Assessment of LES techniques for mitigating the Grey Area in DDES models.

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Turbulence models

Industrial Application

Computational Cost

Degree of Modelling

RANS  HYBRID  WMLES  LES  DNS
Pressure Gradient Magnitude, 2D Snapshot.
Pressure Gradient Magnitude, 2D Snapshot.

RANS-SA
Pressure Gradient Magnitude, 2D Snapshot.

DDES-SA
Detached Eddy Simulation (DES, Spalart 1997)

\[
\frac{D}{Dt} (\rho \tilde{\nu}) = \frac{1}{\sigma_{\nu_t}} \left( \nabla \cdot (\rho (\nu + \tilde{\nu}) \nabla \tilde{\nu}) + C_{b2} \rho |\nabla \tilde{\nu}| \right) + \ldots
\]

\[
+ C_{b1} \rho \tilde{S} \tilde{\nu} (1 - f_{t2}) - \left( C_{\omega} f_\omega - \frac{C_{b1}}{\kappa^2} f_{t2} \right) \rho \left( \frac{\tilde{\nu}}{\tilde{d}} \right)^2
\]

Production Term

Destruction Term

\[\tilde{d} \text{ in “Destruction Term”:}\]

\[\tilde{d} = \min (d, C_{DES} \Delta)\]
Detached Eddy Simulation (DES, Spalart 1997)

\[
\frac{D}{Dt} (\rho \tilde{\nu}) = \frac{1}{\sigma_{\nu_t}} \left( \nabla \cdot (\rho (\nu + \tilde{\nu}) \nabla \tilde{\nu}) + C_{b2} \rho |\nabla \tilde{\nu}| \right) + C_{b1} \rho \tilde{\nu} (1 - f_{t2} - \left( C_{\omega 1} f_\omega - \frac{C_{b1}}{\kappa^2} f_{t2} \right) \rho \left( \frac{\tilde{\nu}}{\tilde{d}} \right)^2 \\
+ \text{Production Term} - \text{Destruction Term}
\]

\[
\tilde{d} \text{ in “Destruction Term”:} \\
\tilde{d} = \min (d, C_{DES} \Delta)
\]
Delayed - DES (DDES, Spalart 2006)

\[
\frac{D}{Dt} \left( \rho \tilde{\nu} \right) = \frac{1}{\sigma_{\nu_t}} \left( \nabla \cdot \left( \rho \left( \nu + \tilde{\nu} \right) \nabla \tilde{\nu} \right) + C_{b2} \rho |\nabla \tilde{\nu}| \right) \ldots
\]

\[
+ C_{b1} \rho \tilde{S} \tilde{\nu} (1 - f_{t2}) - \left( C_{\omega1} f_{\omega} - \frac{C_{b1}}{\kappa^2} f_{t2} \right) \rho \left( \frac{\tilde{\nu}}{\tilde{d}} \right)^2
\]

\tilde{\nu} \text{ in "Production Term"}

\tilde{d} \text{ in "Destruction Term":}

\[
\tilde{d} = d - f_d \max \left( 0, d - \Psi C_{DES} \Delta \right),
\]

\tilde{d} \text{ in "Destruction Term": where } f_d \text{ is the "shielding function"}

\[
f_d = 1 - \tanh \left( [8r_d]^3 \right), \quad r_d = \min \left( \frac{\nu_t + \nu}{|\nabla u| \kappa^2 d^2}, 10 \right)
\]
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\frac{D}{Dt} (\rho \tilde{\nu}) = \frac{1}{\sigma_{\nu_t}} (\nabla \cdot (\rho (\nu + \tilde{\nu}) \nabla \tilde{\nu}) + C_{b2} \rho |\nabla \tilde{\nu}|) \quad \ldots \\
+ \left( C_{b1} \rho \tilde{S} \tilde{\nu} (1 - f_{t2}) - \left( C_{\omega1} f_{\omega} - \frac{C_{b1}}{\kappa^2} f_{t2} \right) \rho \left( \frac{\tilde{\nu}}{\tilde{d}} \right)^2 \right)
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\[
\frac{D}{Dt} (\rho \tilde{\nu}) = \frac{1}{\sigma_{\nu t}} (\nabla \cdot (\rho (\nu + \tilde{\nu}) \nabla \tilde{\nu}) + C_{b2} \rho |\nabla \tilde{\nu}|) \ldots
\]

