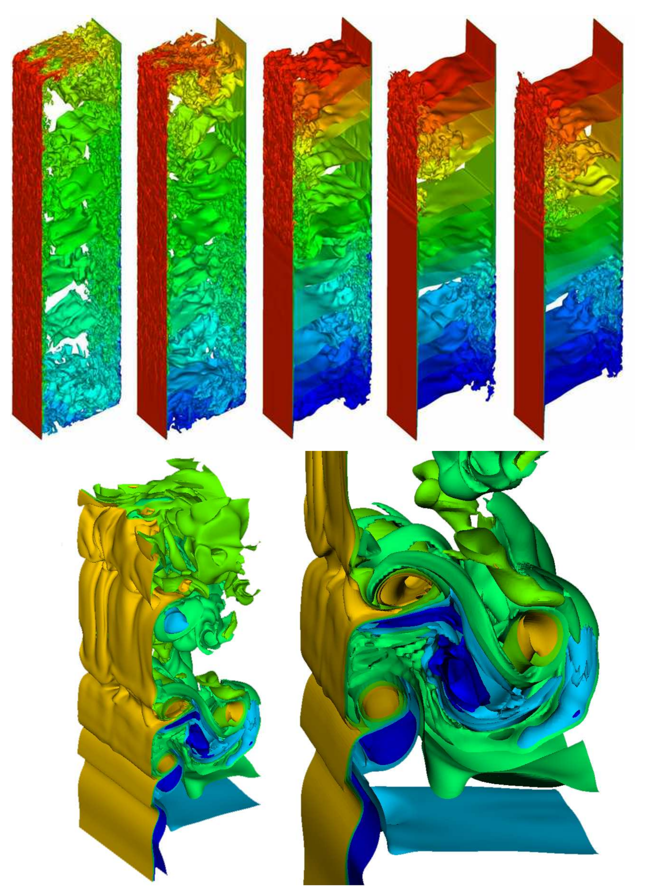
$\Longrightarrow$ Following the same steps as Foias *et al.* (2001)

- $< \widetilde{T}_k >$ is a nonnegative, monotone decreasing function.
- $< \widetilde{T}_k >$ is approximately constant for $k_a < k < k_b$ (existence of inertial range).



$\Longrightarrow$ $-5/3$ **scaling !!!**

DNS results at $Ra = 10^{11}$ and $Pr = 0.71$



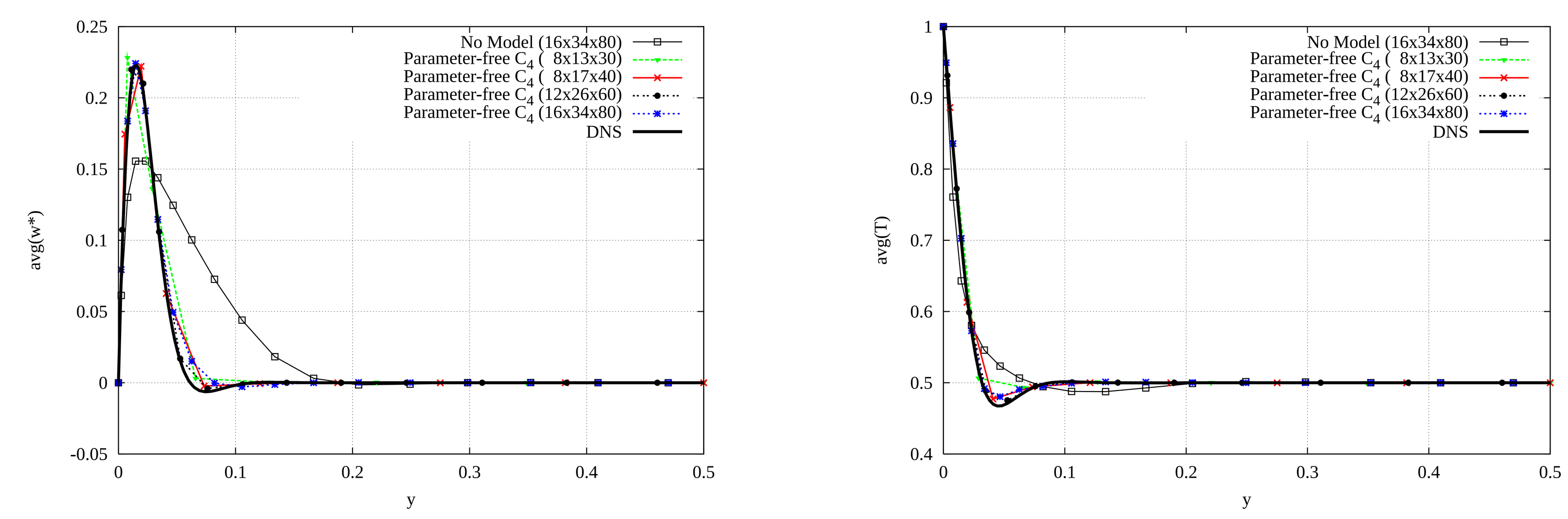
Some details about **DNS simulations**:

