TOWARDS THE REGULARIZATION MODELING OF WIND FARM BOUNDARY LAYERS

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Abstract

The incompressible Navier-Stokes (NS) equations constitute an excellent mathematical modelization of turbulence. Unfortunately, attempts at performing direct simulations are limited to relatively low-Reynolds numbers because of the almost numberless small scales produced by the non-linear convective term. Alternatively, a dynamically less complex formulation is considered here. Namely, regularizations of the NS equations that preserve the symmetry and conservation properties exactly. To do so, the convective is altered in such a way that the production of small scales is effectively restrained. In practice, the only additional ingredient is a self-adjoint linear filter whose local filter length is determined from the requirement that vortex-stretching must stop at the smallest grid scale. Altogether, the proposed method constitutes a parameter-free turbulence model that has already been successfully tested for a variety of natural and forced convection configurations. Large-scale numerical simulation of the atmospheric boundary layer through wind farms constitutes a new challenging application for regularization modeling.

$\mathcal{C}_4$-regularization modeling of turbulence

The incompressible NS equations form an excellent mathematical model for turbulent flows. In primitive variables they read

$$\partial_t \boldsymbol{u} + \mathcal{C}(\boldsymbol{u},\boldsymbol{u}) = \nu\Delta\boldsymbol{u} - \nabla p + f, \qquad \nabla\cdot\boldsymbol{u} = 0,$$

where ν is the kinematic viscosity and the non-linear convective term is defined by $\mathcal{C}(u,v) = (u\cdot\nabla)v$

Since the full energy spectrum, *i.e.* DNS, cannot be computed, a dynamically less complex mathematical formulation is sought. Here, we consider the $\mathcal{C}_4$ approximation: the convective term is replaced by the following $\mathcal{O}(\epsilon^4)$-accurate smooth approximation $\mathcal{C}_4(u,v)$ given by

$$\mathcal{C}_4(u,v) = \mathcal{C}(\bar{u},\bar{v}) + \overline{\mathcal{C}(\bar{u},v')} + \overline{\mathcal{C}(u',\bar{v})}.$$

Note that here the prime indicates the residual of the filter, *e.g.* $u' = u - \bar{u}$, which can be explicitly evaluated, and $\overline{(\cdot)}$ represents a normalized self-adjoint linear filter with filter length ϵ. Therefore, the governing equations result to

$$\partial_t \boldsymbol{u} + \mathcal{C}_4(\boldsymbol{u},\boldsymbol{u}) = \nu\Delta\boldsymbol{u} - \nabla p + f, \qquad \nabla\cdot\boldsymbol{u} = 0.$$

Note that the $\mathcal{C}_4$ approximation is also a skew-symmetric operator like the original convective operator. Hence, the same inviscid invariants -kinetic energy, enstrophy in 2D and helicity- are preserved.

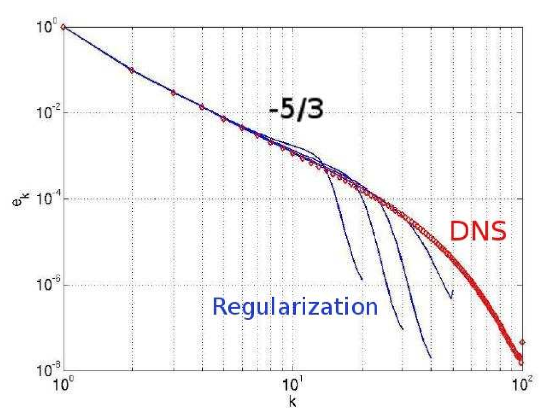
Mathematical foundation

Energy flux equation for the symmetry-preserving regularization resembles the NS

$$\frac{1}{2}\frac{d}{dt}|u_{kk'}|^2 + \nu|\nabla u_{kk'}|^2 = \widetilde{T}_k - \widetilde{T}_{k'} \quad\longrightarrow\quad \nu < |\nabla u_{kk'}|^2 > = < \widetilde{T}_k > - < \widetilde{T}_{k'} >$$