\[
+ C_{b1} \rho \tilde{\Sigma} \tilde{\nu} (1 - f_{t2}) - \left( C_{\omega 1} f_{\omega} - \frac{C_{b1}}{\kappa^2} f_{t2} \right) \rho \left( \frac{\tilde{\nu}}{\tilde{d}} \right)^2
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\[f_d = 1 - \tanh \left( [8r_d]^3 \right), \quad r_d = \min \left( \frac{\nu_t + \nu}{|\nabla u| \kappa^2 d^2}, 10 \right)\]
DDES $\Rightarrow$ LES, Production and Destruction balance

$$
\frac{D}{Dt} (\rho \tilde{v}) = \frac{1}{\sigma_{\nu_t}} (\nabla \cdot (\rho (\nu + \tilde{v}) \nabla \tilde{v})) + C_{b2} \rho |\nabla \tilde{v}|) \\
+ C_{b1} \rho \tilde{S} \tilde{v} (1 - f_{t2}^0) - \left( C_{\omega_1} f_\omega - \frac{C_{b1}}{\kappa^2} f_{t2}^0 \right) \rho \left( \frac{\tilde{v}}{\tilde{d}} \right)^2
$$

Production Term

Destruction Term

balance between production and destruction term,

$$
C_{b1} \rho \tilde{S} \tilde{v} = C_{\omega_1} f_\omega \rho \left( \frac{\tilde{v}}{\tilde{d}} \right)^2
$$

where,

$$
\nu_t = \frac{C_{b1} f_{v1}}{C_{\omega_1} f_\omega} \tilde{d}^2 \tilde{S} \sim \frac{A_{SA}}{C_{\omega_1} f_\omega D_{sgs} (\tilde{u})} \psi^2 \left( C_{DES} \Delta \right)^2 D_{sgs} (\tilde{u})
$$
DDES $\Rightarrow$ LES, Production and Destruction balance

$$\frac{D}{Dt} (\rho \tilde{\nu}) = \frac{1}{\sigma_{\nu_t}} \left( \nabla \cdot (\rho (\nu + \tilde{\nu}) \nabla \tilde{\nu}) + C_{b2} \rho |\nabla \tilde{\nu}| \right) + \ldots$$

$$+ C_{b1} \rho \tilde{S} \tilde{\nu} \left( 1 - f_{t2} \right) - \left( C_{\omega_1} f_{\omega} - \frac{C_{b1}}{\kappa^2} f_{t2} \right) \rho \left( \frac{\tilde{\nu}}{d} \right)^2$$

Balance between production and destruction term,

$$C_{b1} \rho \tilde{S} \tilde{\nu} = C_{\omega_1} f_{\omega} \rho \left( \frac{\tilde{\nu}}{d} \right)^2$$

where,

$$\nu_t = \frac{C_{b1} f_{v1}}{C_{\omega_1} f_{\omega}} \tilde{d}^2 \tilde{S}$$

$$\tilde{S} \sim \frac{C_{b1} f_{v1} \tilde{S}}{C_{\omega_1} f_{\omega} D_{sgs}(\tilde{u})} \psi^2 (C_{DES} \Delta)^2 D_{sgs}(\tilde{u})$$
DDES ⇒ LES, \textbf{Production and Destruction balance}

\[ \frac{D}{Dt} \left( \rho \tilde{v} \right) = \frac{1}{\sigma_{\nu_t}} \left( \nabla \cdot (\rho (\nu + \tilde{v}) \nabla \tilde{v}) + C_{b2} \rho |\nabla \tilde{v}| \right) + \ldots \]