- Mesh size: $128 \times 682 \times 1278$
- Computing Time: ≈ 3 months - 256 CPUs
- 4^{th}-order symmetry-preserving discretization
- $A_z = 4$

Complexity of the flow:

- Boundary layers
- Stratified cavity core
- Internal waves
- Recirculation areas

$\mathcal{C}_4$ results for a DHC at $Ra = 10^{10}$



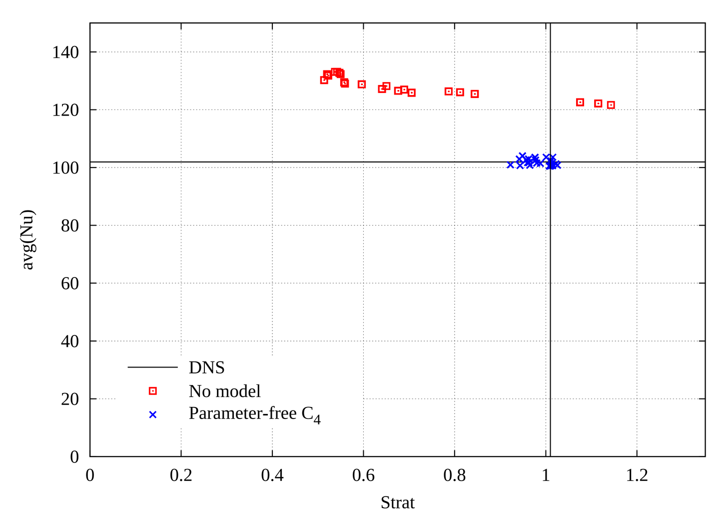
Averaged vertical velocity and temperature profiles at the horizontal mid-height plane at $Ra = 10^{10}$.

Even for a **very coarse** $8 \times 13 \times 30$ grid **reasonable results** are being obtained!

$\Longrightarrow$ Results for different grids show the **robustness** of the method.

Challenging $\mathcal{C}_4$-regularization method

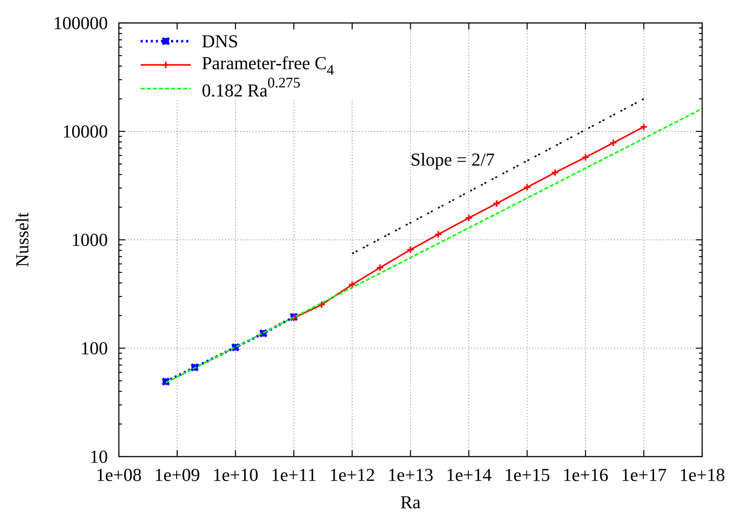
Mesh independence analysis



Nusselt number and centreline stratification for 50 randomly generated coarse grids at $Ra = 10^{10}$.

$\mathcal{C}_4$-method predicts good results irrespective of the meshing!

Performance at very high *Ra*



Meshes have been generated with the criterion of keeping the same number of points in the boundary layer.

Good agreement with a $2/7$ power-law scaling of Nusselt!

Further reading

- F.X. Trias *et al.*, *"Direct numerical simulation of a differentially heated cavity of aspect ratio 4 with Ra-number up to 10^{11} - part I: numerical methods and time-averaged flow"* Int J Heat Mass Trans 2010; 53:665-673.
- F.X. Trias *et al.*, *"Direct numerical simulation of a differentially heated cavity of aspect ratio 4 with Ra-number up to 10^{11} - part II: heat transfer and flow dynamics"* Int J Heat Mass Trans 2010; 53:674-683.
- Roel Verstappen, *"On restraining the production of small scales of motion in a turbulent channel flow"*, Computers & Fluids, 37 (7): 887-897, 2008
- F.X. Trias *et al.*, *"Parameter-free symmetry-preserving regularization modeling of a turbulent differentially heated cavity"*, Computers & Fluids, (published online), 2010.
- F.X. Trias and R.W.C.P. Verstappen, *"On the construction of discrete filters for symmetry-preserving regularization models"*, Computers & Fluids, (under revision).