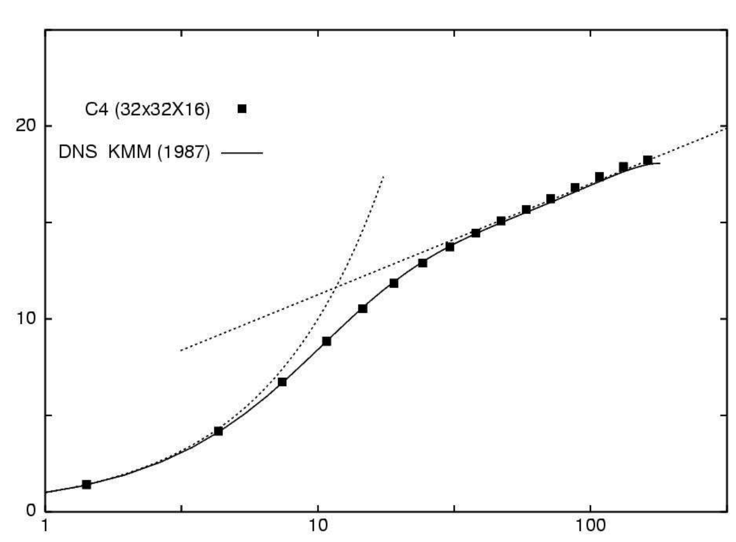
$\Longrightarrow$ Following the same steps as Foias *et al.* (2001)

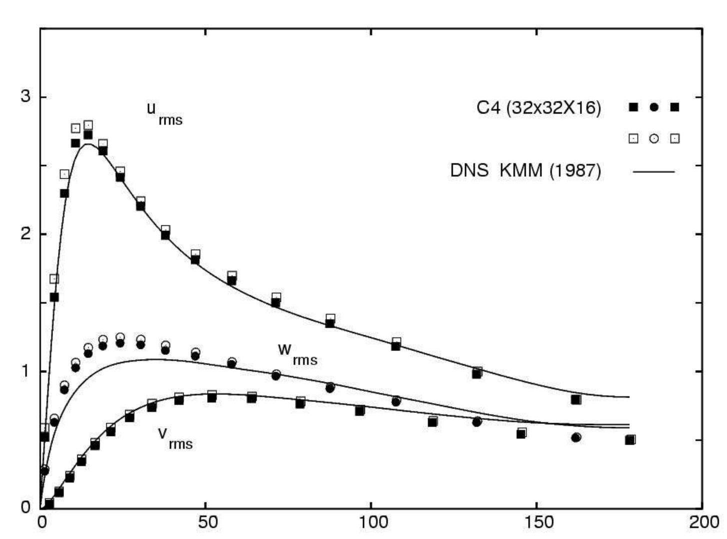
- $< \widetilde{T}_k >$ is a non-negative, monotone decreasing function.
- $< \widetilde{T}_k >$ is approximately constant for $k_a < k < k_b$ (existence of inertial range).