\[ + C_{b1} \rho \tilde{S} \tilde{v} \left( 1 - f_{t2}^0 \right) - \left( C_{\omega 1} f_{\omega} - \frac{C_{b1}}{\kappa^2} f_{t2}^0 \right) \rho \left( \frac{\tilde{v}}{\tilde{d}} \right)^2 \]

balance between production and destruction term,

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where,

\[ \nu_t = \frac{C_{b1} f_{v1}}{C_{\omega 1} f_{\omega}} \tilde{d}^2 \tilde{S} \]

\[ \sim \frac{A_{\text{SA}}}{\frac{C_{b1} f_{v1} \tilde{S}}{C_{\omega 1} f_{\omega} D_{\text{sgs}} (\bar{u})} \Psi^2 (C_{\text{DES} \Delta})^2 D_{\text{sgs}} (\bar{u})} \]

...LESregion...
RANS to LES transition delay (Grey Area)

Where? Free Shear Layer
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2 Grey Mitigation Strategies (GAM)

3 Results & Discussions

4 Conclusions
Grey Area Mitigation Strategies (GAM)

- **Inserting Artificial Oscillations**

- **Decreasing** $\nu_{sgs}$

\[
\nu_{sgs} = (C_m \Delta)^2 D_{sgs} (\bar{u}) \sim A_{SA} (C_{DES} \Delta)^2 D_{sgs} (\bar{u})
\]

Decreasing...

- **Subgrid Length Scale** ($\Delta$) $\Rightarrow$ $\Delta_{max}$, $\tilde{\Delta}_\omega$, $\Delta_{SLA}$ ... $\Delta_{lsq}$

- **Differential Operator** ($D_{sgs} (\bar{u})$) $\Rightarrow \sigma$ – LES, S3PQR ...
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- Subgrid Length Scale ($\Delta$) $\Rightarrow \Delta_{max}, \tilde{\Delta}_\omega, \Delta_{SLA} \ldots \Delta_{lsq}$

- Differential Operator ($D_{sgs} (\bar{u})$) $\Rightarrow \sigma - LES, S3PQR? \ldots$
Subgrid Length Scale (I): State of the art

- In the context of **LES**, most popular (by far) is:
  \[ \Delta_\forall = (\Delta x \Delta y \Delta z)^{1/3} \quad \leftarrow \text{Deardorff (1970)} \]

- In the context of **DES**:
  \[ \Delta_{\text{max}} = \max(\Delta x, \Delta y, \Delta z) \quad \leftarrow \text{Sparlart et al. (1997)} \]

Recent flow-dependant definitions:

\[ \Delta_\omega = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\omega|^2} \quad \leftarrow \text{Chauvet et al. (2007)} \]

\[ \tilde{\Delta}_\omega = \frac{1}{\sqrt{3}} \max_{n,m=1,...,8} |l_n - l_m| \quad \leftarrow \text{Mockett et al. (2015)} \]

\[ \Delta_{\text{SLA}} = \tilde{\Delta}_\omega F_{KH}(\langle VTM \rangle) \quad \leftarrow \text{Shur et al. (2015)} \]
In the context of **LES**, most popular (by far) is:

$$\Delta_V = (\Delta x \Delta y \Delta z)^{1/3} \quad \leftarrow \text{Deardorff (1970)}$$

In the context of **DES**:

$$\Delta_{\text{max}} = \max(\Delta x, \Delta y, \Delta z) \quad \leftarrow \text{Sparlart et al. (1997)}$$

Recent flow-dependant definitions

$$\Delta_\omega = \sqrt{(\omega_x^2 \Delta y \Delta z + \omega_y^2 \Delta x \Delta z + \omega_z^2 \Delta x \Delta y)/|\omega|^2} \quad \leftarrow \text{Chauvet et al. (2007)}$$

$$\tilde{\Delta}_\omega = \frac{1}{\sqrt{3}} \max_{n,m=1,\ldots,8} |I_n - I_m| \quad \leftarrow \text{Mockett et al. (2015)}$$

$$\Delta_{\text{SLA}} = \tilde{\Delta}_\omega F_{\text{KH}}(\langle VTM \rangle) \quad \leftarrow \text{Shur et al. (2015)}$$
Can we find a **simple and robust** definition of $\Delta$ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?
Can we find a simple and robust definition of $\Delta$ that minimizes the effect of mesh anisotropies on the performance of subgrid-scale models?

\[
\tau(\bar{u}) = \frac{\Delta^2}{12} GG^T + O(\Delta^4)
\]

physical space

\[
\tau(\bar{u}) = \frac{1}{12} G_\Delta G_\Delta^T + O(\Delta^4)
\]

computational space
Can we find a **simple and robust** definition of $\Delta$ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?

$$\Delta_{lsq} = \sqrt{\frac{G_\Delta G_\Delta^T}{GG^T : GG^T}}$$
Subgrid Length Scale (II): New approach

Can we find a **simple and robust** definition of $\Delta$ that minimizes the effect of **mesh anisotropies** on the performance of subgrid-scale models?

$$\Delta_{lsq} = \sqrt{\frac{JG^T G : JG^T G}{G^T G : G^T G}}, \quad J = \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \end{pmatrix}$$
Subgrid Length Scale (III): 2D Simple Flow

\[ \Delta_{lsq}(\omega, \beta) = \sqrt{\frac{\Delta x_1^2 (1-2\omega)^4 + \Delta x_2^{-2}}{(1-2\omega)^4 + 1}} = \sqrt{\frac{\beta^2 (1-2\omega)^4 + \beta^{-2}}{(1-2\omega)^4 + 1}} \]

Non-symmetric Kinematics:

\[ \Delta_{lsq}(\omega = 0.5, \beta) = \Delta x_2 = \beta^{-1} \]

\[ \Delta_{lsq}(\omega = 0.5, \beta = 5) = 1/5 \]

\[ \Delta_{lsq}(\omega = 0.5, \beta = 1/5) = 5 \]
Subgrid Length Scale (III): 2D Simple Flow

\[
\begin{align*}
\Delta_{\text{vol}} & \\
\Delta_\omega & \text{ with } \beta = 5, 1/5 \\
\Delta_{\text{lsq}} & \text{ with } \beta = 1/5 \\
\Delta_{\text{lsq}} & \text{ with } \beta = 5
\end{align*}
\]
Differential Operators \( (D_{sgs}(\bar{u})) \)

\[ \Delta_{SLA} = F_{KH}(\langle VTM \rangle) \text{ turns off} \text{ in 2D flow regions, but } \ldots \text{ is it an appropriate } \Delta \text{ feature?} \]

\[ \nu_{sgs} = (C_m \Delta_{SLA})^2 D_{sgs}(\bar{u}) \]
\[ = (C_m \tilde{\Delta}_\omega)^2 (F_{KH}(\langle VTM \rangle)^2 D_{sgs}(\bar{u})) \]
\[ = (C_m \tilde{\Delta}_\omega)^2 D_{sgs}^{2D}(\bar{u}). \]

The importance of using 2D sensitive \( D_{sgs}(\bar{u}) \) is \underline{remarked!},

such as

\[ \sigma - LES, S3PQR \ldots \]
Differential Operators \( (D_{\text{sgs}}(\bar{u})) \)

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Differential Operators \( (D_{\text{sgs}}(\bar{u})) \)

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$$= \left( C_m \tilde{\Delta}_\omega \right)^2 (F_{KH}(\langle VTM \rangle)^2 D_{sgs}(\bar{u}))$$

$$= \left( C_m \tilde{\Delta}_\omega \right)^2 D_{sgs}^{2D}(\bar{u}).$$

The importance of using 2D sensitive $D_{sgs}(\bar{u})$ is remarked!,

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Applicability

**Square cylinder**

**Airfoil FX 77-W2-500**

**Wall-mounted Hump**

**BFS (present DNS)**
Applicability

Square cylinder

Wall-mounted Hump

Airfoil FX 77-W2-500

BFS (present DNS)
DNS of a Backward-Facing Step (BFS)