$\Longrightarrow$ $-5/3$ **scaling !!!**

Test-case 1: Turbulent channel flow at $Re_\tau = 180$

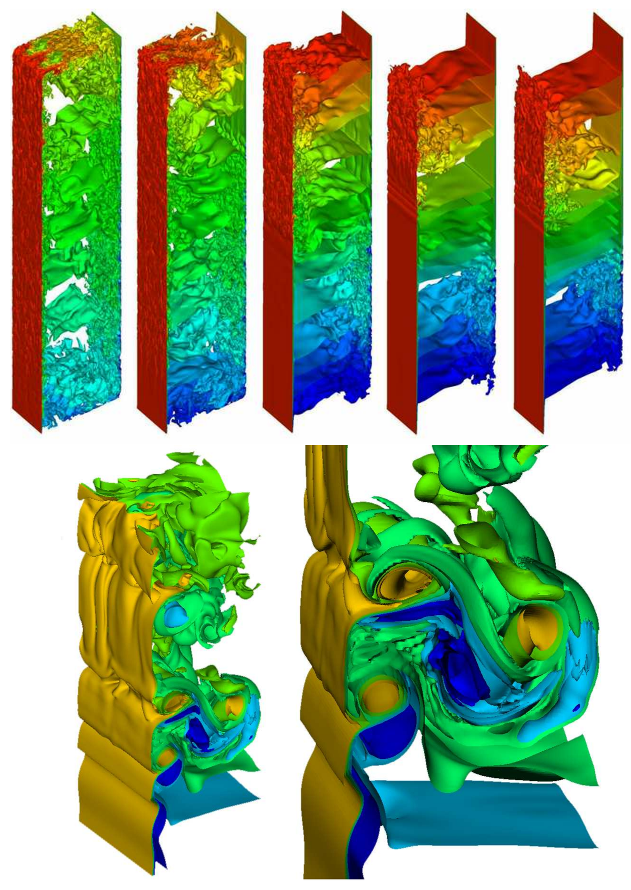


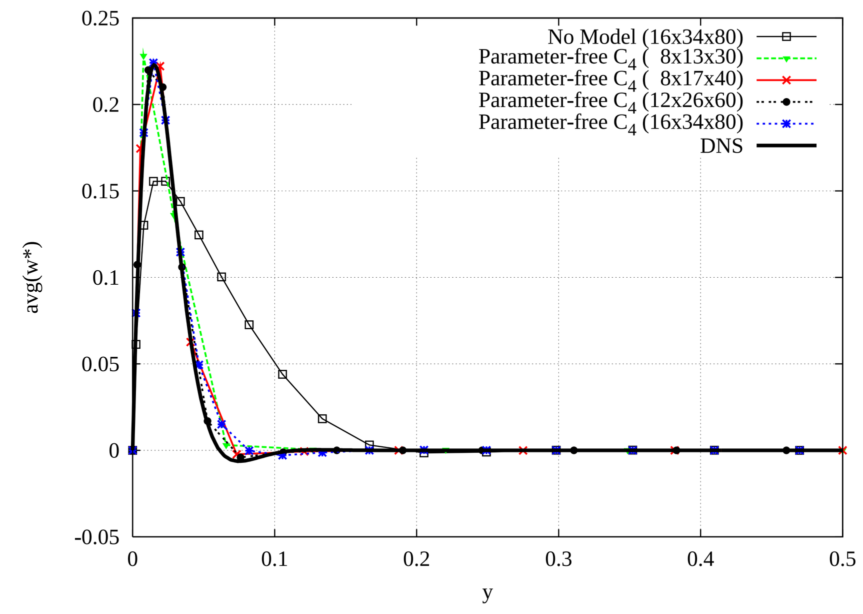


Mean streamwise velocity profile (left) and turbulent statistics (right).

Results are in good agreement with the DNS reference data

Test-case 2: air-filled differentially heated cavity





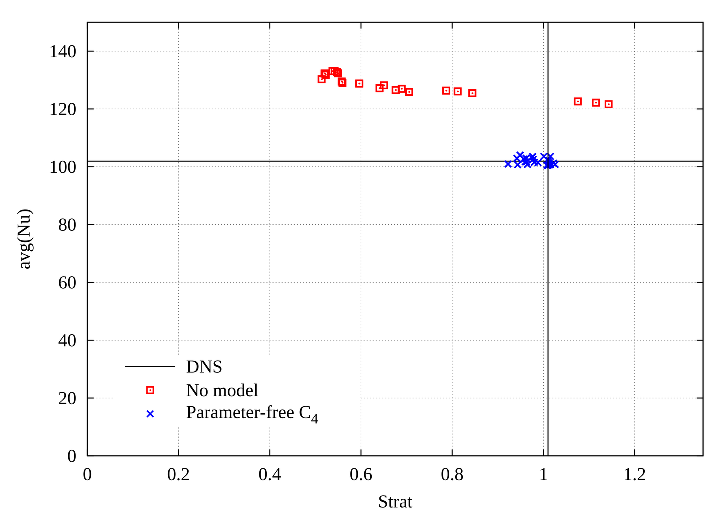
Averaged vertical velocity and temperature profiles at the horizontal mid-height plane at $Ra = 10^{10}$.