Geometry

- $L_u = 6h$, $L_d = 32h$
- $ER = 2$, $H = 2h$

BoCos

- **Inflow**: Channel Flow at $Re_T = 395$ (Previous Simulation)
- **Outflow**: Convective
- **Span-wise**: Periodic
DNS of a Backward-Facing Step (BFS)

$L_u = 6h \quad L_d = 32h$

$H - h$

INFLOW

$x_2$

$x_1$

$x_3$

$H$

References

- Published in: Journal of Fluid Mechanics.
- Link: http://www.cttc.upc.edu/downloads/BFS_Ret395_ER2
- Computational Resources: Marenostrum4.
Studied Cases

### DNS

- **Code:**
  - STG\(^1\) (In-house code)
- **Mesh:**
  - Structured Non-Uniform
  - \(\sim 165M\) grid points
  - \(L_x = 2\pi h\)
- **Spatial Integration:**
  - 4\(^{th}\)-order Symm-Pres
- **Time Integration:**
  - Explicit Adams-Bashforth
  - \(\sim 1.57e - 4 \Delta th / u_T\)
- **Inflow:**
  - Unsteady

---

### DDES-SA

- **Code:**
  - OpenFOAM
- **Mesh:**
  - Structured Body-Fitted
  - \(\sim 0.7M\) grid points
  - \(L_x = 2h\)
- **Spatial Integration:**
  - Hybrid Scheme (UDS-CDS)
- **Time Integration:**
  - Implicit 2\(^{nd}\)-order
  - \(\sim 5.76e - 4 \Delta th / u_T\)
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**Studied Cases**

### DNS

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High order statistics, $u_{rms}$
High order statistics, $u_{rms}$
High order statistics, $u_{rms}$
Shear Layer Growth: Scheme.
Shear Layer Growth, $\Delta \delta_1$. 

![Graph showing shear layer growth](image)
Shear Layer Growth, $\Delta \delta_2$. 

\[
\Delta \delta_2 = \Delta U_1 / (\partial \langle u_1 \rangle / \partial x_2)_{\text{max}}
\]

$\Delta_{\text{SLA-SMG}}, \sim 2.2e-01$

$\Delta_{\text{lsq-SMG}}, \sim 2.5e-01$

$\Delta_{\tilde{\omega}-\text{SMG}}, \sim 1.6e-01$

DNS, $\sim 2.5e-01$

DNS, Fitting Line

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Differential Operators \( \left( D_{sgs}^{2D} (\bar{u}) \right) \)
Comparison of high order statistics, $u_{rms}$. $\Delta$ vs $D_{sgs}^{2D}(\bar{u})$.

$\Delta$, Smagorinsky Model:

![Graph showing $\Delta$, Smagorinsky Model](image)

$\Delta$, $D_{sgs}^{2D}(\bar{u})$:

![Graph showing $\Delta$, $D_{sgs}^{2D}(\bar{u})$](image)
Shear Layer Growth: Scheme.
Shear Layer Growth, $\Delta \delta_2$.

$$\Delta \delta_2 = \Delta U_1 / (\partial \langle u_1 \rangle / \partial x_2)_{\text{max}}$$

![Graph showing the relationship between $\Delta \delta_2$ and $x_1/h$.]
Conclusions

Take away Message

- $\Delta_{lsq}$ has proved to be a promising candidate for DES applications as a GAM technique.

- The use of differential operators sensitive to 2D flows, $D_{sgs}^{2D}(\bar{u})$, is recommended. It improves both results and the mesh resilience capabilities.

Further work

- $\Delta_{lsq}$ has to be tested with more challenging flow configurations and unstructured meshes.

- Computational performance analysis is desirable.
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Thank you for your attention.