Even for a **very coarse** $8\times13\times30$ grid **reasonable results** are being obtained!

$\Longrightarrow$ Results for different grids show the **robustness** of the method.

Challenging $\mathcal{C}_4$-regulatization method

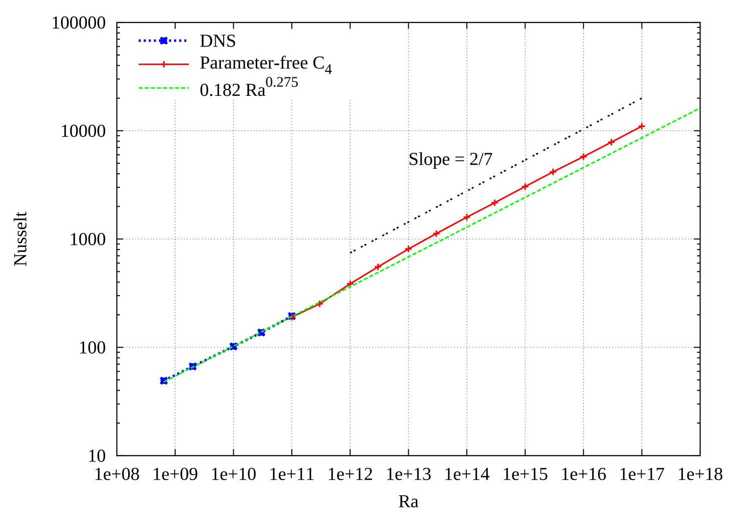
Mesh independence analysis



Nusselt number and centreline stratification for 50 randomly generated coarse grids at $Ra = 10^{10}$.

$\mathcal{C}_4$-method predicts good results irrespective of the meshing!

Performance at very high *Ra*



Meshes have been generated with the criterion of keeping the same number of points in the boundary layer.

Good agreement with a $2/7$ power-law scaling of Nusselt!

Next step: simulation of wind farm boundary layers

The $\mathcal{C}_4$ regularization is a promising new simulation shortcut for the numerical simulation of turbulent flows. The main advantages can be summarized:

- **Robustness.** As the smoothed governing equations preserve the symmetry properties of the original NS equations the solution cannot blow up (in the energy-norm, in 2D also: enstrophy-norm). It seems that even for very coarse meshes reasonably results can be obtained.
- **Universality.** No *ad hoc* phenomenological arguments that cannot be formally derived for the NS equations are used.
- The proposed method constitutes a **parameter-free** turbulence model.

We can conclude that, with the current state of development, $\mathcal{C}_4$ regularization modeling is ready for more challenging applications. The numerical simulation of atmospheric boundary layers through wind farms is an example of thereof. This is an active area of the research group and it is expected that the first numerical studies will be published soon.

Further reading

- Roel Verstappen, *"On restraining the production of small scales of motion in a turbulent channel flow"*, Computers & Fluids, 37 (7): 887-897, 2008
- F.X. Trias, R.W.C.P. Verstappen, A. Gorobets, M. Soria, and A. Oliva, *"Parameter-free symmetry-preserving regularization modeling of a turbulent differentially heated cavity"*, Computers & Fluids, 39: 1815-1831, 2010.
- F.X. Trias and R.W.C.P. Verstappen, *"On the construction of discrete filters for symmetry-preserving regularization models"*, Computers & Fluids, 40: 139-148, 2011.